QCD equation of state with improved precision from lattice simulations

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Abstract. The equation of state of Quantum Chromodynamics has been in recent years the focus of intense effort from first principle methods, mostly lattice simulations, with particular interest to the finite baryon density regime. Because of the sign problem, various extrapolation methods have been used to reconstruct bulk properties of the theory up to as far as $\mu_B/T \simeq 3.5$. However, said efforts rely on the equation of state at vanishing baryon density as an integration constant, which up to $\mu_B/T \simeq 2 - 2.5$ proves to be the dominant source of uncertainty at the level of precision currently available. In this contribution we present the update of our equation of state at zero net baryon density from 2014, performing a continuum limit from lattices with $N_\tau = 8, 10, 12, 16$. We show how the improved precision is translated in a lower uncertainty on the extrapolated equation of state at finite chemical potential.

1 Introduction

The equation of state of Quantum Chromodynamics (QCD) is a quantity of crucial importance, both for its fundamental interest, as well as for its use in the modeling of heavy-ion collisions. At vanishing baryon density, the equation of state - pressure, baryon density, entropy density, energy density, speed of sound - is now known for about a decade thanks to continuum extrapolated lattice simulations at physical quark masses, with results from different collaborations showing very good quantitative agreement [1, 2].

With the advent of the beam energy scan program at RHIC, and the general aim to explore the QCD phase diagram at different densities, the attention has largely moved to the finite-density regime. Because of the fermion sign problem, direct simulations at finite baryon density cannot be easily performed, although significant progress has been made recently

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in this regard. A number of extrapolation methods have been employed in recent years to determine the equation of state at finite chemical potential, exploiting simulations carried out in the accessible parameter space. Taylor expansion allows for a reconstruction of finite-density thermodynamics up to $\mu_B/T \simeq 2-2.5$ [3], while with a novel expansion scheme based on imaginary chemical potential simulations, a value of $\mu_B/T \simeq 3.5$ was reached [4, 5]. Moreover, new advances in reweighting techniques have produced the first direct results at finite baryon density, although in a small volume [6–8]. Independently of the method employed to access thermodynamics at finite baryon chemical potential, the equation of state at $\mu_B = 0$ is required as an integration constant.

In this contribution, we present new continuum extrapolated results for the equation of state at $\mu_B = 0$ with significantly improved precision, and show how that is translated into better precision at finite density, too.

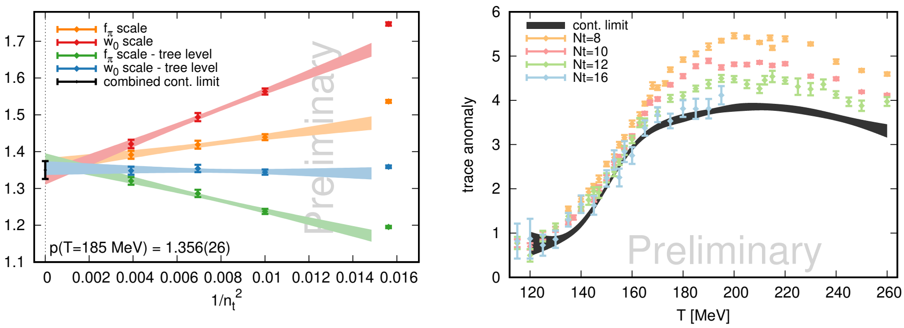

Figure 1. Left: continuum extrapolation of the integration constant in Eq. (1). Right: continuum extrapolation of the trace anomaly in the temperature range $T = 120 - 260$ MeV.

2 Results

The pressure cannot be directly determined via lattice simulations, so it is customary to obtain it as an integral over the temperature of the trace anomaly $I(T)$:

$$\frac{p(T)}{T^4} = \frac{p(T_0)}{T_0^4} + \int_{T_0}^{T} \frac{dT'}{T'} \frac{I(T')}{T'^4} \tag{1}$$

where $p(T_0)$ is an integration constant, namely the pressure at a temperature T_0 chosen at will. The trace anomaly can instead be determined directly on the lattice via:

$$\frac{I(T)}{T^4} \frac{dT}{T} = N_\tau^4 \left(d\beta \langle -s_G \rangle_R + \sum_f dm_f \left\langle \bar{\psi}_f \psi_f \right\rangle_R \right) \tag{2}$$

where the sum in the second term runs over the quark flavors, $\beta = 6/g^2$ is the gauge coupling and m_f are the fermion masses, while $\langle -s_G \rangle_R$ and $\left\langle \bar{\psi}_f \psi_f \right\rangle_R$ are the renormalized gauge action and chiral condensates.

As already mentioned, it is not possible to directly determine the pressure from lattice simulations for a given choice of parameters. Hence, in order to determine the integration constant in Eq. (1), it is necessary to calculate an integral in some parameter, starting from a point where the pressure has a known value. We choose to calculate the pressure at $T_0 = 185$ MeV, and we obtain it via an integral of the chiral condensates, integrating down in the

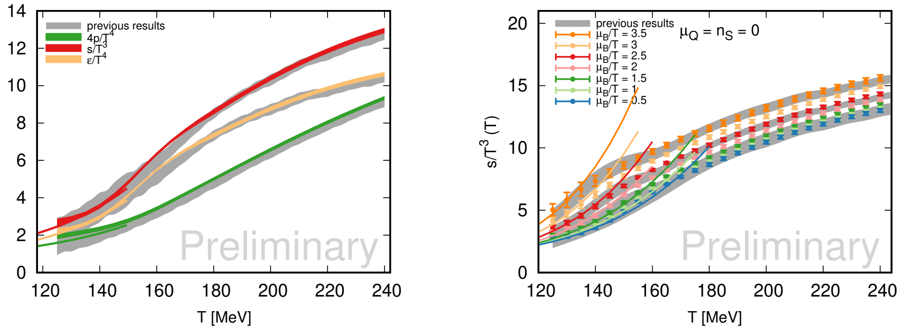

Figure 2. Left: Equation of state at $\mu_B = 0$ (colored bands), compared to our old results (grey bands). Right: entropy density at strangeness neutrality, calculated from the alternative expansion scheme at different values of μ_B/T, with our new results at $\mu_B = 0$. In both panels, hadron resonance gas model results are shown as solid lines.

quark mass(es) from infinity, where the pressure is equal to 0:

$$\frac{p(T_0)}{T_0^4} = \int_{\infty}^{m_s} dm_3 \, \langle \bar{\psi}\psi \rangle_{R,3} (m_3) + \int_{m_s}^{m_l} dm_2 \, \langle \bar{\psi}\psi \rangle_{R,2} (m_2) \; . \tag{3}$$

The first term is an integral of the 3-flavour theory, whereby the quark mass runs from infinity down to the strange quark mass. In the second term, the strange quark mass is fixed to its physical value, and the light quark masses are integrated down to their physical (degenerate) mass. The left panel of Fig. 1 shows our continuum extrapolation of this integration constant, obtained from lattices with $N_\tau = 10, 12, 16$. Results at $N_\tau = 8$ are shown, but not included in the continuum fit. In order to estimate the systematic uncertainties, two ways of setting the scale were considered (with f_π and w_0 [9]), and we considered both the case with and without tree level correction of the pressure. The final result $p(T = 185\,\mathrm{MeV})/T^4 = 1.356(26)$ has roughly a factor 2x improvement in precision over our result from 2014 [1]. In the right panel of Fig. 1, we show the trace anomaly at finite N_τ (colored points), together with our continuum extrapolation (black band). This is obtained via a combined fit including a linear dependence on $1/N_\tau^2$ and a spline in the temperature. This result also has roughly a factor 2-3x improvement in precision over our previous determination.

The continuum extrapolated results shown in both panels of Fig. 1 are the necessary ingredients to completely determine the thermodynamics at $\mu_B = 0$. Through Eq. (1), the pressure can be obtained in the available range $T = 120 - 260\,\mathrm{MeV}$, and from it other thermodynamic quantities follow from standard identities. The results are shown in the left panel of Fig. 2 (colored bands), where they are compared with our previous results (grey bands). The improvement in precision is dramatic. We note that, at low temperature, our result for the pressure shows a slight tension with the prediction of the hadron resonance gas model, shown as a solid line. However, a full analysis of the systematic uncertainties is in progress, and additional statistics will be collected in this regime on our $N_\tau = 16$ lattice.

At the existing level of precision the equation of state at $\mu_B = 0$ was the dominant source of uncertainty in the equation of state at finite chemical potential, up to $\mu_B/T \simeq 2.5$. In order to observe the improvement at finite density, we show in the right panel of Fig.2 the entropy density at increasing values of μ_B/T, obtained with our alternative expansion scheme, and using the new $\mu_B = 0$ equation of state (colored points). The corresponding results obtained with our previous $\mu_B = 0$ equation of state are shown as grey bands for comparison. The new determination shows much improved uncertainties except for largest value of the chemical potential, namely $\mu_B/T = 3.5$, where the extrapolation errors remain dominant.

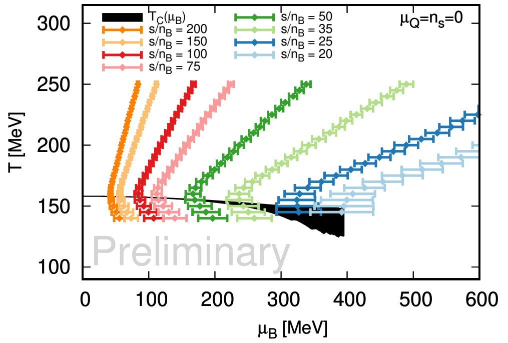

Figure 3. Isentropic lines in the density range covered by the RHIC beam energy scan, in the case of strangeness neutrality (colored points). The QCD transition line is shown as a black band [10].

The improved precision achieved over the whole range accessible to our alternative expansion, allows us to draw a more precise than ever picture of the phase diagram, at least in the regime accessible to the RHIC beam energy scan. In Fig. 3 we show isentropic trajectories for different values of s/n_B, obtained with our new results. The transition line is also shown, as a black band. With the precision achieved, no trace of critical lensing – a focusing effect on isentropic lines towards a critical point – is present.

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References

[1] S. Borsanyi, Z. Fodor, C. Hoelbling, S.D. Katz, S. Krieg, K.K. Szabo, Phys. Lett. B **730**, 99 (2014), `1309.5258`

[2] A. Bazavov et al. (HotQCD), Phys. Rev. D **90**, 094503 (2014), `1407.6387`

[3] D. Bollweg, D.A. Clarke, J. Goswami, O. Kaczmarek, F. Karsch, S. Mukherjee, P. Petreczky, C. Schmidt, S. Sharma (HotQCD), Phys. Rev. D **108**, 014510 (2023), `2212.09043`

[4] S. Borsányi, Z. Fodor, J.N. Guenther, R. Kara, S.D. Katz, P. Parotto, A. Pásztor, C. Ratti, K.K. Szabó, Phys. Rev. Lett. **126**, 232001 (2021), `2102.06660`

[5] S. Borsanyi, J.N. Guenther, R. Kara, Z. Fodor, P. Parotto, A. Pasztor, C. Ratti, K.K. Szabo, Phys. Rev. D **105**, 114504 (2022), `2202.05574`

[6] S. Borsanyi, Z. Fodor, M. Giordano, S.D. Katz, D. Nogradi, A. Pasztor, C.H. Wong (2021), `2108.09213`

[7] M. Giordano, K. Kapas, S.D. Katz, D. Nogradi, A. Pasztor, JHEP **05**, 088 (2020), `2004.10800`

[8] S. Borsanyi, Z. Fodor, M. Giordano, J.N. Guenther, S.D. Katz, A. Pasztor, C.H. Wong, Phys. Rev. D **107**, L091503 (2023), `2208.05398`

[9] S. Borsanyi et al. (BMW), JHEP **09**, 010 (2012), `1203.4469`

[10] S. Borsanyi, Z. Fodor, J.N. Guenther, R. Kara, S.D. Katz, P. Parotto, A. Pasztor, C. Ratti, K.K. Szabo, Phys. Rev. Lett. **125**, 052001 (2020), `2002.02821`