Signatures for tetraquark mixing from partial decay widths of the two light-meson nonets [*]

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Abstract. In this talk, we present successful aspects of the tetraquark mixing model for the two light-meson nonets in the $J^{PC} = 0^{++}$ channel, the light nonet [$a_0(980)$, $K_0^*(700)$, $f_0(500)$, $f_0(980)$] and the heavy nonet [$a_0(1450)$, $K_0^*(1430)$, $f_0(1370)$, $f_0(1500)$]. In particular, we focus on how their experimental partial decay widths extracted from Particle Data Group (PDG) can support this mixing model. Currently, the experimental data exhibit an unnatural tendency that partial widths of the light nonet are consistently larger than those of the heavy nonet. This unnatural tendency can be explained if the coupling into two pseudoscalar mesons is enhanced in the light nonet and suppressed in the heavy nonet as predicted by the tetraquark mixing model. Therefore, this could be strong evidence to support for the tetraquark mixing model.

In PDG [2], there are two nonets in the $J^{PC} = 0^{++}$ channel: the light nonet [$a_0(980)$, $K_0^*(700)$, $f_0(500)$, $f_0(980)$] and the heavy nonet [$a_0(1450)$, $K_0^*(1430)$, $f_0(1370)$, $f_0(1500)$]. About six years ago, we proposed a tetraquark mixing model that treats the two nonets as tetraquarks generated by mixing the two tetraquark types, $|000\rangle$ and $|011\rangle$ [3–7]. The first type, $|000\rangle$, represents the spin-0 tetraquarks formed by combining the spin-0 diquark of the structure $(\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_f)$ and its antidiquark. The second type, $|011\rangle$, represents the spin-0 tetraquarks constructed by the spin-1 diquark of the structure $(\mathbf{6}_c, \bar{\mathbf{3}}_f)$ and its antidiquark. Their mixtures that diagonalize the color-spin interaction, V_{CS}, have been identified as two physical nonets,

$$|\text{Heavy nonet}\rangle = -\alpha|000\rangle + \beta|011\rangle\,, \tag{1}$$

$$|\text{Light nonet}\rangle = \beta|000\rangle + \alpha|011\rangle\,. \tag{2}$$

The mixing parameters $\alpha \approx \sqrt{2/3}, \beta \approx 1/\sqrt{3}$ are also fixed by the diagonalization process.

This model has been tested in several occasions [3–7] and appears to have successful aspects, such as qualitatively explaining the mass of the two nonets and the difference in mass between the two nonets. Specifically, the mixing creates a large hyperfine mass for the light nonet, of the order of −500 MeV, which can explain why the light nonet has masses smaller than 1 GeV even though its members are composed of 4 constituent quarks. The mixing makes a hyperfine mass for the heavy nonet to around ~ -20 MeV, which can explain qualitatively why the heavy nonet has masses not far from $4m_q$, four times of the constituent quark mass. More interestingly, the mass difference between the two nonets, which is around 500 MeV or more, can be explained by the the hyperfine mass difference, $\Delta M \approx \Delta\langle V_{CS}\rangle$. It all comes from the fact that the two types of tetraquark mix together to create the two nonets.

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Table 1. Decay modes and the coupling strengths of the light nonet (heavy nonet), G (G'), calculated from the tetraquark mixing model.

Light nonet		Heavy nonet	
Decay mode	G	Decay mode	G'
$a_0(980) \to \pi\eta$	0.6076	$a_0(1450) \to \pi\eta$	0.1406
$a_0(980) \to K\bar{K}$	0.7441	$a_0(1450) \to K\bar{K}$	0.1722
$K_0^*(700) \to \pi K$	0.5253	$K_0^*(1430) \to \pi K$	0.1251
$f_0(500) \to \pi\pi$	−0.3310	$f_0(1370) \to \pi\pi$	−0.0785
$f_0(980) \to \pi\pi$	−0.1690	$f_0(1500) \to \pi\pi$	−0.0394
$f_0(980) \to K\bar{K}$	−0.4685	$f_0(1500) \to K\bar{K}$	−0.1093

The most striking prediction of the tetraquark mixing model is that the coupling strengths of the two nonets decaying into two pseudoscalar mesons must satisfy the following inequality [1, 4]

$$|G| \text{ (light nonet)} \gg |G'| \text{ (heavy nonet)} . \tag{3}$$

It says that the coupling strength (G) of the light nonet is much larger than that (G') of the heavy nonet. To illustrate this prediction, we notice that tetraquarks ($q_1q_2\bar{q}_3\bar{q}_4$) either in $|000\rangle$ or in $|011\rangle$ are composed of diquark (q_1q_2) and antidiquark ($\bar{q}_3\bar{q}_4$) with definite color, spin, and flavor states. If the tetraquark is recombined into quark-antiquark pairs, $q_1\bar{q}_3$, $q_2\bar{q}_4$, its wave function in color space has a component consisting of two color-singlets and additional component of two color-octets. This can be schematically written as,

$$[q_1q_2\bar{q}_3\bar{q}_4]_{\mathbf{1}_c} \sim (q_1\bar{q}_3)_{\mathbf{1}_c} \otimes (q_2\bar{q}_4)_{\mathbf{1}_c} + [(q_1\bar{q}_3)_{\mathbf{8}_c} \otimes (q_2\bar{q}_4)_{\mathbf{8}_c}]_{\mathbf{1}_c} . \tag{4}$$

Tetraquarks can decay into two pseudoscalar mesons through the first component containing two color-singlets. Of course, to make two-meson modes, the tetraquarks has to be recombined in spin and flavor space also. The coupling strengths of our concern, which are basically the coefficients of the two-meson modes, can be calculated from the recombination factors from color, spin and flavor. The tetraquark type, $|000\rangle$, can decay into two mesons through this component and the other type, $|011\rangle$, can decay into the same mesons through this component also. However, due to the opposite signs in the heavy nonet, Eq. (1), two-meson modes from $|000\rangle$ and $|011\rangle$ partially cancel out to suppress the coupling strengths, $|G'|$ (heavy nonet). On the other hand, the two-meson modes have the same sign in the light nonet, Eq. (2), and they add up each other to enhance the coupling strengths, $|G|$ (light nonet). This leads to the inequality like Eq. (3). A more detailed explanation can be found also in Refs. [1, 4].

Table 1 shows the coupling strengths obtained by recombining the wave functions of Eqs. (1),(2) in terms of quark-antiquark pairs. As advertised in Eq. (3), we clearly see that the tetraquark mixing model predicts that $|G|$ is much larger than $|G'|$. More interestingly, this prediction can be experimentally verified by examining the partial decay width because the partial decay width is given as

$$\Gamma_{partial} = (\text{coupling strength})^2 \Gamma_{kin} . \tag{5}$$

Here, the kinematical decay width, Γ_{kin}, which depends only on kinematical factors in decay processes, should satisfy

$$\Gamma_{kin}(\text{light nonet}) \ll \Gamma_{kin}(\text{heavy nonet}) , \tag{6}$$

Table 2. Partial decay modes and their widths that are extracted from PDG 2022 [2].

Light nonet		Heavy nonet	
Decay mode	Γ_{exp}(MeV)	Decay mode	Γ_{exp}(MeV)
$a_0(980) \to \pi\eta$	60	$a_0(1450) \to \pi\eta$	15.4–20.5
$a_0(980) \to K\bar{K}$	10.6	$a_0(1450) \to K\bar{K}$	13.5–18.0
$K_0^*(700) \to \pi K$	468	$K_0^*(1430) \to \pi K$	251.1
$f_0(500) \to \pi\pi$	Not conclusive	$f_0(1370) \to \pi\pi$	Not conclusive
$f_0(980) \to \pi\pi$	50	$f_0(1500) \to \pi\pi$	38.1
$f_0(980) \to K\bar{K}$	9.5–46.2	$f_0(1500) \to K\bar{K}$	9.5

because the heavy nonet has much more phase space for its decay than the light nonet. By equating Eq. (5) with the experimental partial decay widths, one can extract some information of the coupling strengths, G, G', of the two nonets decaying into two pseudoscalar mesons.

Table 2 shows the experimental partial decay widths extracted from PDG 2022 [2]. Except for $a_0(980)$, $a_0(1450)$ decaying into $K\bar{K}$, there seems to be a general tendency that the partial decay width of the light nonet is larger than that of the heavy nonet,

$$\Gamma_{exp}(\text{light nonet}) \geq \Gamma_{exp}(\text{heavy nonet})\,, \tag{7}$$

For $f_0(500)$, $f_0(1370)$, their partial widths for the modes, $f_0(500) \to \pi\pi$, $f_0(1370) \to \pi\pi$, are not conclusive from the present PDG data. But the tendency like Eq. (7) is still expected to hold in these cases as well. The $f_0(500)$ is a resonance famous for its broad width and the $\pi\pi$ mode probably represents the full width of $f_0(500)$. But for the $f_0(1370)$, the $\pi\pi$ mode is just one mode among various modes and the total width of $f_0(1370)$ is shared by all the decay modes. Even though we do not have specific numbers for their widths, it is quite likely that their partial widths also follow the general trend of Eq. (7).

The general tendency represented by Eq. (7) is not natural because, kinematically, heavy resonances are expected to have larger partial widths as in Eq. (6). What is interesting is that this unnatural trend of Eq. (7) can be explained if we have $|G| \gg |G'|$ as predicted by the tetraquark mixing framework. Since partial decay widths are given by Eq. (5), the tendency of experimental partial widths, Eq. (7), between the two nonets must be reproduced if we multiply (coupling strength)^2 on both sides of Eq. (6). That is, multiplying G^2 on the left-hand side and G'^2 on the right-hand side of Eq. (6), we should have

$$G^2\Gamma_{kin}(\text{light nonet}) \geq G'^2\Gamma_{kin}(\text{heavy nonet})\,, \tag{8}$$

with the opposite inequality from Eq. (6) in order to reproduce the tendency in the experimental partial width, Eq. (7). Only way to get this opposite inequality from Eq. (6) is to have $|G| \gg |G'|$ as predicted by the tetraquark mixing framework.

One exceptional case is the isovector resonances decaying into $K\bar{K}$. In this case, $\Gamma_{exp}[a_0(980) \to K\bar{K}] < \Gamma_{exp}[a_0(1450) \to K\bar{K}]$ so this does not follow the general trend of Eq. (7). But, $\Gamma_{exp}[a_0(980) \to K\bar{K}]$ is smaller by the kinematical cutoff. The central mass of $a_0(980)$, ~ 980 MeV, is smaller than the $K\bar{K}$ threshold, ~ 990 MeV so the $a_0(980)$ can decay into $K\bar{K}$ only through high tail of the mass distribution broaden by the total width. Even if this decay mode is amplified by the coupling strength, more than half of the partial width is blocked by the kinematical cutoff. This is in contrast to the heavy nonet case, $a_0(1450) \to K\bar{K}$, where all the mass region of $a_0(1450)$ broadened by the total width can decay to $K\bar{K}$ without suffering from the kinematical cutoff. In fact, it can be demonstrated quantitatively that the partial widths explicitly calculated including the mass distribution as well as the coupling strengths given by Table 1 agree reasonably well with the experimental partial widths [1, 4]. So this exceptional case does not undermine our conclusion, Eq. (3).

In conclusion, our prediction from the tetraquark mixing model, $|G|(\text{light nonet}) \gg |G'|(\text{heavy nonet})$, is clearly supported by the experimental partial decay widths satisfying $\Gamma_{exp}(\text{light nonet}) \geq \Gamma_{exp}(\text{heavy nonet})$. Kinematically, the inequality in the experimental widths is very unnatural and the tetraquark mixing model is likely the only one that can explain this unnatural trend. The enhanced coupling strength in the light nonet also helps in part to understand why the $K_0^*(700)$ and $f_0(500)$ have very broad decay widths. Since our prediction crucially relies on the fact that two types of tetraquark mix each other, the two nonets in PDG cannot be treated separately when their physical properties are investigated. Our predictions on the couplings hold for all members of the two nonets. This contrasts with non-tetraquark models, where applications are often limited to some members of the two nonets. Non-tetraquark models include meson molecular picture [8], $q\bar{q}$ picture with hadronic intermediate states [9, 10] and so on. In this sense, it is quite unlikely that this prediction on the couplings can be reproduced from those models other than tetraquarks. Therefore, our results strongly support that the two nonets in $J^{PC} = 0^{++}$, the light nonet [$a_0(980)$, $K_0^*(700)$, $f_0(500)$, $f_0(980)$] and the heavy nonet [$a_0(1450)$, $K_0^*(1430)$, $f_0(1370)$, $f_0(1500)$], are tetraquarks generated from the admixture of two tetraquark types, $|000\rangle$ and $|011\rangle$.

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