Revisiting prompt emission calculations for $^{252}\text{Cf(SF)}$ with focus on post-neutron fragment distributions and different correlations

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Abstract. The impact of energy partition in fission and the $Y(A,TKE)$ distribution of pre-neutron fragments on the independent FPY and different correlations between pre- and post-neutron fragment quantities is investigated in the frame of a deterministic approach of prompt emission in fission.

1 Short introduction

A part of prompt emission results for $^{252}\text{Cf(SF)}$ provided by both deterministic models PbP (Point-by-Point) and DSE (Deterministic Sequential Emission) by using the $Y(A,TKE)$ distributions of pre-neutron fragments available at that moments [1, 2] were already reported (e.g. of PbP in Ref.[3] and of both DSE and PbP in Refs.[4, 5]. In this investigation the DSE approach is used to keep track of post-neutron fragment distributions and different correlations between pre- and post-neutron fragment quantities and of how they are influenced by the energy partition in fission and the $Y(A,TKE)$ distribution.

2 DSE model calculations

The deterministic construction of the fragmentation and TKE ranges (which is usually employed in the PbP and DSE treatments) consists of a large pre-neutron fragment mass range with $A$ going from symmetric fission up to a very asymmetric split, with 5 charge numbers $Z$ per each $A$ taken as the nearest integers above and below the most probable charge $Z_p(A)=Z_{UCD}(A)+\Delta Z(A)$ and Gaussian isobaric charge distributions $p(Z,A)$ centred on $Z_p(A)$ with the charge polarization $\Delta Z(A)$ and rms$(A)$ of the Zp model. A large TKE range is taken for each fragmentation. By excluding the unphysical cases, the total number of $[A,Z,TKE]$ configurations taken in the present calculations was of about 17000-18000.

The DSE model is based on a detailed treatment of successive prompt neutron emission (sequence by sequence), its primary results consisting of matrices for numerous quantities characterizing the fragments and prompt emission (generically labelled $q_k(A,Z,TKE)$, with $k$ going from 1 to the number of emission sequences $n(A,Z,TKE)$), so that this approach is appropriate for the present investigation because the post-neutron fragment is well

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determined as the last residual fragment (at the end of prompt emission), its mass number \( A_p = A - n(A,Z,TKE) \) being an integer. This allows the calculation of independent fission product yields (FPY) and different distributions of the kinetic energy of post neutron fragments (KE\(_p\)), the investigation of different correlations between pre- and post-neutron fragment quantities and different distributions related to each emission sequence.

The distributions of post-neutron fragments (Y(Z,A), Y(A), KE\(_p\)(A,Z) etc.), the single distributions related to prompt emission (of prompt neutrons e.g. \( n(A)\), \( n(TKE)\), \( <\gamma>(A)\), \( \gamma\)-rays e.g. E\(_\gamma\)(A), E\(_\gamma\)(TKE), My(A), M\(_y\)(TKE)) and the total average values of prompt neutron and \( \gamma\)-ray quantities depend on the Y(A,TKE) distribution (which is an input data in a great part of prompt emission model codes, including DSE). Consequently the influence of Y(A,TKE) on the DSE results of prompt emission and post-neutron fragment distributions is investigated by considering three Y(A,TKE) data measured at JRC-Geel by Hambsch and Oberstedt [1], Göök et al. [2] and recently in the VESPA experiment [6]. Visible differences between these Y(A,TKE) data exist.

It is known that any prompt emission modelling is strongly dependent on the energy partition in fission (which is known as “TXE partition” even if the sharing of excitation energy takes place at scission or even before scission). The methods of TXE partition are classified into the following two categories (differing as principle)

1. methods based on modelling at scission and
2. methods based on modelling at acceleration of fragments, being based on different parameterizations. In some cases such parameterizations are obtained from the calculation based on a modelling at scission, in other cases they are empirical (being obtained by fitting \( n(A)\) etc.

Our method based on modelling at scission (see e.g. Ref.[3, 4, 5, 7] and references therein) consists of the calculation of the extra-deformation energy (as difference between the absolute deformation energy of a fragment at scission and at its full acceleration) and the sharing of the available excitation energy at scission under the assumptions of statistical equilibrium at scission and fragment level densities in the Fermi-gas regime. Different prescriptions for the level density parameters of fragments at scission and full acceleration can be used. A result of this method is exemplified in the left part of Fig.1 for the case of non-energy dependent level density parameters (provided by the Egidy-Bucurescu systematic for BSFG, see Refs.[4, 7]) and the three Y(A,TKE) distributions [1, 2, 6].

**Fig.1:** left part \( E^*(A)\) results of the TXE partition based on modeling at scission, right part \( R(T)\) obtained from the TXE partition based on modeling at scission using the level density parameter prescriptions of the super-fluid model (red symbols) and the systematic of Egidy-Bucurescu (blue symbols) together with their parameterizations by jointed segments (red and blue lines).
In contradistinction to the previous investigated case of $^{235}\text{U}(n_{th},f)$ [7] when the use of an unique $R_T$ value for all fragmentations (i.e. the total average $<R_T>$=1.2 obtained for different prescriptions of level density parameters used in the frame of our method based on modelling at scission) is working well (i.e. prompt emission results describing well the experimental data), in the case of $^{252}\text{Cf}(SF)$ this is not happening. So that $R_T$ was taken as a function of $A$, by using a parameterization (by a few jointed segments) which approximates well the shape of $R_T(A_{HI})$ resulting from modelling at scission with the super-fluid model prescription for the level density parameters of fragments at scission and full acceleration. This is shown in the right part of Fig.1 by the red line approximating the calculated $R_T(A_{HI})$ given with red symbols. Others $R_T(A)$ results (obtained with another prescription for the level density parameter) are plotted with different blue symbols and the corresponding parameterization with a dashed blue line.

3 Results and discussions

DSE results for $^{252}\text{Cf}(SF)$ being already reported (e.g. Refs.[4, 5]), only the prompt neutron multiplicity $\nu(A)$ is given as example, because it plays the most important role in the calculation of independent FPY. See the upper left part of Fig.2 where the $\nu(A)$ results (obtained with $Y(A,TKE)$ of Refs.[1, 2, 6]) describe well the experimental data (full and open black symbols), differences between these $\nu(A)$ results are visible only at $A$ of about 130 and for far asymmetric fragmentations. Another example refers to the DSE results of prompt $\gamma$-ray multiplicity $M_\gamma(A)$ (obtained with $Y(A,TKE)$ of Refs.[2, 6]) given in the upper right part of Fig.2 (with orange and green symbols, respectively), which describe reasonably well the recent prompt $\gamma$-ray data of the VESPA experiment [9] (black symbols).

Fig.2 Upper part: DSE results of $\nu(A)$ (left frame) and $M_\gamma(A)$ (right frame) in comparison with the experimental data (different black and gray symbols). Lower part: the multiplicity ratios as a function of $A_{HI}$; that referring to prompt neutrons $\nu_H/(\nu_L+\nu_H)$ (experimental data with different open and full black symbols and a DSE result with red circles) and that referring to prompt $\gamma$-rays $M_\gamma^H/(M_\gamma^L+M_\gamma^H)$ (experimental data of VESPA with violet diamonds and DSE results with green and orange symbols).
It is interesting to note that both multiplicity ratios as a function of $A_H$ (i.e. of prompt neutrons $\nu_H/(\nu_H+\nu_L)$ and of prompt $\gamma$-rays $M_{\gamma_H}/(M_{\gamma_H}+M_{\gamma_L})$) exhibit the same behaviour: see the lower part of Fig.2 showing the multiplicity ratios of prompt neutrons (experimental data with black and gray symbols and the DSE result with red circles) and prompt $\gamma$-rays (experimental data of VESPA [9] with violet diamonds and DSE results with orange and green symbols).

The DSE results of independent FPY are given in Fig.3: the $Y(A_p)$ results obtained with $Y(A,TKE)$ of Refs.[1, 2, 6] (green, orange and red symbols respectively, connected with lines to guide the eye) by using the modelling at scission and $R_T(A_H)$ are given in the left upper and lower frames, respectively. They describe well the experimental data (different coloured symbols). The $Y(A_p)$ structure seems to be less pronounced than in the case of $^{235}$U(n$_{th}$,f) [7]. The use of three $Y(A,TKE)$ and two TXE partitions leads to changes in the $Y(A_p)$ structure of both the position and the magnitude of peaks and dips. Nevertheless the position of most pronounced peaks (e.g. at $A_p = 142$, 138, 104, 108) and dips (e.g. at $A_p = 141$, 126, 107, 105) does not change. The influence of TXE partition is also visible in the $Y(Z,A_p)$ results (given in the right upper part with full and open small circles connected with lines to guide the eye), which describe well the experimental data found in EXFOR.

By investigating the charge and mass distributions of pre-neutron fragments which lead to $A_p$ values where visible peaks and dips appear in the $Y(A_p)$ structure (using the same procedure as in Ref.[7]) we have observed that pronounced peaks at e.g. $A_p = 142$, 138, 108, 104 are due to high maxima of post-neutron fragment yields of $^{56}$Ba, $^{54}$Xe, $^{44}$Ru, $^{42}$Mo (i.e. even Z), while the yields of e.g. $^{55}$Cs and $^{43}$Tc (odd-Z) contribute to dips at $A_p = 141$, 139 and 107. Consequently the even-odd effect in fragment charge plays a role in this case, too. But this role is less pronounced than in the previous investigated case of $^{235}$U(n$_{th}$,f) [7], because the global even-odd effect in $Y(Z)$ is 10 times lower in the case of $^{252}$Cl(SF) compared to $^{235}$U(n$_{th}$,f).
Looking at the example of isotonic yield \(Y_{N_p}\) given with red symbols in the lower right part of Fig.3, it can be seen that pronounced peaks at e.g. \(N_p = 82, 84, 86, 64, 62\) correspond to the pronounced peaks in the \(Y(A_p)\) structure at e.g. \(A_p = 138, 140, 142, 108, 106, 104\).

The nice correlation between the excitation energy \(E^*\) of fully accelerated pre-neutron fragments and the kinetic energy \(K_{E_p}\) of post-neutron fragments, ascertained in the case of \(^{235}\text{U}(\text{n}_{th},\text{f})\) [10], is maintained in the case of \(^{252}\text{Cf}(\text{SF})\), too, see Fig.4. It can be observed that \(E^*(K_{E_p})\) exhibits a well delineated sawtooth shape which looks as a mirror reflection of the well-known sawtooth shape of \(\nu(A)\) and \(E^*(A)\). The influence on \(E^*(K_{E_p})\) of \(Y(A,TKE)\) is illustrated in the left part of Fig.4 and that of \(\text{TXE} \) partition in the right part.

![Fig.4](image1.png)

**Fig.4**: \(E^*\) of fully accelerated pre-neutron fragments as a function of \(K_{E_p}\) of post-neutron fragments. The influence on \(E^*(K_{E_p})\) of \(Y(A,TKE)\) is illustrated in the left part and that of \(\text{TXE}\) partition in the right part.

The pre-neutron fragment distributions of the excitation energy at full acceleration and of mass, charge and TKE, which lead to each number “n” of emission sequences \((Y_d(E^*), Y_d(A), Y_d(Z), Y_d(TKE))\) are illustrated in Fig.5.

![Fig.5](image2.png)

**Fig.5**: \(Y_d(E^*), Y_d(A), Y_d(Z)\) and \(Y_d(TKE)\) exemplified for the \(\text{TXE}\) partition based on modeling at scission and the \(Y(A,TKE)\) distributions of Refs.[1, 6]
The first moments of these distributions have shown almost linear increases of the average excitation energies \(<E^*_{L,H}>_n\) and slight increases of average fragment masses \(<A_{L,H}>_n\) with increasing number of sequences, as well as an expected linear decrease of \(<\text{TKE}>_n\) with increasing number of sequences (because lower TKE values mean higher TXE values, allowing the emission of more neutrons). The average fragment charges \(<Z_{L,H}>_n\) remain almost constant with increasing number of sequences, being close to the values of 42 (Mo) and 56 (Ba) of the most probable charge fragmentation. Looking at Fig.5 it can be seen that the highest \(Y_n(E^*), Y_n(A), Y_n(Z)\), and \(Y_n(\text{TKE})\) are those corresponding to \(n = 2, 3, 4\), confirming again the values of total average prompt neutron multiplicity \(<\nu>\) between 3.7 and 3.8, on which the experimental \(P(\nu)\) distributions are centred.

The neutron excesses of pre- and post-neutron fragments of \(^{252}\text{Cf(SF)}\) are investigated in comparison with those of \(^{235}\text{U}\). The average neutron excesses \(<N/Z>\) and \(<N_p/Z>\) as a function of \(Z\) and of \(A\) (obtained by averaging \(N/Z\) and \(N_p/Z\) corresponding to \((A,Z,\text{TKE})\) configurations over \(Y(A,Z,\text{TKE})\)) are illustrated in Fig.6 for \(^{235}\text{U}\) in the left part and for \(^{252}\text{Cf(SF)}\) in the right part. \(<N/Z>\) are plotted with full symbols and \(<N_p/Z>\) with open symbols. The use of different \((A,\text{TKE})\) distributions is indicated by different coloured symbols. The neutron excesses corresponding to the magic shells \(N=50\) and 82 are plotted with gray full and open circles in the lower frames. \(N/Z\) of the fissioning nucleus is indicated by a horizontal black solid line.

Fig.6: Average neutron excesses of pre- and post-neutron fragments as a function of \(Z\) (upper part) and as a function of \(A\) (lower part) for \(^{235}\text{U}\) ((left frames) and \(^{252}\text{Cf(SF)}\) (right frames).

The most interesting behaviour consists of oscillations with a periodicity of about 5 mass units exhibited by both neutron excesses of pre- and post-neutron fragments as a function of \(A\), which are clearly visible in the case of \(^{235}\text{U}\) and almost invisible for \(^{252}\text{Cf(SF)}\). They are due to the charge polarization \(\Delta Z(A)\) (entering the most probable charge \(Z_p(A)\) on which the construction of fragmentation range is based). Both \(\Delta Z(A)\) and \(\text{rms}(A)\) (of the isobaric charge distribution) always oscillates with a periodicity of about 5 mass units due the periodicity of nuclear properties [11]. Only the magnitude of these oscillation amplitudes reflect the magnitude of the even-odd effect in charge distribution (for details
see Ref.[11]). This explains why the oscillations of both neutron excesses of pre- and post neutron fragments as a function of A are clearly visible in the case of $^{235}$U(n_{th},f) only.

4 Conclusions

The prompt emission results of the DSE model are validated by their good description of experimental data. The independent FPY results, $Y(Z,A_p)$ and $Y(A_p)$, are obtained in good agreement with the experimental data, too.

In the present investigation of $^{252}$Cf(SF) the influence of $Y(A,TKE)$ on independent FPY is more pronounced than in the previous investigated case of $^{235}$U(n_{th},f) [7] leading to visible differences in the magnitude of pronounced peaks and dips of the $Y(A_p)$ structure and also in the position of some less pronounced peaks. Again the pronounced peaks are due to even-Z fragments and the pronounced dips to odd-Z fragments. But the role of the even-odd effect in fragment charge is less pronounced than in the previous studied case of Ref.[7] because the global even-odd effect in the fragment charge yield is almost 10 times lower in the case of $^{235}$U(n_{th},f). The correlation between the excitation energy $E*$ of fully accelerated pre-neutron fragments and the kinetic energy $KE_p$ of post-neutron fragments, ascertained in the case of $^{235}$U(n_{th},f) [10], is maintained in the case of $^{252}$Cf(SF), too.

The highest $Y_n(E*)$, $Y_n(A)$, $Y_n(Z)$, $Y_n(TKE)$ distributions are those corresponding to the number of emission sequences of 2, 3 and 4, confirming again the experimental values between 3.7 and 3.8 of the total average prompt neutron multiplicity. The first moments of $Y_n(E*)$ have shown that for the emission of the same number of prompt neutrons the light fragments need more excitation energy than the heavy ones.

The oscillations with a periodicity of about 5 mass units exhibited by the neutron excesses of pre- and post neutron fragments as a function of A are due to the charge polarization $\Delta Z(A)$ which exhibit similar oscillations [11]. Consequently, the magnitude of the even-odd effect in fragment charge is reflected not only in the magnitude of oscillation amplitudes of the charge polarization $\Delta Z(A)$ but also in the magnitude of oscillation amplitudes of the neutron excesses of pre- and post-neutron fragments as a function of A.

References