

A dispersive estimate of the $a_0(980)$ contribution to hadronic light-by-light scattering in $(g - 2)_\mu$

Oleksandra Deineka^{1,*}, Igor Danilkin¹, and Marc Vanderhaeghen¹

¹Institut für Kernphysik & PRISMA⁺ Cluster of Excellence, Johannes Gutenberg Universität, D-55099 Mainz, Germany

Abstract. A dispersive implementation of the $a_0(980)$ resonance to $(g - 2)_\mu$ requires the knowledge of the double-virtual S -wave $\gamma^*\gamma^* \rightarrow \pi\eta/K\bar{K}_{I=1}$ amplitudes. To obtain these amplitudes we used a modified coupled-channel Muskhelishvili–Omnès formalism, with the input from the left-hand cuts and the hadronic Omnès function. The latter were obtained using a data-driven N/D method in which the fits were performed to the different sets of experimental data on two-photon fusion processes with $\pi\eta$ and $K\bar{K}$ final states. This yields the preliminary dispersive estimate $a_\mu^{\text{HLbL}}[a_0(980)]_{\text{resc.}} = -0.46(2) \times 10^{-11}$.

1 Introduction

The tension between the presently ultra-precise measurements of the anomalous magnetic moment of the muon $(g - 2)_\mu$ and the theoretical calculations amounts to around 5.0σ difference [1] when compared to the theoretical value from the 2020 White Paper [2]. The source of the current theoretical error solely arises from contributions from hadronic vacuum polarization (HVP) and hadronic light-by-light scattering (HLbL). Apart from the pseudo-scalar pole contributions, further nontrivial contributions to HLbL arise from the two-particle intermediate states such as $\pi\pi$, $\pi\eta$, and $K\bar{K}$. Currently, only the contributions from the $\pi\pi_{I=0,2}$ and $K\bar{K}_{I=0}$ channels have been considered in a dispersive manner [3, 4]. The isospin-0 part of this result can be understood as a model-independent implementation of the contribution from the $f_0(500)$ and $f_0(980)$ resonances. The contribution from the $a_0(980)$ resonance arises from the rescattering of the $\pi\eta/K\bar{K}_{I=1}$ states and necessitates knowledge of the double-virtual processes $\gamma^*\gamma^* \rightarrow \pi\eta/K\bar{K}_{I=1}$. On the experimental side, currently, data is only available for the real photon case from the Belle Collaboration [5, 6]. The measurement of the photon-fusion processes with a single tagged photon is a part of the two-photon physics program of the BESIII Collaboration [7]. To describe the currently available data and provide theoretical predictions for the single- and double-virtual processes, we opt for the dispersive approach, which adheres to the fundamental properties of the S -matrix, namely, analyticity and coupled-channel unitarity.

*e-mail: deineka@uni-mainz.de

2 Formalism

To compute the HLbL contribution of $a_0(980)$ to $(g-2)_\mu$, we adopt the formalism outlined in [3]. This approach yields the following master formula:

$$a_\mu^{HLbL} = \frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2} Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1, Q_2, Q_3) \bar{\Pi}_i(Q_1, Q_2, Q_3), \quad (1)$$

where $\bar{\Pi}_i$ are scalar functions containing the dynamics of the HLbL amplitude, T_i denote known kernel functions, and τ is defined as $Q_3^2 = Q_1^2 + 2Q_1Q_2\tau + Q_2^2$. For the S -wave, the only contributing scalar functions can be written as

$$\begin{aligned} \bar{\Pi}_3^{J=0} &= \frac{1}{\pi} \int_{s_{th}}^\infty ds' \frac{-2}{\lambda_{12}(s')(s' + Q_3^2)^2} \left(4s' \text{Im} \bar{h}_{++++}^{(0)}(s') - (s' - Q_1^2 + Q_2^2)(s' + Q_1^2 - Q_2^2) \text{Im} \bar{h}_{00,++}^{(0)}(s') \right), \\ \bar{\Pi}_9^{J=0} &= \frac{1}{\pi} \int_{s_{th}}^\infty ds' \frac{4}{\lambda_{12}(s')(s' + Q_3^2)^2} \left(2 \text{Im} \bar{h}_{++++}^{(0)}(s') - (s' + Q_1^2 + Q_2^2) \text{Im} \bar{h}_{00,++}^{(0)}(s') \right), \end{aligned} \quad (2)$$

plus crossed versions. Here $\lambda_{12}(s) \equiv \lambda(s, Q_1^2, Q_2^2)$ is a Källén triangle function.

Since $a_0(980)$ is known to have a dynamical coupled-channel $\pi\eta/K\bar{K}$ origin, the inclusion of $K\bar{K}$ intermediate states is necessary. In this case, the unitarity relation implies

$$\text{Im} \bar{h}_{1,\lambda_1\lambda_2,\lambda_3\lambda_4}^{(0)}(s) = \bar{h}_{1,\lambda_1\lambda_2}^{(0)}(s) \rho_{\pi\eta}(s) \bar{h}_{1,\lambda_3\lambda_4}^{(0)*}(s) + \bar{k}_{1,\lambda_1\lambda_2}^{(0)}(s) \rho_{K\bar{K}}(s) \bar{k}_{1,\lambda_3\lambda_4}^{(0)*}(s), \quad (3)$$

where $\rho_{\pi\eta}(\rho_{K\bar{K}})$ is the phase space factor of $\pi\eta(K\bar{K})$ system, and $\bar{h}_{1,\lambda\lambda'}^{(0)}$ ($\bar{k}_{1,\lambda\lambda'}^{(0)}$) denotes the $I = 1, J = 0$ Born subtracted (e.g. $\bar{k} \equiv k - k^{\text{Born}}$) partial-wave (p.w.) amplitude of the $\gamma^*(Q_1^2)\gamma^*(Q_2^2) \rightarrow \pi\eta(K\bar{K})$ process. These p.w. amplitudes contain kinematic constraints and therefore it is important to find a transformation to a new basis of amplitudes which can be used in a modified Muskhelishvili-Omnès (MO) method [8]. For the S -wave, the amplitudes which are free from kinematic constraints can be written as [3]¹

$$\bar{h}_{i=1,2}^{(0)} = \frac{\bar{h}_{++}^{(0)} \mp Q_1 Q_2 \bar{h}_{00}^{(0)}}{s - s_{\text{kin}}^{(\mp)}}, \quad s_{\text{kin}}^{(\pm)} \equiv -(Q_1 \pm Q_2)^2, \quad (4)$$

with $Q_i \equiv \sqrt{Q_i^2}$. In Eq.(4) we omitted the isospin index for simplicity. In the case of a single virtual or real photons, this constraint arises from the requirement of the soft-photon theorem. Similarly to $\gamma^*\gamma^* \rightarrow \pi\pi/K\bar{K}$ process [9, 10], the coupled-channel dispersion relation for the $\gamma^*\gamma^* \rightarrow \pi\eta/K\bar{K}$ process with $J = 0, I = 1$ can be written as follows

$$\begin{pmatrix} \bar{h}_i^{(0)}(s) \\ \bar{k}_i^{(0)}(s) \end{pmatrix} = \begin{pmatrix} 0 \\ k_i^{(0), \text{Born}}(s) \end{pmatrix} + \Omega^{(0)}(s) \left[- \int_{s_{th}}^\infty \frac{ds'}{\pi} \frac{\text{Disc}(\Omega^{(0)}(s'))^{-1}}{s' - s} \begin{pmatrix} 0 \\ k_i^{(0), \text{Born}}(s') \end{pmatrix} \right], \quad (5)$$

where only kaon-pole left-hand cut is currently taken into account. The generalization of the kaon-pole left-hand contribution $k_i^{(0), \text{Born}}$ to the case involving off-shell photons is achieved by the product of the scalar QED result with the electromagnetic kaon form factors [11]. The latter is parameterized using the VMD model. We have verified that within the $Q^2 \lesssim 1 \text{ GeV}^2$

¹To maintain consistency with Eq.(2) we follow the conventions from [3], which slightly differ from those in [9].

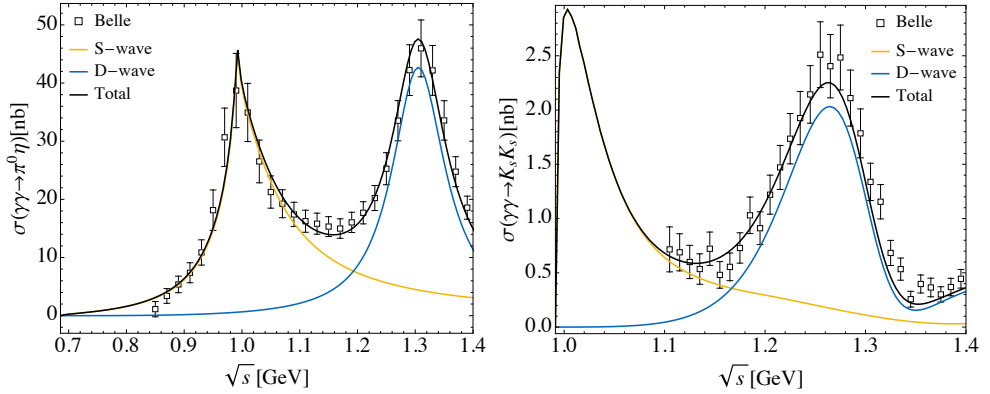


Figure 1. Total cross sections ($|\cos\theta| < 0.8$) for the $\gamma\gamma \rightarrow \pi^0\eta$ (left) and $\gamma\gamma \rightarrow K_S K_S$ (right) processes compared to the fit results. The data are taken from [5, 6].

range, which is crucial for the a_μ calculation, VMD is consistent with a simple monopole fit to the existing data and the dispersive estimation from [12].

To obtain the Omnès matrix $\Omega^{(0)}(s)$, which encodes the hadronic $\pi\eta/K\bar{K}$ rescattering effects, we utilize the coupled-channel dispersion relation for the partial wave amplitude. The latter is numerically solved using the N/D ansatz [13], with input from the left-hand cuts. When bound states or Castillejo-Dalitz-Dyson (CDD) poles are absent, the Omnès matrix is the inverse of the D -matrix. We parameterize the left-hand cuts in a model-independent manner, expressing them as an expansion in a suitably constructed conformal mapping variable [14, 15], which is chosen to map the left-hand cut plane onto the unit circle. In the absence of experimental $\pi\eta/K\bar{K}$ data, the coefficients of this conformal expansion can be estimated theoretically from χ PT, as demonstrated in [16–18]. However, for the $\pi\eta/K\bar{K}$ system, it is necessary to rely on the slowly convergent $SU(3)$ χ PT. Instead, we directly determine the unknown coefficients by fitting to $\gamma\gamma \rightarrow \pi\eta/K_S K_S$ data [5, 6] and use χ PT predictions only as additional constraints. Particularly, for the $\pi\eta \rightarrow K\bar{K}$ channel, we impose an Adler zero and ensure that the $\pi\eta \rightarrow K\bar{K}$ amplitude remains consistent with χ PT at $s_{th} = (m_\pi + m_\eta)^2$. Furthermore, for the $\pi\eta \rightarrow \pi\eta$ channel, we employ the χ PT scattering length as a constraint. In all cases, the NLO result with low-energy coefficients from [19] is considered as the central value, with an error range defined by the spread between LO and NLO results.

3 Results and Outlook

To reconstruct the physical $\gamma\gamma \rightarrow K_S K_S$ cross section, the input for $I = 0$, S -wave amplitude $k_{0,++}^{(0)}(s)$ is taken from the coupled-channel $\pi\pi/K\bar{K}_{I=0}$ analysis [20]. Since we are aiming to describe $\gamma\gamma \rightarrow \pi\eta/K_S K_S$ data in the region from threshold up to 1.4 GeV, we also incorporate the D -wave resonances $f_2(1270)$ and $a_2(1320)$ using the Breit-Wigner parametrization, similar to the approach in [21]. We find that with as few as $(2, 2, 2)$ S -wave parameters in $(11, 12, 22)$ channels ($1 = \pi\eta, 2 = K\bar{K}$) we obtain the fit with $\chi^2/\text{d.o.f.} = 0.83$. The resulting total cross sections for $\gamma\gamma \rightarrow \pi\eta/K_S K_S$ processes are illustrated in Fig. 1. Through analytical continuation into the complex plane we find the pole on the Riemann sheet II, corresponding to the $a_0(980)$ resonance with $\sqrt{s_{a_0(980)}} = 1.06 - i0.058$ GeV.

With the obtained $\gamma^*\gamma^* \rightarrow \pi\eta/K\bar{K}$ amplitudes in hand, we can now proceed to calculate the $a_0(980)$ contribution to the HLbL in $(g-2)$. The preliminary result is

$$a_\mu^{\text{HLbL}}[a_0(980)]_{\text{rescattering}} = -0.46(2) \times 10^{-11}, \quad (6)$$

where the uncertainty currently covers only the sum-rule violation (reflecting the choice of the HLbL basis [3]). It is useful to compare the obtained dispersive result with the outcome from the narrow width approximation $a_\mu^{\text{HLbL}}[a_0(980)]_{\text{NWA}} = -([0.3, 0.6]_{-0.1}^{+0.2}) \times 10^{-11}$ [4], where the range reflects the variation in the scale of transition form factor parametrisation taken from the quark model [22].

It is planned to further add new experimental data into the current analysis, in particular, $\gamma\gamma \rightarrow K^+K^-$ data from BESIII [23]. In addition, the hadronic $\pi\eta/K\bar{K}$ rescattering will be further constrained by including the existing data for the $\phi \rightarrow \gamma\pi\eta$ [24] and $\eta' \rightarrow \pi\pi\eta$ [25] decays.

References

- [1] D. P. Aguillard *et al.* [Muon $g-2$], Phys. Rev. Lett. **131** (2023) no.16, 161802
- [2] T. Aoyama, N. Asmussen, M. Benayoun, J. Bijnens, T. Blum, M. Bruno, I. Caprini, C. M. Carloni Calame, M. Cè and G. Colangelo, *et al.* Phys. Rept. **887** (2020), 1-166
- [3] G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, Phys. Rev. Lett. **118** (2017) no.23, 232001, JHEP **04** (2017), 161
- [4] I. Danilkin, M. Hoferichter and P. Stoffer, Phys. Lett. B **820** (2021), 136502
- [5] S. Uehara *et al.* [Belle], Phys. Rev. D **80** (2009), 032001
- [6] S. Uehara *et al.* [Belle], PTEP **2013** (2013) no.12, 123C01
- [7] C. F. Redmer [BESIII], EPJ Web Conf. **166** (2018), 00017
- [8] R. Garcia-Martin and B. Moussallam, Eur. Phys. J. C **70** (2010), 155-175
- [9] I. Danilkin, O. Deineka and M. Vanderhaeghen, Phys. Rev. D **101** (2020) no.5, 054008
- [10] I. Danilkin and M. Vanderhaeghen, Phys. Lett. B **789** (2019), 366-372
- [11] G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, JHEP **09** (2015), 074
- [12] D. Stamen, D. Hariharan, M. Hoferichter, B. Kubis and P. Stoffer, Eur. Phys. J. C **82** (2022) no.5, 432
- [13] G. F. Chew and S. Mandelstam, Phys. Rev. **119** (1960), 467-477
- [14] A. Gasparyan and M. F. M. Lutz, Nucl. Phys. A **848** (2010), 126-182
- [15] I. V. Danilkin, A. M. Gasparyan and M. F. M. Lutz, Phys. Lett. B **697** (2011), 147-152
- [16] I. V. Danilkin, L. I. R. Gil and M. F. M. Lutz, Phys. Lett. B **703** (2011), 504-509
- [17] I. V. Danilkin, M. F. M. Lutz, S. Leupold and C. Terschusen, Eur. Phys. J. C **73** (2013) no.4, 2358
- [18] I. Danilkin, O. Deineka and M. Vanderhaeghen, Phys. Rev. D **96** (2017) no.11, 114018
- [19] J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. **64** (2014), 149-174
- [20] I. Danilkin, O. Deineka and M. Vanderhaeghen, Phys. Rev. D **103** (2021) no.11, 114023
- [21] J. Lu and B. Moussallam, Eur. Phys. J. C **80** (2020) no.5, 436
- [22] G. A. Schuler, F. A. Berends and R. van Gulik, Nucl. Phys. B **523** (1998), 423-438
- [23] M. Küßner, *Coupled channel partial wave analysis of two-photon reactions at BESIII* (Ruhr U., Bochum, 2022)
- [24] F. Ambrosino *et al.* [KLOE], Phys. Lett. B **681** (2009), 5-13
- [25] M. Ablikim *et al.* [BESIII], Phys. Rev. D **97** (2018) no.1, 012003