

# Charge-dependent anisotropic flow in relativistic resistive magneto-hydrodynamic expansion

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**Abstract.** We construct a dynamical model for high-energy heavy-ion collisions based on the relativistic resistive magneto-hydrodynamic framework. Using the code, we investigate the charge-dependent anisotropic flow in  $\sqrt{s_{NN}} = 200$  GeV Au+Au collisions and Cu+Au collisions at RHIC. We conclude that the charge-dependent anisotropic flow is a good probe to extract the electrical conductivity of QGP medium.

## 1 Introduction

In high-energy heavy-ion collisions, the production of ultraintense electromagnetic fields by two colliding nuclei is one of the hottest topics. For example, in  $\sqrt{s_{NN}} = 200$  GeV Au-Au collisions at the BNL Relativistic Heavy Ion Collider (RHIC), the highest intensity of the magnetic field in our universe may be reached, e.g.,  $|eB| \sim 10^{15}$  T. The intensity of the magnetic field in the transverse plane increases approximately linearly with the center of mass collision energy. The corresponding electric field in the transverse plane is also enhanced by a Lorentz factor of colliding nuclei. Such intense electromagnetic fields can affect the hadron distribution detected in high-energy heavy-ion collision experiments at RHIC and the CERN Large Hadron Collider (LHC). As the electromagnetic response of the quark-gluon plasma (QGP), electromagnetic fields alter the electric charge of quarks. As a consequence of it, the charge dependence is found in directed flows of hadrons at RHIC and the LHC [1–3].

To describe time evolution of the initial electromagnetic fields in dynamics of high-energy heavy-ion collisions, we construct the relativistic resistive magneto-hydrodynamics (RRMHD) [4]. Then we investigate the effect of electromagnetic fields on the charge-dependent anisotropic flow based on RRMHD [5, 6].

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## 2 Relativistic Resistive Magneto-Hydrodynamics (RRMHD)

The RRMHD equation consists of the conservation laws for the charged current  $N^\mu$  and for the total energy-momentum tensor of the plasma  $T^{\mu\nu}$  in the dynamics of the whole system,

$$\nabla_\mu N^\mu = 0, \quad (1)$$

$$\nabla_\mu T^{\mu\nu} = 0, \quad (2)$$

where  $\nabla_\mu$  is the covariant derivative. The electromagnetic fields satisfy Maxwell equations,

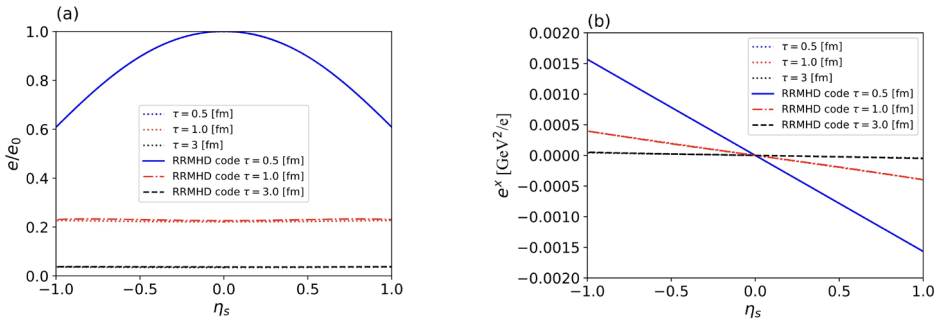
$$\nabla_\mu F^{\mu\nu} = J^\nu. \quad (3)$$

To close the equations, we employ the simplest form of Ohm's law in Ref. [7],

$$J^\mu = \sigma F^{\mu\nu} u_\nu + q u^\mu, \quad (4)$$

where  $\sigma$  is electrical conductivity,  $u^\mu$  is the four-velocity, and  $q = -J^\mu \mu_\mu$  is electric charge density of the fluid in the comoving frame.

We construct a RRMHD numerical simulation code for high-energy heavy-ion collisions as a first designed code in the Milne coordinates [4]. Figures 1 (a) and (b) show the numerical results in comparison with the semi-analytic solution. Our results are in good agreement with the semi-analytic solutions. In Fig. 1 (a), the energy density decays and expands to the longitudinal direction by the resistive effects. Figure 1 (b) shows the electric field component measured in the coming frame  $e^x$  as a function of  $\eta_s$ . The electric field has a positive value in the backward rapidity region and it decreases with rapidity. The electric field changes its sign at  $\eta_s = 0$ . This feature describes that the electric field is produced by the two colliding nuclei in high-energy heavy-ion collisions. Our results capture these features and its diffusion which is consistent with the semi-analytic solutions.



**Figure 1.** (a) The ratio of the energy density of the fluid to the initial energy density and (b) the electric field component measured in the comoving frame. The blue solid, red long dashed-dotted and black dashed lines show the numerical results at  $\tau = 0.5, 1.0,$  and  $3$  fm, respectively. The blue, red, and black dotted lines show the semi-analytic solutions.

## 3 Charge Dependent Flow

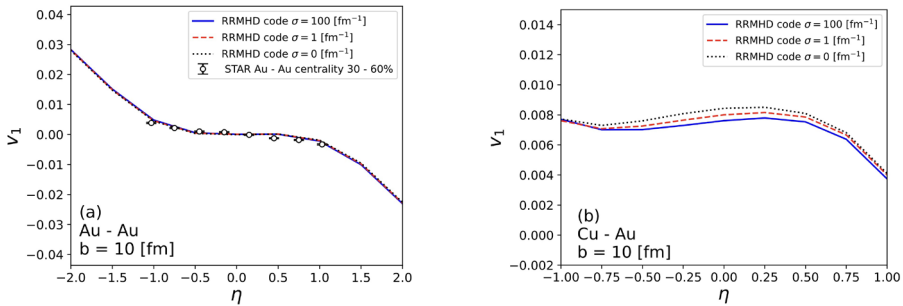
We apply the newly developed RRMHD simulation code to analysis of Au+Au and Cu+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV. The initial conditions of the medium are built up with the

optical Glauber model. The parameters for the initial condition of our model are chosen based on the ECHO-QGP simulation [8]. Also, we take the solution of the Maxwell equations as the initial condition of electromagnetic fields [9]. We consider the system in which the electric charge is moving along parallel to the beam axis in the laboratory frame.

We investigate the effect of electromagnetic fields on the directed flow of hadrons,

$$v_1(\eta) = \frac{\int dp_T d\phi \cos(\phi) \frac{dN}{dp_T d\phi}}{\int dp_T d\phi \frac{dN}{dp_T d\phi}}. \quad (5)$$

We terminate the hydrodynamic expansion at  $e = 0.15 \text{ GeV/fm}^3$ . To extract the purely hydrodynamic response of electromagnetic fields, we neglect the final state interactions. Figure 2 shows the directed flow for the charged  $\pi$  in Au-Au and Cu-Au collisions. The blue solid, red dashed, and black dotted lines show the cases of  $\sigma = 100, 1, \text{ and } 0 \text{ fm}^{-1}$ , respectively. In Fig. 2 (a), our results of the directed flow in Au-Au collisions are consistent with the STAR data in 30-60 % centrality class [1]. The electrical conductivity dependence of the directed flow is not clearly observed. We show the directed flow for charged  $\pi$  in Cu-Au collisions in Fig. 2 (b). The directed flow of our RRMHD simulation exhibits the clear dependence of the electrical conductivity of the QGP. The amplitude of the directed flow decreases with the electrical conductivity.

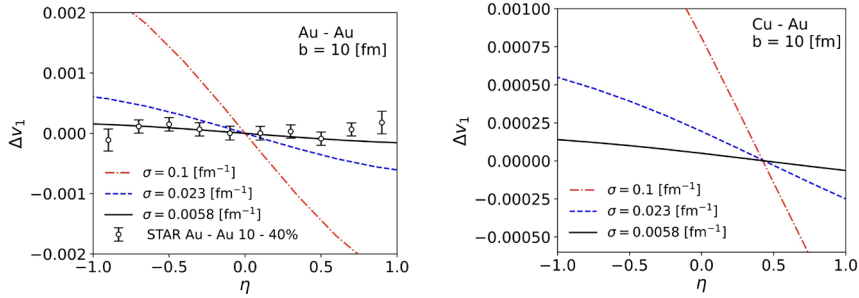


**Figure 2.** The directed flow as a function of rapidity for different electrical conductivities  $\sigma$ . The blue solid, red dashed, and black dotted lines show  $\sigma = 100, 1, \text{ and } 0 \text{ fm}^{-1}$ . We display the cases of Au-Au collisions (a) and Cu-Au collisions (b) at  $\sqrt{s_{NN}} = 200 \text{ GeV}$ .

Next we show the charge-odd contribution to the directed flow for  $\pi$ ,

$$\Delta v_1(\eta) = v_1^{\pi^+}(\eta) - v_1^{\pi^-}(\eta). \quad (6)$$

Figure 3 (a) displays the charge-odd contribution to the directed flow of  $\pi$  in Au-Au collisions. The electrical conductivity dependence is clearly observed in the forward and backward rapidity regions. The charge-odd contribution to the directed flow is approximately proportional to electrical conductivity. In the lowest conductive medium with  $\sigma = 0.0058 \text{ fm}^{-1}$ , our result is consistent with the STAR data [10] within the error bar. The charge-odd contribution to the directed flow in Cu+Au is shown in Fig. 3 (b). It has the nonzero value at  $\eta = 0$  in finite electrical conductivity case. Our result indicates that the precise measurement of the charge-odd contribution to the directed flow is appropriate for the determination of the value of the electrical conductivity of the QGP. This measurement sheds light on the electromagnetic response of the QGP medium.



**Figure 3.** The charge-odd contribution to the directed flow as a function of  $\eta$  in Au+Au (left panel) and Cu-Au (right panel) collisions. The black solid, blue dashed, and red long dashed-dotted lines represent the charge-odd contribution to the directed flow in the cases of  $\sigma = 0.0058, 0.023,$  and  $0.1 \text{ fm}^{-1}$ .

## 4 Summary

We have constructed a RRMHD numerical simulation code for high-energy heavy-ion collisions as a first designed code in the Milne coordinates. Utilizing our newly developed RRMHD model, we have investigated the charge-dependent anisotropic flow in  $\sqrt{s_{NN}} = 200$  GeV Au+Au collisions and Cu+Au collisions at RHIC. We conclude that the charge-dependent anisotropic flow is a good probe to extract the electrical conductivity of QGP medium.

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