Pion absorption from the lowest atomic orbital in $^2\text{H}$, $^3\text{H}$ and $^3\text{He}$

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Abstract. The $\pi^- + ^2\text{H} \rightarrow n + n$, $\pi^- + ^3\text{H} \rightarrow n + n + n$, $\pi^- + ^3\text{He} \rightarrow n + d$ and $\pi^- + ^3\text{He} \rightarrow p + n + n$ capture reactions from the lowest atomic orbitals are investigated under full inclusion of final state interactions and employing the single-nucleon and two-nucleon transition operators derived at leading order in chiral effective field theory.

1 Introduction

Pion production and absorption reactions involving nuclei have been extensively studied, especially since the mid 1960s, when the so-called “meson factories” became available. Early experimental and theoretical research conducted by many groups was summarized in several review papers and books (see for example Refs. [1–6]).

In the early 1990s, the precise experimental data for the total cross section of neutral pion production in proton-proton collisions in the threshold kinematics measured at IUCF [7] revealed a serious disagreement with the theoretical calculations [8, 9]. Also the phenomenological model for charged pion production involving the Weinberg-Tomozawa vertex [9] failed to describe the data. Various phenomenological attempts to find the missing physics [10] triggered theoretical research in the framework of chiral effective field theory (EFT) [11].

The studies of the $N N \rightarrow N N\pi\pi$ reactions uncovered in particular an important role played by the momentum scale $p \sim \sqrt{M_\pi M}$ ($M_\pi$ and $M$ are the pion and nucleon masses, respectively) bound with real pion production [12–15]. The emergence of this scale required a modification of the standard chiral power counting used to describe few-nucleon reactions below pion-production threshold [16–18]. The use of this momentum counting scheme (MCS) led to

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Table 1. Absorption rates for the $\pi^- + ^2\text{H} \rightarrow n + n$ reaction calculated [32] with the chiral SMS nucleon-nucleon force [36] at N$^4$LO$^+$ and with selected cutoff values $\Lambda$ with the single-nucleon transition operator (SN) and including two-nucleon contributions (SN+2N) at leading order in the MCS. Plane wave results (PW) and results obtained with the inclusion of 2N rescattering (Full) are shown.

<table>
<thead>
<tr>
<th>$\Lambda$ (MeV)</th>
<th>SN PW</th>
<th>SN Full</th>
<th>SN PW</th>
<th>SN Full</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.0028</td>
<td>0.0125</td>
<td>2.057</td>
<td>1.484</td>
</tr>
<tr>
<td>450</td>
<td>0.0142</td>
<td>0.0070</td>
<td>1.836</td>
<td>1.292</td>
</tr>
<tr>
<td>500</td>
<td>0.0305</td>
<td>0.0032</td>
<td>1.644</td>
<td>1.224</td>
</tr>
<tr>
<td>550</td>
<td>0.0460</td>
<td>0.0007</td>
<td>1.508</td>
<td>1.247</td>
</tr>
</tbody>
</table>

a very good description of the threshold charge pion production data already at leading order in the MCS (LO-MCS) [19]. In Refs. [20–22] the calculation was elevated to the N$^2$LO-MCS. Although experimental data for pion production and pion absorption reactions involving more than two nucleons are available [23–29], theoretical efforts are very scarce compared to the two-nucleon sector. See however Refs. [30, 31]. In this contribution we present a sample of results published in [32], where we performed an exploratory study of stopped $\pi^-$ absorption from the lowest orbitals of $^2\text{H}$, $^3\text{H}$ and $^3\text{He}$ pionic atoms. To the best of our knowledge, this is the first calculation in the framework of chiral EFT framework.

2 Formalism

Recently, we investigated muon capture as well as pion radiative capture on $^2\text{H}$, $^3\text{He}$ and $^3\text{H}$ [33–35]. We refer the reader especially to Appendices A and B of Ref. [35] for details of our momentum space framework, which enables us to use many models of nuclear forces or currents, and to identify effects of various dynamical ingredients, such as final state interactions or three-nucleon (3N) forces. Results presented in this contribution are obtained with the chiral SMS nucleon-nucleon potentials up to N$^3$LO$^+$ [36] and the N$^2$LO 3N forces [37].

The transition operator is limited to the LO-MCS contributions. The explicit expressions for the single-nucleon (SN) and two-nucleon (2N) parts of the absorption operator, which are used in [32], come from Refs. [38] and [19], respectively. We do not employ any regulator for the 2N transition operator in this study. Dealing with the reactions with three nucleons, we neglect 3N contributions in the transition operator, since 3N operators are suppressed compared to the SN and 2N ones by the power counting.

3 Selected results

Here we restrict ourselves to showing only the total absorption rates and refer the reader to [32] for other observables. In Tab. 1 we display our results for the $\pi^- + ^2\text{H} \rightarrow n + n$ reaction calculated under different dynamical assumptions. Our most complete predictions (last column) are in good agreement with the experimental value $1.306_{-0.055}^{+0.026} \times 10^{15}$ s$^{-1}$ from the hadronic ground-state broadening in pionic deuterium [39, 40].

For the reactions with three nucleons we compare in Tabs. 2–4 four different calculations: (1) symmetrized plane wave with the single-nucleon and two-nucleon parts in the transition operator and 3N force effects included in the initial 3N bound state, (2) calculation with the initial and final states calculated with the same Hamiltonian comprising 2N and 3N forces.
Table 2. Absorption rates for the $\pi^- + ^3\text{He} \rightarrow n + d$ reaction calculated [32] with the chiral SMS 2N potentials [36] at N\textsuperscript{4}LO\textsuperscript{+} augmented by the consistently regularized 3N force at N\textsuperscript{2}LO [37] for selected values of the cutoff parameter $\Lambda$. The four calculations are defined in the text.

<table>
<thead>
<tr>
<th>$\Lambda$ (MeV)</th>
<th>Calc. (1)</th>
<th>Calc. (2)</th>
<th>Calc. (3)</th>
<th>Calc. (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>8.3158</td>
<td>0.0172</td>
<td>3.6566</td>
<td>3.028</td>
</tr>
<tr>
<td>450</td>
<td>6.6961</td>
<td>0.0231</td>
<td>2.5466</td>
<td>2.089</td>
</tr>
<tr>
<td>500</td>
<td>5.4398</td>
<td>0.0666</td>
<td>1.9909</td>
<td>1.595</td>
</tr>
<tr>
<td>550</td>
<td>4.6015</td>
<td>0.1840</td>
<td>1.8029</td>
<td>1.371</td>
</tr>
</tbody>
</table>

Table 3. Absorption rates for the $\pi^- + ^3\text{He} \rightarrow p + n + n$ reaction calculated [32] with the same forces and with the same four types of dynamics as in the case of $\Gamma_{nd}$ in Table 2.

<table>
<thead>
<tr>
<th>$\Lambda$ (MeV)</th>
<th>Calc. (1)</th>
<th>Calc. (2)</th>
<th>Calc. (3)</th>
<th>Calc. (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>38.378</td>
<td>0.675</td>
<td>16.346</td>
<td>15.686</td>
</tr>
<tr>
<td>450</td>
<td>35.212</td>
<td>0.612</td>
<td>13.237</td>
<td>12.733</td>
</tr>
<tr>
<td>500</td>
<td>32.343</td>
<td>0.601</td>
<td>11.849</td>
<td>11.367</td>
</tr>
<tr>
<td>550</td>
<td>30.170</td>
<td>0.650</td>
<td>12.039</td>
<td>11.421</td>
</tr>
</tbody>
</table>

Table 4. Absorption rates for the $\pi^- + ^3\text{H} \rightarrow n + n + n$ reaction calculated [32] with the same combinations of 2N and 3N potentials and with the same four types of dynamics as in the case of $\Gamma_{nd}$.

<table>
<thead>
<tr>
<th>$\Lambda$ (MeV)</th>
<th>Calc. (1)</th>
<th>Calc. (2)</th>
<th>Calc. (3)</th>
<th>Calc. (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>2.352</td>
<td>0.086</td>
<td>1.360</td>
<td>1.375</td>
</tr>
<tr>
<td>450</td>
<td>2.264</td>
<td>0.074</td>
<td>1.103</td>
<td>1.110</td>
</tr>
<tr>
<td>500</td>
<td>2.179</td>
<td>0.065</td>
<td>0.999</td>
<td>1.002</td>
</tr>
<tr>
<td>550</td>
<td>2.120</td>
<td>0.057</td>
<td>1.056</td>
<td>1.061</td>
</tr>
</tbody>
</table>

but retaining only the single-nucleon contribution in the transition operator, (3) calculation with the initial and final states calculated with the same Hamiltonian comprising only 2N forces and including the single-nucleon and two-nucleon parts in the transition operator, and (4) calculation with the initial and final states calculated with the 2N and 3N forces and the complete transition operator.

We see that for all the studied reactions the absorption rates depend strongly on the nuclear pion absorption operator used, since its two-body parts change the rates by a few orders of magnitude. The final state interactions between nucleons generated by the two-nucleon forces are also important, while the three-nucleon interaction plays a visible role only in the $\pi^- + ^3\text{He} \rightarrow n + d$ reaction. For further results, comparisons with experimental data and an analysis of the theoretical uncertainties the reader is referred to [32].

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References