Light front approach to axial meson photon transition form factors: probing the structure of $\chi_{c1}(3872)$

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Abstract. We propose to study the structure of the enigmatic $\chi_{c1}(3872)$ axial vector meson through its $\gamma^*_L \gamma \rightarrow \chi_{c1}(3872)$ transition form factor. We use our recently derived light-front wave function representation of the form factor for the lowest $c\bar{c}$ Fock-state. We found that the reduced width of the state is well within the current experimental bound recently published by the Belle collaboration. This strongly suggests a crucial role of the $c\bar{c}$ Fock-state in the photon-induced production. Our predictions for the $Q^2$ dependence can be tested by future single tagged $e^+e^-$ experiments, giving further insights into the short-distance structure of this meson.

1 Introduction

The $\chi_{c1}(3872)$ (or $X(3872)$) axial vector meson state discovered by the Belle collaboration some 20 years ago [1] still poses a number of puzzles. Its mass is close to the $D^0\bar{D}^0$ threshold which suggests a picture based on a very weakly bound meson molecule (for a review, see e.g. Ref.[2] and references therein). On the other hand, in high-energy proton-proton collisions, its production at large transverse momenta proceeds at rates comparable to the quarkonium $\psi'(2S)$ [3–5]. This appears counterintuitive for such a weakly bound, very large, strongly interacting system. Indeed, our recent calculations [6] suggest, that a compact $c\bar{c}$ component may play a decisive role in the production mechanism. Here, the production in a virtual photon-photon mode suggests itself as a much cleaner environment to study the role of the possible $c\bar{c}$ component. Here the photon virtuality serves as a handle to zoom in on the short-distance structure of the meson. First results in $e^+e^-$ collisions have been reported by the Belle Collaboration [7]. Here, we give a brief review of our recent work [8].

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2 $\gamma^*\gamma^*$ transition form factors for the $J^{PC} = 1^{++}$ state

In the coupling of two virtual photons to an axial vector meson, three invariant form factors appear. The relevant $\gamma^*\gamma^*$-meson amplitude can be written in covariant form as [9, 10]:

\[
\frac{1}{4\pi\alpha_{em}} M_{\mu\nu} = i(q_1 - q_2 + \frac{Q_1^2 - Q_2^2}{(q_1 + q_2)^2}(q_1 + q_2)) \rho \tilde{G}_{\mu\nu} \frac{M}{2X} F_{TT}(Q_1^2, Q_2^2) + ie^{+}_{\mu}(q_1)G_{\mu\nu} \frac{1}{\sqrt{X}} F_{LT}(Q_1^2, Q_2^2) + ie^+_{\nu}(q_2)G_{\mu\nu} \frac{1}{\sqrt{X}} F_{TL}(Q_1^2, Q_2^2).
\]

Here we have introduced the mass $M$ of the meson, and

\[
G_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta, \quad X = (q_1 \cdot q_2)^2 - q_1^2 q_2^2,
\]

as well as the polarization vectors of longitudinal photons

\[
e^{+}_{\mu}(q_1) = \sqrt{-q_1^2} X (q_2 - q_1 q_2^\mu) / (q_1^2 - q_1^2 q_2^\mu), \quad e^{+}_{\nu}(q_2) = \sqrt{-q_2^2} X (q_1 - q_2^\nu q_2^\mu) / (q_2^2 - q_2^2 q_1^\mu).
\]

Above, the Lorentz indices $\mu, \nu$ correspond to the photon legs, while $\rho$ is to be contracted with the meson’s polarization vector. We have denoted $Q_i^2 = -q_i^2$ for the photon virtualities, and are only interested in the spacelike (and on-shell) region $Q_i^2 \geq 0$. A spin-1 particle cannot decay into two photons (by the Landau-Yang “theorem”), which means that $F_{TT}(0,0) = 0$. On the other hand, to avoid a kinematical singularity, the combination

\[
f_{LT}(Q^2) = \frac{F_{LT}(Q^2,0)}{Q}
\]

has a definite limit at $Q^2 \to 0$. Its value $f_{LT}(0)$, therefore, serves to quantify the strength of the two-photon coupling to the axial vector and gives rise to the so-called “reduced width”:

\[
\Gamma = \frac{\pi \alpha_{em}^2 M}{3} f_{LT}^2(0).
\]

We thus need at least one virtual photon to produce an axial vector in photon-photon collisions. This excludes ultraperipheral heavy ion collisions, where photons are quasi-real. Electron scattering gives us access to finite $Q^2$ and a whole polarization matrix of virtual photons. Here, feasible options are (see the diagram in the left panel of figure 1):

1. single tag $e^+e^-$ collisions. Here the tagged lepton couples to the virtual photon, while photons from the lepton “lost in the beampipe” are quasi-real [7].

2. electron-proton or electron-ion scattering. Here especially heavy ions such as Gold, the large charge $Z = 79$ of which give rise to a large quasi-real photon flux enhanced by $Z^2$, are of interest. For the example of $\eta_c$ production, see [11].

2.1 Light front representation of transition form factors

We evaluate the $\gamma^*\gamma^* \to \chi_{c1}$ amplitude in the Drell-Yan frame where $q_{1\mu} = q_{1+} n_{1+} + q_{1-} n_{1-}$ and $q_{2\mu} = q_{2+} n_{2+} + q_{2-} n_{2-}$, using the light front plus-component of the electromagnetic current, (compare the diagram in the right panel of figure 1):

\[
\langle \chi_{c1}(\lambda)|J_+(0)|\gamma_{\mu}^+(Q^2) \rangle = 2q_{1+}e e_c \sqrt{N_c} \int \frac{dzd^2k}{z(1-z)16\pi^3} \sum_{\sigma,\bar{\sigma}} \eta_{\sigma\bar{\sigma}}(z, k)(q_2 \cdot \nabla)_{\sigma\bar{\sigma}}(z, k, Q^2).
\]
The reduced width of the $\chi_{c1}(2P)$ state for several models of the charmonium wave functions with specific $c\bar{c}$-quark mass.

<table>
<thead>
<tr>
<th>$c\bar{c}$ potential</th>
<th>$m_c$ (GeV)</th>
<th>$f_{LT}(0)$</th>
<th>$\Gamma_{\gamma\gamma}$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>harmonic oscillator</td>
<td>1.4</td>
<td>0.041</td>
<td>0.36</td>
</tr>
<tr>
<td>power-law</td>
<td>1.334</td>
<td>0.033</td>
<td>0.24</td>
</tr>
<tr>
<td>Buchmüller-Tye</td>
<td>1.48</td>
<td>0.029</td>
<td>0.18</td>
</tr>
<tr>
<td>logarithmic</td>
<td>1.5</td>
<td>0.025</td>
<td>0.14</td>
</tr>
<tr>
<td>Cornell</td>
<td>1.84</td>
<td>0.018</td>
<td>0.07</td>
</tr>
<tr>
<td>BLFQ [13]</td>
<td>1.6</td>
<td>0.044</td>
<td>0.42</td>
</tr>
</tbody>
</table>

The form factor $f_{LT}$ when expressed through the $c\bar{c}$ Fock state light front wave function (LFWF) then takes the form

$$f_{LT}(Q^2) = -\frac{2}{Q^2 + M^2_c} \int \frac{d^3k}{(2\pi)^3} \left[ k_\perp + i k_y \right] \sqrt{(1-z)^{k_1^+(+)}(z, k) + k_1^{(+)\gamma}(z, k)}.$$

We note that the fully transverse form factor for the case of a quark-antiquark state is obtained as:

$$F_{TT}(Q^2, 0) = -\frac{Q^2}{M} f_{LT}(Q^2).$$

Regarding the LFWF of the meson, we adopt two options. The first one is based on solutions of the radial Schrödinger equation in the meson rest frame and a prescription on how to transform the obtained wave function into a radial LFWF. The spin-orbit part is obtained by means of a Melosh transformation, see Ref.[10] for details. A second approach is based on a solution of the bound state problem directly on the light front [12, 13], labelled BLFQ.

![Figure 1](image)

Figure 1. Left: a typical $\gamma^*\gamma$ fusion process in the large angle scattering of an electron off a positron, proton, or nucleus. Right: the $\gamma^*_{LT} \rightarrow \chi_{c1}$ transition form factor in the kinematics of the Drell–Yan frame.

Modelling the $\chi_{c1}(3872)$ as a $2P_c c\bar{c}$ state, we obtain the values of the reduced width shown in table 1, and the form factor $f_{LT}(Q^2)$ shown in figure 2. In the case of $\chi_{c1}(3872)$, the values obtained for a $2^1P_1$ charmonium are well within the range of the first Belle data, compare the updated value of Ref.[14]:

$$0.024 \text{ keV} < \tilde{\Gamma}(\chi_{c1}(3872)) < 0.615 \text{ keV}.$$ (7)
Figure 2. The form factor $f_{LT}(Q^2)$ for different models of the LFWF of the $\chi_{c1}(3872)$.

This suggests an important role of the $c\bar{c}$ Fock state for production in the $\gamma^*\gamma$ mode. (Of course, there is still room for additional contributions.) Electroproduction of $\chi_{c1}(1P), \chi_{c1}(3872)$ in the Coulomb field of a heavy nucleus may give access to the form factor $f_{LT}(Q^2)$. This is additional information on the structure. We know how to calculate it for $c\bar{c}$ states.

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References