Proton-neutron pairing, quartet condensation and $\alpha-$ transfer in N=Z nuclei

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Abstract. In the past years various studies have shown that the proton-neutron pairing correlations in N=Z nuclei can be accurately described by a condensate of α-like quartets. It was also shown that the α-like quartets, defined as 4-body collective structures built with two neutrons and two protons, are also able to describe the correlations induced by general two-body interactions of shell-model type. In this paper we summarise the studies mentioned above and then we discuss how the fingerprints of the quartet condensation could be investigated by α-transfer reactions.

1 Introduction

In the nuclei close to the N=Z line are usually considered two kinds of pairing correlations, corresponding to the isovector pairs, i.e. , neutron-neutron, proton-proton and proton-neutron pairs of isospin T=1 and spin S=0, and the isoscalar proton-neutron pairs of T=0 and S=1. One question addressed in many theoretical and experimental studies is whether in nuclei the proton-neutron pairs act coherently in the form of a pair condensate, as in the case of neutron-neutron and proton-proton pairs [1]. In particular, the question of great interest is whether a condensate of deuteron-like isoscalar proton-neutron pairs could exist in nuclei. In spite of many studies, at present there is not a clear answer to this question.

The majority of the theoretical studies on proton-neutron pairing have been done in the framework of the HFB approach [2, 3]. The advantage of this approach is that the isovector and the isoscalar pairs are treated together in a simple manner through the generalised Bogoliubov transformation. The main drawback is that in HFB are not conserved exactly the particle number, the spin and the isospin. To restore these broken symmetries in the framework of the HFB approach is a difficult task which was worked out only for special cases [4, 5].

There is however another alternative to treat the proton-neutron pairing, namely to construct trial wave functions which conserve from the beginning the most important symmetries. For the isovector pairing such alternative was proposed many years ago by Flowers and Vujici [6]. They pointed out that in order to conserve the isospin, the simplest manner is to work with 4-body α-like structures built be two neutrons and two protons coupled to the

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total isospin T=0. With these 4-body structures they constructed a BCS-like state, in which the pairs are replaced by the α-like operators. This approach, which conserves exactly the isospin but not the particle number, is rather complicated and it was never applied for realistic calculations. A solution of the isovector pairing in terms of α-like quartets was found much later for the particular case of degenerate single-particle levels [7] . It was shown that in this case the ground state of $N = Z$ systems interacting by an isovector force can be expressed exactly by an α-like quartet condensate. A realistic quartet condensation model (QCM) for non-degenerate levels and general pairing forces was proposed later in Refs. [8–10]. Recently it was also shown that the ground states and the low-lying excited states of N=Z nuclei, described by realistic two-body forces of shell-model type, can be also approximated rather well in the framework of the QCM approach [11–14]. The QCM approach will be summarized below. Then we will discuss how the quartet condensation could be probed by α- transfer reactions and, finally, we will present some preliminary results on α-transfer spectroscopic factors calculated with the QCM wave functions [15].

2 QCM approach for the isovector-isoscalar proton-neutron pairing Hamiltonians

The isovector and isoscalar pairing correlations are described usually by the Hamiltonian

$$\begin{aligned} H &= \sum_i \epsilon_i N_i + \sum_{i,j} V^{T=1}_{J=0}(i,j) \sum_{T_z} P^+_{i,T_z} P_{j,T_z} \\ &+ \sum_{i\leq j,k\leq l} V^{T=0}_{J=1}(ij,kl) \sum_{J_z} D^+_{ij,J_z} D_{kl,J_z}. \end{aligned} \tag{1}$$

In the first term ϵ_i and N_i are, respectively, the energy and the particle number operator relative to the single-particle state $i = \{n_i, l_i, j_i\}$, where l_i is the orbital angular momentum and τ_i is the isospin projection. The Coulomb interaction between the protons is not taken into account so that the single-particle energies of protons and neutrons are assumed to be equal. The second and the third terms are, respectively, the isovector ($T = 1, J = 0$) and isoscalar ($T = 0, J = 1$) pairing interactions. They are written in terms of the pair operators

$$P^+_{j,T_z} = \frac{1}{\sqrt{2}}[a^+_j a^+_j]^{T=1,J=0}_{T_z} \tag{2}$$

$$D^+_{j_1 j_2 J_z} = \frac{1}{\sqrt{1+\delta_{j_1 j_2}}}[a^+_{j_1} a^+_{j_2}]^{J=1,T=0}_{J_z}. \tag{3}$$

where J and T are the angular momentum and the isospin of the pairs, respectively.

In Ref. [10], which is summarized here, the ground state of the Hamiltonian (1) was described by a condensate of quartets defined by

$$|\Psi_{gs}\rangle = (Q^+)^{n_q}|0\rangle. \tag{4}$$

The quartet operator Q^+ is taken as a sum of two quartets

$$Q^+ = Q^+_1 + Q^+_0, \tag{5}$$

where Q^+_1 is the collective isovector quartet formed by coupling two isovector pairs to total $T = 0$, i.e.,

$$Q^+_1 = \sum_{j_1 j_2} x_{j_1 j_2} [P^+_{j_1} P^+_{j_2}]^{T=0} \tag{6}$$

and Q_0^+ is the collective isoscalar quartet built by coupling two isoscalar pairs to total $J = 0$, i.e.,

$$Q_0^+ = \sum_{j_1 j_2 j_3 j_4} y_{j_1 j_2 j_3 j_4} [D^+_{j_1 j_2} D^+_{j_3 j_4}]^{J=0}. \tag{7}$$

In order to relate the QCM wave function to the collective isosvector and isoscalar pairs, we use the procedure applied in Ref. [8], i.e., we assume that $x_{j_1 j_2} = \bar{x}_{j_1} \bar{x}_{j_2}$ and $y_{j_1 j_2 j_3 j_4} = \bar{y}_{j_1 j_2} \bar{y}_{j_3 j_4}$. With this assumption the collective quartets can be written as

$$\bar{Q}_1^+ = 2\Gamma_1^+ \Gamma_{-1}^+ - (\Gamma_0^+)^2 \tag{8}$$

$$\bar{Q}_0^+ = 2\Delta_1^+ \Delta_{-1}^+ - \Delta_0^{+2}. \tag{9}$$

These quartets are expressed in terms of the collective isoscalar and isovector pairs

$$\Gamma^+_{T_z} = \sum_j \bar{x}_j P^+_{j,T_z} \tag{10}$$

$$\Delta^+_{J_z} = \sum_{j_1 j_2} \bar{y}_{j_1 j_2} D^+_{j_1 j_2 J_z}. \tag{11}$$

The collective isovector and isoscalar pairs defined above can be used to construct various PBCS-type states for $N = Z$ systems. Thus, with the isovector pairs (10) can be formed the following PBCS states with well-defined numbers of protons and neutrons [8]:

$$|PBCS\,1\rangle = (\Gamma_1^+ \Gamma_{-1}^+)^{n_q} |0\rangle \tag{12}$$

$$|PBCS\,0_{iv}\rangle = (\Gamma_0^+)^{2n_q} |0\rangle. \tag{13}$$

Both states have, as required, $J = 0$ and $T_z = 0$, but they do not have a well-defined total isospin. Similar PBCS states can be constructed with the isoscalar pairs (11). Of physical interest is the PBCS state

$$|PBCS\,0_{is}\rangle = (\Delta_0^+)^{2n_q} |0\rangle. \tag{14}$$

This state has $T = 0$ and $J_z = 0$, but it has not a well-defined angular momentum. Since the states (13) and (14) are condensates, respectively, of $T = 1$ and $T = 0$ proton-neutron pairs, one might think that a comparison of their correlation energies could give a clear evidence on what type of proton-neutron pairing is prevailing in $N = Z$ nuclei. However, a conclusion based only on this comparison would be misleading because, as shown below, the PBCS approximation is not accurate enough to describe properly the isovector and isoscalar pairing correlations.

To probe the accuracy of the QCM approximaion we performed calculations for various N=Z nuclei from the *sd* and *pf* shells as well as for the nuclei above ^{100}Sn. The single-particle energies and the pairing interactions for the three sets of nuclei are extracted from the shell model forces, respectively USDB, KB3G and G-matrix two-body force. With these inputs we have calculated the correlation energies predicted by the QCM approach and we compare them with the results obtained by exact digonalisation. The errors relative to the exact values are given in Table 1. As can be seen, in all cases the errors are very small, under 1%. From Table 1 one can also see that the PBCS approximations (12-13) and (14), which do not conserve the isospin and the angular momentum, respectively, are much less accurate than the ones provided by QCM. In fact, as shown in Table 1, a proper description of the competition between the isovector and isoscalar pairing correlations requires a ground state in which all types of pairs are mixed together in order to conserve exactly the spin and the angular momentum.

Table 1. The accuracy of correlation energies predicted by the QCM (6) and the PBCS (12-14) states. The errors are relative to the exact results obtained by diagonalisation.

	QCM	$PBCS\,1$	$PBCS\,0_{iv}$	$PBCS\,0_{is}$
^{20}Ne	-	12.35%	14.52%	12.99%
^{24}Mg	0.24%	23.35%	28.50%	19.22%
^{28}Si	0.57%	23.58%	28.95%	22.19%
^{44}Ti	-	18.62%	28.25%	40.22%
^{48}Cr	0.21%	16.85%	25.97%	46.81%
^{52}Fe	0.42%	15.21%	23.73%	51.95%
^{104}Te	-	10.58%	19.16%	53.19%
^{108}Xe	0.20%	11.61%	19.49%	55.82%
^{112}Ba	0.34%	12.82%	20.17%	57.13%

3 QCM approach for the excited states of proton-neutron pairing Hamiltonians

Recently the QCM approach presented in the previous section was extended for describing the low-lying excited states of $N = Z$ systems interacting by isovector-isoscalar pairing forces [12]. Below we present briefly this extension and the most important results.

In correspondence with the QCM ansatz (6) for the ground state, we have constructed a class of excited states by replacing a quartet of the condensate with an “excited" quartet. For the case of a spherically-symmetric mean field, these states take the form

$$|\Phi_{\nu,JJ_z}\rangle = \tilde{Q}_{\nu,JJ_z}(Q^+_{ivs})^{n_q-1}|-\rangle, \tag{15}$$

where

$$\tilde{Q}^+_{\nu,JM_\nu} = \sum_{ijkl} Y^{(\nu)}_{ijkl,J}[a^+_i a^+_j a^+_k a^+_l]^{T=0,J} \tag{16}$$

are the excited collective quartets. In order to define its coefficients $Y^{(\nu)}_{JJ_z}$, one has now to diagonalize the Hamiltonian (1) in the basis of non-orthogonal states

$$[P^+_{J_1,T'}(i_1, j_1)P^+_{J_2,T'}(i_2, j_2)]^{J,T=0}_{J_z}(Q^+_{ivs})^{n_q-1}|-\rangle. \tag{17}$$

To illustrate the accuracy of the approximation (15) we take as example the valence nucleons in ^{28}Si and assume an isovector-isoscalar pairing force corresponding to the ($J = 0, T = 1$) and ($J = 1, T = 0$) channels of the the USDB interaction. Exact and approximate spectra are shown in Fig. 2. It can be seen that the overall agreement is good. For more details see Ref. [12].

4 QCM approach for a general two-body interaction of shell-model type

In Ref. [11], summarized below, the QCM approach was extended for the most general two-body Hamiltonian commonly used in shell model calculations, i.e.,

$$H = \sum_i \epsilon_i N_i + \sum_{i,i',k,k',J,T} V^{JT}(ii';kk')[\mathcal{A}^{+JT}(i,i')\widetilde{\mathcal{A}}^{JT}(k,k')]^{(0,0)}. \tag{18}$$

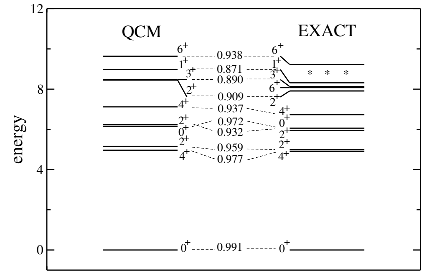

Figure 1. The low-lying spectrum provided by the QCM approximation (15) for the valence nucleons of ^{28}Si interacting by an isovector-isoscalar pairing force extracted from the USDB interaction. The numbers are the overlaps between the QCM and the exact wave functions. Energies are in MeV.

The second term in Eq. (18) is the two-body interaction written in terms of particle-particle operators

$$\mathcal{A}^{+JT}_{J_z,T_z}(i,i') = \frac{1}{\sqrt{(1+\delta(i,i'))}}[a_i^+ a_{i'}^+]^{J,T}_{J_z,T_z}, \tag{19}$$

where J, T are the angular momentum and the isospin of the pair, respectively. The other pair operator in Eq.(18) has the standard definition $\widetilde{A}^{JT}_{J_z,T_z}(i,i') = (-1)^{J-J_z+T-T_z}A^{JT}_{-J_z,-T_z}(i,i')$. The notation $(0,0)$ in the second term of the Hamiltonian, as well as in the Eq.(20) below, means that the two pair operators are coupled to total $J=0$ and $T=0$.

In Ref. [11] the Hamiltonian (18) was employed to investigate a certain class of 0^+ states, namely those which can be expressed in terms of collective α-like quartets. A collective α-like quartet is defined as

$$Q^+ = \sum_{i,i',k,k',J,T} x_{ii'kk';J,T}[\mathcal{A}^{+JT}(ii')\mathcal{A}^{+JT}(k,k')]^{(0,0)}. \tag{20}$$

This collective quartet is more general than the ones that we have used previously in the case of pairing forces since it is built by all possible isovector ($T=1$) and isoscalar ($T=0$) non-collective pairs which can be formed. It should be observed that, by definition, the α-like quartet operator (20) is not a boson operator and does not represent an alpha particle.

With the collective quartet (20) we construct the quartet condensate model (QCM) state

$$|QCM\rangle = (Q^+)^{n_q}|-\rangle, \tag{21}$$

where n_q is the number of quartets. In the applications discussed in this study, the quartets are built only with the valence nucleons which move outside a double magic core. This is represented by the vacuum state $|-\rangle$. The main issue that we address is to what extent the trial state (21) can represent the correlations generated by the two-body Hamiltonian (18) in the ground state of even-even $N=Z$ systems.

It is worth mentioning that, since the quartet (20) is not a boson, the state (21) is not a boson condensate. Here the term "condensation" has the same meaning as "pair condensation" in BCS-like theories: a product of many-body substructures (pairs in BCS, quartets in QCM) which are all in the same many-body state.

The state (21) depends on the mixing amplitudes x which define the collective quartet. These amplitudes are determined variationally by minimizing the expectation value $\langle QCM|H|QCM\rangle$ under the constraint $\langle QCM|QCM\rangle = 1$. To calculate the average of the Hamiltonian and the norm we apply standard many-body techniques.

Table 2. Correlation energies, in MeV, predicted with the QCM approach in comparison with the shell model (SM) results. In brackets we show the differences,in percentage, between the SM results and the QCM predictions. In the last two columns we report the overlaps between SM and QCM states.

	$E_{corr}(SM)$	$E_{corr}(QCM)$	$\langle SM\|QCM\rangle$
^{20}Ne	24.77	24.77	1
^{24}Mg	55.70	53.04 (4.77%)	0.85
^{28}Si	88.75	86.52 (2.52%)	0.86
^{32}S	122.51	122.02 (0.40%)	0.98

We have applied the QCM approximation described above for the *sd*-shell nuclei. For the two-body force we use the USDB interaction. The ground state correlation energies predicted by the QCM approximation are shown in Table 2 . They are compared to the exact SM results. In the last column are given the overlaps between the SM and the QCM states. One can see that the QCM approach is giving a rather good description of the ground state correlations. The deviations from the SM results have a maximum for ^{24}Mg and they are seen to decrease significantly in the heavier nuclei. As expected, the results of the dynamical QCM approach applied here, in which the quartets are determined variationally for each nucleus, are significantly better than those which were found within the quartet model (QM) approach of Ref. [16] where, as $J = 0, T = 0$ quartets, we assumed those describing the ground state ^{20}Ne. For example, in the case of ^{28}Si we observed a deviation of about 6.6% from the SM ground state energy while, in the present QCM calculation, this deviation is seen to drop to 2.52%. The quality of the QCM results of Table 2 indicates that a significant part of the the ground state correlations of the even-even $N = Z$ *sd* shell nuclei can be represented by a condensate of α-like quartets with J=0 and T=0. This is especially the case for the nucleus ^{32}S, for which the QCM and SM states have an overlap close to one.

5 QCM approach for the excited states of N=Z nuclei

Very recently it was shown that the QCM framework, based on dynamical quartets, can be extended in order to describe the low-lying excited states of N=Z nuclei [13, 14]. In particular, in Ref. [14] it was shown that the band structure of N=Z nuclei can be associated to a set of intrinsic states expressed in term of quartets. This study is summarised below.

To describe the structures of $N = Z$ nuclei, in which the valence nucleons interact by a general two-body force, we have used quartets of isospin $T = 0$ and of angular momentum J defined by

$$q^+_{JM} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a^+_{i_1} a^+_{j_1}]^{J_1 T'} [a^+_{i_2} a^+_{j_2}]^{J_2 T'}]^{JT=0}_M, \tag{22}$$

where $a^+_{i\tau}$ creates either a proton or a neutron (depending on the isospin projection τ) on the spherically-symmetric state $i \equiv \{n_i, l_i, j_i\}$. No restrictions on the intermediate couplings $J_1 T'$ and $J_2 T'$ are introduced and the amplitudes $q_{i_1 j_1 J_1, i_2 j_2 J_2, T'}$ are supposed to guarantee the normalization of the operator.

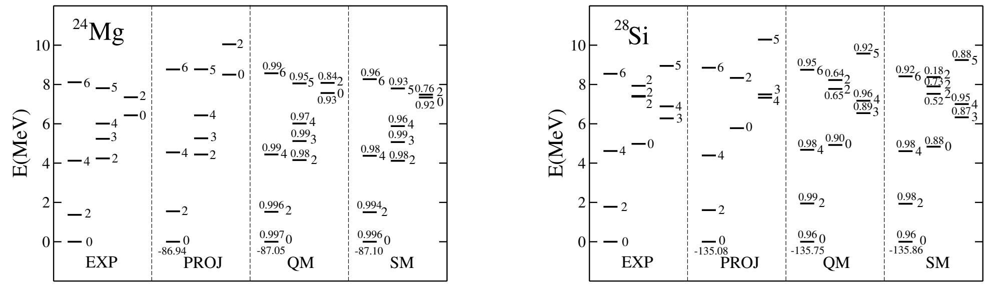

Figure 2. The experimental (EXP), projected (PROJ), QM and SM spectra for ^{24}Mg (left) and ^{28}Si (right). See the text for the details.

In the representation spanned by the quartets (22) we construct a set of intrinsic quartet states. Thus, we introduce the ground intrinsic state

$$|\Theta_g\rangle = \mathcal{N}_g (Q_g^+)^n |0\rangle, \tag{23}$$

where by n is denoted the number of quartets which can be formed with the valence nucleons outside the closed core, denoted by $|0\rangle$. As can be noticed, $|\Theta_g\rangle$ is a condensate of the intrinsic quartet Q_g^+ defined by

$$Q_g^+ = \sum_J \alpha_J^{(g)} (q_g^+)_{J0}, \tag{24}$$

where

$$(q_g^+)_{J0} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q^{(g)}_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a^+_{i_1} a^+_{j_1}]^{J_1 T'} [a^+_{i_2} a^+_{j_2}]^{J_2 T'}]_0^{JT=0} \tag{25}$$

In order to fix Q_g^+, we minimize the energy of the state $|\Theta_g\rangle$ with respect to the coefficients $q^{(g)}_{i_1 j_1 J_1, i_2 j_2 J_2, T'}$ and $\alpha_{g,J}$.

In addition to the ground intrinsic state, we have introduced a set of " excited " intrinsic states which are generated by promoting one of the quartets Q_g^+ of $|\Theta_g\rangle$ to an excited $T = 0$ configuration. These states have the general form

$$|\Theta_k\rangle = \mathcal{N}_k Q_k^\dagger (Q_g^\dagger)^{(n-1)} |0\rangle, \tag{26}$$

with

$$Q_k^\dagger = \sum_J \alpha_J^{(k)} (q_k^\dagger)_{Jk}, \tag{27}$$

$$(q_k^+)_{Jk} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q^{(k)}_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a^+_{i_1} a^+_{j_1}]^{J_1 T'} [a^+_{i_2} a^+_{j_2}]^{J_2 T'}]_k^{JT=0} \tag{28}$$

Assuming that the quartet Q_g^+ has already been fixed, we construct the new quartet Q_k^+ by minimizing the energy of $|\Theta_k\rangle$ with respect to the coefficients $q^{(k)}_{i_1 j_1 J_1, i_2 j_2 J_2, T'}$ and $\alpha_J^{(k)}$ (under the constraint of orthogonality when various states with the same k are involved). The states (26) will be identified with the value of the quantum number k. It can be seen that these states, as well as the state (23), have an undefined angular momentum.

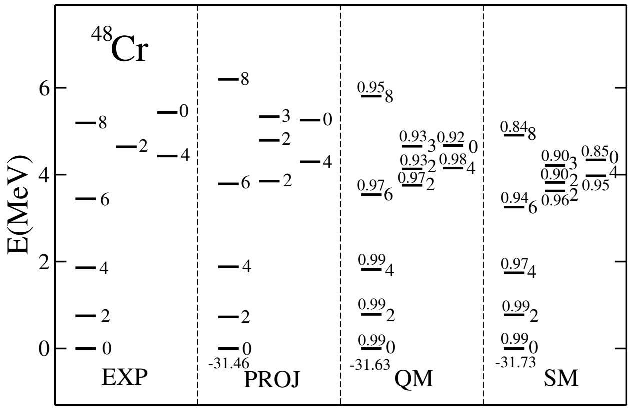

Figure 3. The same as in Fig. 2 but for ^{48}Cr.

In Ref.[14] we have explored the ability of the trial states (23) and (26) to represent proper intrinsic states of $N = Z$ nuclei. To this end we generate the spectrum of these nuclei by projecting states of good angular momentum from them. The projection technique which has been used employs standard tensor coupling rules which do not deserve special explanations. What is instead worth of being noticed is the fact that when projecting $\mathcal{N}$ intrinsic states associated to a given nucleus one can generate up to $\mathcal{N}$ states with the same angular momentum J. This projection does not guarantee neither that these states are orthogonal with each other nor that the Hamiltonian matrix is diagonal with respect to them. Thus, in these cases, a proper definition of the spectrum implies that we have first to build an orthonormal basis out of these projected states and then to diagonalize the Hamiltonian in such a basis. The maximum size of these basis in the calculations that we are going to present has been 6.

The formalism presented above is applied for the even-even $N = Z$ nuclei with the valence nucleons in the *sd* and pf major shells. Thus, the vacuum state $|0\rangle$ of the previous expressions stands for the nuclei ^{16}O and ^{40}Ca. The calculations for the *sd* and pf shell nuclei have been performed with the USDB and KB3G interactions, respectively. In Figs. 2-3, the low-lying states obtained within the projected approach are compared with the experimental spectra, the QM calculations of Ref. [13] and the shell model (SM) calculations. For the experimental spectra we have shown only the states with certain angular momenta and parities. The numbers next to each level of the QM and SM spectra give the overlaps with the corresponding projected states while those at the bottom represent the ground state energies. At this point it is worth mentioning that a simple SM calculation provides only a sequence of states. Associating them with specific band-like structures, such as ground, β and γ-like bands, requires additional analysis. In Figs. 2-3 we have split the SM states in groups of levels following the correspondence with the band-like structures associated to the QM and the projected intrinsic states.

From Figs. 2-3 one can notice that, in general, there is a clear correspondence among projected, QM and SM states. The only exception is in the case of ^{28}Si, where one generates only two $J = 2$ projected states and, in correspondence with the second of them, one finds two QM and three SM $J = 2$ states. It can be also observed that the overlap of the projected states with the QM and SM states are significant, especially for the ground band. Since the QM results are obtained by making a diagonalisation of the Hamiltonian in the space spanned by the quartets, without the restriction imposed on them by the projection, the QM levels are closer to the SM ones and, since the SM interaction is fitted to the data, the QM levels are also closer to the experimental spectrum.

As seen from Fig. 2-3, the agreement between projected states with the experimental data is rather good for all the nuclei. We find quite surprising that this agreement is obtained within such a simple approach, based only on projected states and a subsequent diagonalization in very reduced spaces (only a few units). These results support the definition of the states (23) and (26) as proper intrinsic states and show that the quartet structure of these intrinsic states is able to encapsulate the most important correlations which determine the spectra of even-even $N = Z$ nuclei.

6 Probing the quartet condensation by α-transfer reactions

As shown above, there is an interesting analogy between the treatment of proton-neutron pairing in N=Z nuclei and the like-particle pairing. Thus, in the first case the ground states is described by a condensate of quartets while in the second case by a condensate of pairs. To probe the pair condensation are usually performed pair transfer reactions along a chain of isotopes. A similar alternative can be used to search for the fingerprints of the quartet condensation in N=Z nuclei. In the present case one needs to study how the α-transfer cross sections are changing when the α particles are transferred on a chain of even-even N=Z nuclei, starting from a double magic core. As in the case of the pair transfer, at the beginning of the shell it is expected a vibrational regime, in which the cross section is increasing rapidly. For heavier N=Z nuclei the cross section is expected to enter into a plateau region. This plateau would indicate that the structure of the ground state is not changing much when a new α particle is added or removed, which is a specific behaviour of a condensate.

At present the only chain of N=Z nuclei for which there are data on α- transfer are the *sd*-shell nuclei, i.e the N=Z nuclei with the atomic mass between A=16 and A=40. For these nuclei there are data for the (^{6}Li,d) reactions and the (p,α p) knockout reactions [17, 18]. The spectroscopic factors (SFs) corresponding to the (p, α p) knockout reactions, which are supposed to be more accurate than the ones for the (^{6}Li,d) reactions, are summarised in Fig. 4. Are shown the SFs extracted from the cross sections calculated with a Woods Saxon optical potential. For A=28 are shown two values for the SF, the larger one corresponding to the peak around 65 MeV [18]. The SFs have large errors, of about 20-25 %, related both to the uncertainties of experimental data and to the errors in the reaction models [18]. It is worth stressing that these errors are bigger for ^{28}Si, which affect the conclusions on the trend of the SFs towards the middle of the *sd*-shell.

For a long time the theoretical calculations could not reproduce the general trend of the experimental data. This was the case of the shell model calculations, which predicted much smaller values for the SFs (e.g., see [19]). Recently it was shown that by using properly renormalized cluster form factors, the shell model based calculations are able to describe reasonably well the SFs in sd-shell nuclei [20]. These SM results, obtained with the USDB interaction, are shown in Fig. 4 by red squares. It can be seen that the SM values follows nicely the general trend of SFs, i.e a decrease towards the middle of the *sd*-shell and then an increase at the end of the shell.

In order to investigate the role of α-like quartet correlations on α- transfer reactions, we have estimated the SFs with the ground states obtained by diagonalising the shell model

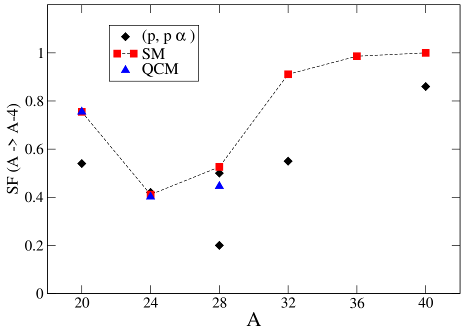

Figure 4. Spectroscopic factors for the α transfer between the ground states of *sd*-shell nuclei with N=Z. By black diamonds are shown the data extracted from (p,p α) knockout reactions [17, 18] and the read squares represent the results of the SM calculations by Volya et al [20]. The blue triangle show the SFs obtained with the quartets extracted from the QCM state (23).

Hamiltonian in the space spanned by the quartets (25) associated to the quartet condensate (23). The SFs corresponding to these states are estimated with the same calculation scheme employed in Ref.[20]. The results are shown in Fig 4 by blue triangles. It can be seen that the SFs obtained with the wave functions based on quartets are very close to the ones obtained with the SM wave functions. This means that the most important correlations in the exact SM wave function are of quartet type.

The calculations and the data show that the SF decrease fast from A=20 to A=24 and then increase for A > 28. Since for ^{28}Si the data are ambiguous, one cannot draw any conclusion on the evolution of SFs from A=24 to A=28. For A>28 the increase of the SFs is most probably related to the crossing on the second half of the *sd*-shell. Therefore, since this chain of N=Z isotopes is too small, it is not possible to observe if one could reach a plateau region at midshell, which would indicate the presence of a quartet condensate.

7 Summary and Conclusions

The QCM approach summarised here shows an interesting analogy between this approach and the projected-BCS (PBCS) approximation employed for describing the correlations between like-particles, neutrons or protons. Thus, in PBCS the ground state of nuclei with the nucleons in different major shells are described by a condensate of like-particle pairs while in QCM the ground states of N=Z nuclei are described by a condensate of quartets. In particular, both the PBCS pair condensate and the QCM quartet condensate are exact solutions when the like-particle pairing and the isovector pairing forces act on degenerate levels. It is also interesting to notice the analogy between the treatment of the excited states in PBCS and QCM. Thus, in PBCS the excited states are generated by breaking a pair form the gorund state condensate and replacing it with an excited pairs. Likewise, in QCM the excited states are generated by breaking a quartet from the quartet condensate and replacing it with an excited quartet. These analogies show that in N=Z nuclei the quartets play the same role as the pairs in nuclei with the protons and neutrons in different major shell.

In N>Z nuclei the condensation of neutron pairs is probed by analysing how the pair transfer cross section is evolving when pairs are transferred on a chain of isotopes. More precisely, a pair condensate is associated to the occurrence of a plateau region in the pair transfer cross sections. Taking into account the analogies mentioned above, one expects that the α-like quartet condensation could be probed by α transfer on a chain of N=Z nuclei. We have discussed this possibility for the *sd*-shell nuclei. Since this chain of isotopes is too small, it is not possible to see a plateau region in the spectroscopic factors , which could indicate the presence of a quartet condensate. To be able to observe if a a plateau region could develop in the α- transfer reactions one needs data and calculations on longer chains of N=Z nuclei.

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