

Temperature measurements with the relativistic Sunyaev-Zel'dovich effect

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Abstract. At temperatures above ~ 5 keV, the non-relativistic approximation used to derive the classical thermal Sunyaev-Zel'dovich effect spectrum begins to fail. When relativistic effects are included, the spectrum becomes temperature-dependent. This leads to both a problem and an opportunity: a problem, because when the temperature dependence is not accounted for the Compton- y estimate is biased; and an opportunity, because it represents a new way to measure the temperature of the intracluster medium independently of X-ray observations. This work presents current results from investigating the impact of relativistic effects on *Planck* cluster observations, and projections for future measurements of cluster temperatures using the Atacama Large Aperture Sub-millimetre Telescope.

1 Introduction

The standard thermal SZ spectrum is a non-relativistic limit, however it has been shown that at temperatures $\gtrsim 5$ keV (typical for massive clusters), electrons start to move at non-negligible fractions of the speed of light and therefore higher-order, temperature-dependent corrections become necessary [1–3]. This is known as the relativistic SZ (rSZ) effect. The rSZ frequency spectrum is temperature-dependent (see figure 1): at frequencies less than ≈ 500 GHz, the main change is a decrease in (absolute) amplitude of the signal with temperature, meaning that without strong constraints from higher frequency bands, there is a degeneracy between temperature and signal strength. When the non-relativistic approximation is assumed, the effect is an underestimate of the overall cluster signal, which we refer to as the rSZ bias. [4] detected the relativistic correction to the SZ spectrum in a stacked sample of *Planck* clusters at $\approx 2\sigma$ level and predicted an rSZ bias of up to 14% in the integrated Compton- y parameter for the most massive clusters; [5] considered the effect of the rSZ spectrum on the power spectrum of the Compton- y parameter and concluded it could shift the constraint on σ_8 by $\approx 1\sigma$, partially alleviating the tension with σ_8 measurements from the *Planck* primary CMB anisotropy data.

It is well-known that X-ray temperatures can be biased due to the presence of dense, cold clumps due to the density-squared weighting of the signal; the rSZ effect, in contrast, is pressure-weighted and may be less biased (eg [6]). rSZ temperature therefore offers an independent way to measure the temperature of the ICM, and perhaps to improve our understanding of the ICM astrophysics.

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Computation of the rSZ effect is now tractable with software packages such as SZPACK [7, 8] so it has become relatively straightforward to incorporate in SZ data analysis. Some attempts have been made to try to constrain rSZ temperatures on individual clusters (eg [9]) although the constraints so far have been weak.

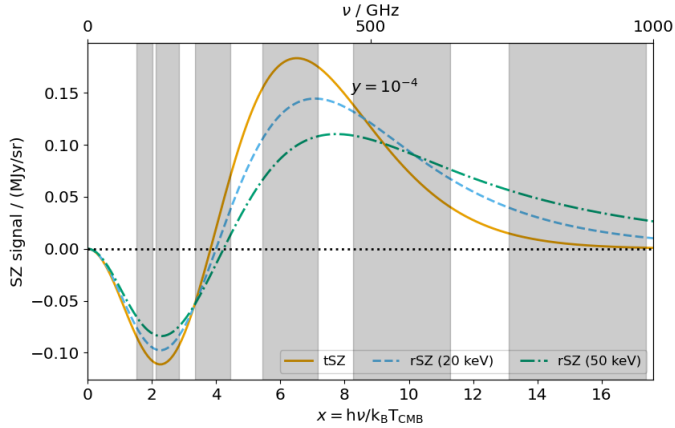


Figure 1. Change in the rSZ spectrum with increasing temperature, compared to the non-relativistic approximation (tSZ). Grey bands show *Planck* frequency bands.

2 rSZ with *Planck*

The *Planck* frequency bands span an appropriate frequency range to distinguish between the rSZ effect and a decrease in Compton- y . We investigated whether it was possible to constrain rSZ temperatures with *Planck* data by creating simulations using a realistic physical model for the cluster thermodynamic profiles [10, 11], then generating rSZ signal images using SZPACK. The simulated clusters were injected into real *Planck* data in areas away from real clusters, and the resulting images analyzed using a version of the POWELLSNAKES software [12, 13] adapted to also fit for a single average temperature. We used a simple cluster parameterisation defined by the characteristic angular scale of the cluster (θ_s) and its total integrated Compton- y parameter (Y_{tot}) to fit the simulated data.

In figure 2 we show example results from some realizations of two of the simulated clusters. The first is a high-SNR cluster with parameters chosen to be similar to Coma, while the second is more representative of the median of the *Planck* population. When assuming the tSZ spectrum (left-hand column), the recovered Y_{tot} values are significantly offset from the true value for the Coma-like cluster, by an average of $\approx 13\%$. For the lower-SNR cluster, the bias is not significant on the level of an individual realization, but over the whole simulation population Y_{tot} is biased down by $\approx 5\%$.

We attempted to constrain the rSZ temperature by putting a uniform prior on temperature in the analysis (middle column of figure 2). In the case of Coma, this does successfully produce a (weak) constraint on temperature and an unbiased Y_{tot} value. However, in the case of the low-SNR cluster, the temperature is not constrained and the degeneracy between Y_{tot} and temperature leads to a large *upward* bias in Y_{tot} . We therefore conclude that at the sensitivity level of *Planck*, for most clusters it will not be possible to constrain temperature and attempting to do so will lead to Y_{tot} constraints that are biased up rather than down. We

can, however, avoid the downward rSZ bias by putting an informative prior on temperature (right-hand column of figure 2) such as a Gaussian centred at the true value. In the case of the simulations, the ‘true’ (pressure-weighted average) value is known; in the case of real data an external estimate of the temperature from X-ray could be used, although care should be taken in considering the different weightings involved in averaging over the cluster volume. Alternatively, a scaling relation between, eg, Y_{tot} and temperature such as those calibrated from numerical simulations by [14, 15] can also be used.

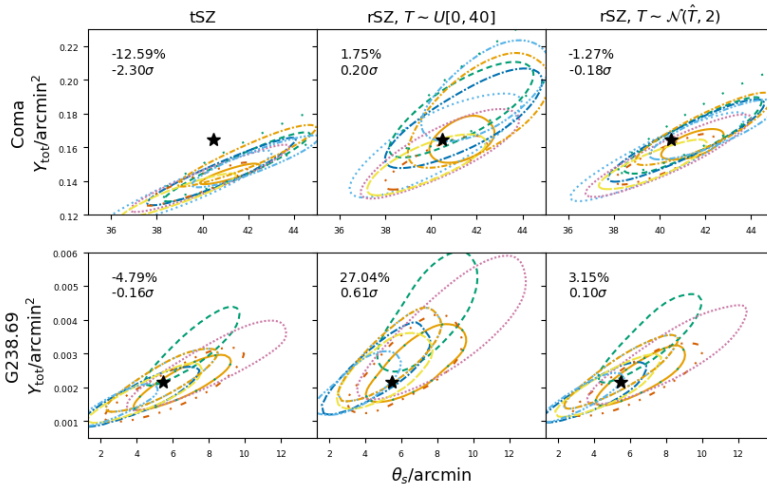


Figure 2. Posterior constraints on the cluster θ_s and Y_{tot} model parameters derived from analysing relativistic simulations with the non-relativistic approximation to the SZ signal (left column) and isothermal relativistic signal (other columns; prior on T_{SZ} given in title). For a given cluster, all plots have the same axis ranges and the true value is marked with a black star. The 68% probability contour is shown with different colours/line styles for a random selection of ten realizations of the simulation. Average 1D Y_{tot} bias values for the whole simulation set are given on each figure, both as a percentage and as an average significance level.

Although these biases are small and not necessarily significant for individual clusters, when considering the population as a whole (i.e. for constructing scaling relations), the rSZ bias produces a mass-dependent decrease in Y_{tot} which skews the scaling relation. When calibrating a scaling relation *on Planck* data for applying *to Planck* data the bias cancels out; however when comparing scaling relations between SZ measurements made by different instruments and/or simulations, this can have a significant effect. A corrected *Planck* scaling relation will be presented in a forthcoming publication (Perrott et al, in prep).

3 rSZ with AtLAST

The Atacama Large Aperture Sub-millimetre Telescope (AtLAST; [16]) is a proposed 50m single dish telescope to be situated in the Atacama desert, allowing the high-frequency observations necessary for constraining rSZ temperature. Its high angular resolution will allow both spatial separation of confusing foreground/background emission, and the possibility to resolve SZ effect signals.

Table 1. Bands assumed to make rSZ predictions for AtLAST.

Lower limit / GHz	Upper limit / GHz
30	54
66	117
120	182
183	252
252	325
325	375
384	422
595	713
786	905

We assume the bands in table 1 to make the following predictions, and the current sensitivity calculator estimates¹. The band structure and sensitivity calculator are under development so these estimates should be taken as preliminary only.

We first ask the question, what sensitivity is required to constrain rSZ temperature? For a given temperature, we generate an rSZ spectrum as in Fig. 1 and average within the band-passes shown in Table 1 to simulate an AtLAST observation. We assume the same observing time for all bands to generate error estimates in each band, taking into account the variation both in point source sensitivity and in beam sizes amongst the frequency bands, i.e.

$$\frac{\Delta I(n)}{\Delta I(\text{ref})} = \frac{\sigma(n)}{\sigma(\text{ref})} \times \frac{\Omega(\text{ref})}{\Omega(n)}, \quad (1)$$

where I is the SZ signal in MJy sr⁻¹, n denotes the band index, and $\sigma(n)$ and $\Omega(n)$ are the point source flux density sensitivity and beam size in a given band n , respectively. An example simulated spectrum is shown in Fig. 3 (left).

We then fit the simulated flux density measurements for y and T_e to test the error on the recovered T_e estimate. By doing this for a range of temperatures and observing times, we can test how well temperature is constrained as a function of SNR in a reference band, chosen to be the ≈ 400 GHz band. The results are shown in figure 3; we conclude a reference SNR of 40 is sufficient for a < 1 keV temperature constraint in all cases, and a reference SNR of 20 is sufficient for a < 1 keV temperature constraint if there is no correlated dust component, or < 2 keV temperature constraint if there is a spatially correlated dust component which must be separated spectrally, as detected by, eg [4].

Given these results, we next predict what observing time will be needed to obtain the required SNR for a global temperature estimate, using a simple physically motivated model to create signal maps as in Section 2. Based on the signal maps, we estimate the cylindrically-integrated signal within r_{200} in the reference band and use the sensitivity calculator to estimate the observing time required to reach the required SNR of 20 or 40 for a given cluster mass and redshift. These results are shown in figure 4. We see that with modest amounts of observing time, rSZ constraints should be possible for all low-redshift ($z < 0.1$) clusters, and clusters with $M_{200} \gtrsim 3 \times 10^{14} M_\odot$ across the whole redshift range.

Given AtLAST's angular resolution, we would also like to test whether resolved rSZ temperature profiles will be possible. To do this, we divide each cluster simulation into three annular bins inside θ_{200} and check the observing time required to reach an SNR of 20 in the reference band, with respect to the signal in each annulus. Two example results are shown in figure 5. For a high mass cluster of $M_{200} = 10^{15} M_\odot$ at $z = 0.2$, resolved

¹https://github.com/lucadimascolo/AtLAST_sensitivity_calculator.git

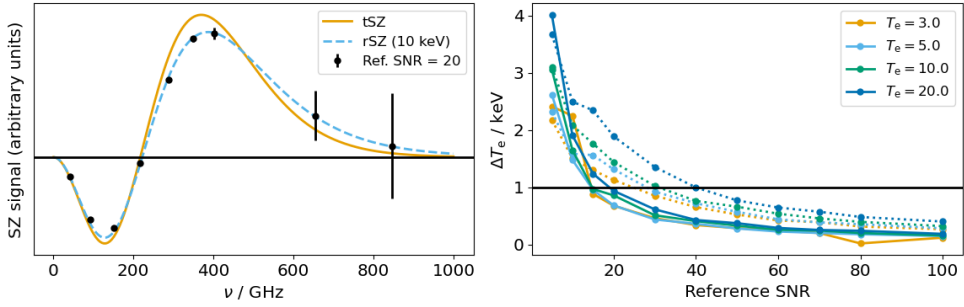


Figure 3. Left: simulated AtLAST observation of a $T_e = 10$ keV cluster, with SNR=20 in the reference band. Right: rSZ temperature 68% confidence limits as a function of reference band SNR for different input temperatures. Dotted (solid) lines show the case that there is (not) a correlated dust component. The horizontal black line shows a desirable constraint level of 1 keV, which would be reached by fitting to the spectrum on the left.

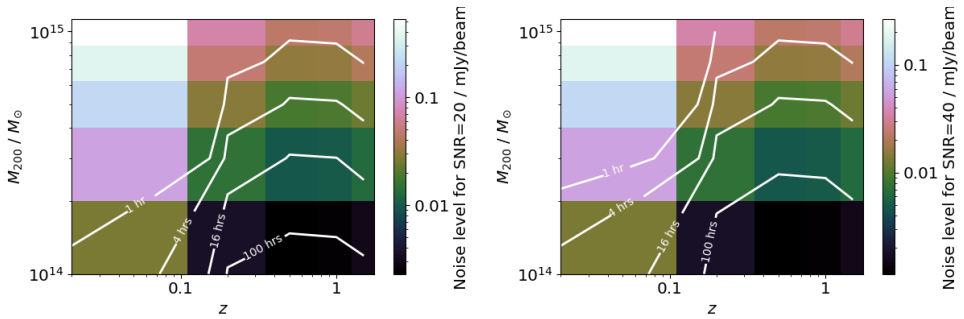


Figure 4. Noise levels, and corresponding predicted observing times, required to get to a reference band SNR = 20 (left) and 40 (right) with AtLAST.

temperatures are constrained within θ_{500} after 4 hours observing time. For a lower-mass cluster of $M_{200} = 5 \times 10^{14} M_{\odot}$ at $z = 1.5$, resolved temperatures are constrained within θ_{200} after 1 hour observing time. The improvement at high redshift (also seen in figure 4) is likely due to the angular diameter distance turnover.

4 Conclusion

With the precision and sensitivity of current and forthcoming instruments, the non-relativistic SZ spectrum is no longer an adequate approximation. With current instruments such as *Planck*, this mainly presents a problem in that Compton- y estimates are biased when it is neglected, but sensitivity limits (*Planck*) and/or lack of high-frequency channels (ACT, SPT) make it difficult to constrain. If external temperature estimates are used, for example from X-ray measurements or scaling relations calibrated using numerical simulations, the bias can be accounted for and removed. This is particularly necessary when comparing SZ scaling relations between different instruments and/or with numerical simulations.

Forthcoming instruments with higher sensitivity and the required high-frequency capability (AtLAST, FYST) will be able to do better. Simulations suggest that AtLAST will be

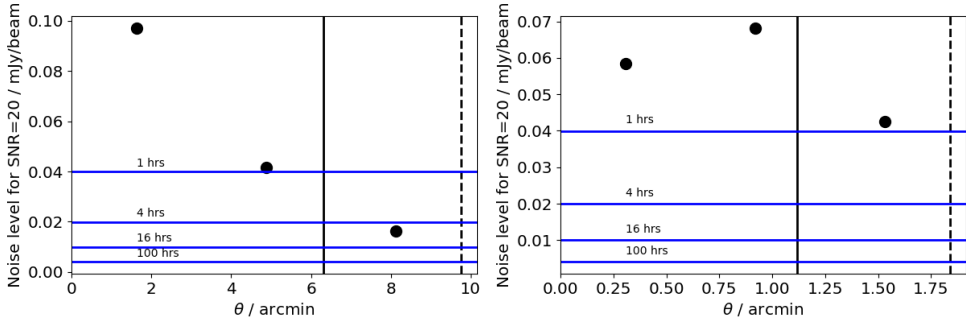


Figure 5. Noise levels required to get to a reference band SNR = 20 in angular bins for a cluster of $M_{200} = 10^{15} M_{\odot}$ at $z = 0.2$ (left) and $M_{200} = 5 \times 10^{14} M_{\odot}$ at $z = 1.5$ (right). Predicted observing times are shown with horizontal blue lines, and θ_{500} and θ_{200} with vertical black solid and dashed lines respectively.

able to measure global and resolved rSZ temperature estimates in reasonable amounts of observing time. For these instruments, the rSZ effect becomes an opportunity since it offers an alternative way to measure temperature that simulations suggest should be less biased than X-ray temperature measurements. This offers an exciting new avenue for probing cluster astrophysics in the future.

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