Global angular momentum generation in heavy-ion reactions within a hadronic transport approach

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Abstract. In this study, we utilize the SMASH transport model to explore the generation of global angular momentum in heavy-ion collisions. Our results corroborate previous models, highlighting a pronounced peak in angular momentum transfer during mid-central collisions at \( b = 4 - 6 \) fm across \( \sqrt{s_{NN}} = 2.41 - 200 \) GeV. Additionally, we thoroughly investigate the impact of system size and centrality on angular momentum. Intriguingly, we observe a distinct trend towards higher relative angular momentum transfer in smaller systems and more central collisions. To address local angular momentum conservation intricacies, we propose tailored setups for varying energy regimes, relying on the test particle method and Fermi motion treatment.

1 Introduction

Exploring heavy-ion collisions across varied energies provides unique insights into the QCD phase diagram. A breakthrough arose from the observation of \( \Lambda \) hyperon polarization at lower energies by the STAR collaboration [1], unveiling a vortical structure indicative of the system’s dynamic response to non-zero impact parameters [2, 3]. Collisions with non-zero impact parameters lead to a system with significant orbital angular momentum, evolving into smaller rotational domains characterized by vorticity. Employing the hadronic transport approach SMASH [4, 5], this study comprehensively investigates the dynamic deposition of angular momentum across a broad energy spectrum, providing insights into the dependence of angular momentum on the size of the colliding system and impact parameter. Additionally, it encompasses studies of microscopic angular momentum conservation and its dynamics over time. These proceedings are based on [6], to which we refer for a more detailed discussion.

2 Setup

In this investigation of angular momentum transfer in heavy-ion collisions, we employ the hadronic transport approach SMASH (version 2.0) [4, 5]. This model has demonstrated its efficacy across a multitude of scenarios, successfully capturing particle production dynamics.
The approach provides an effective solution of the relativistic Boltzmann equation, offering the complete phase-space evolution for each particle [6]. As degrees of freedom, SMASH incorporates most of the established hadrons as well as hadronic resonances from the Particle Data Group, reaching up to a mass $m = 2.35$ GeV. High-energy processes like hard scattering and string fragmentation, on the other hand, are realized by employing the string model Pythia 8 [8]. The collision dynamics in SMASH are governed by a geometric collision criterion

$$d_T < d_{int} = \sqrt{\frac{\sigma}{\pi}},$$

which determines whether a collision occurs, based on the transverse distance of closest approach $d_T$ between two particles. Nevertheless, a geometric collision criterion also embodies an immediate finite-range interaction, breaking Poincaré invariance and thus angular momentum conservation locally. This can be mitigated by the test particle method under which particle number and cross sections are rescaled, following $N \rightarrow N_{\text{test}}$ and $\sigma \rightarrow N_{\text{test}}^{-1}\sigma$.

In coordinate space, nucleon positions follow a Woods-Saxon distribution [9], while the momentum-space ground state gives rise to the so-called Fermi motion important at low beam energies. In this study, we employ the frozen Fermi approximation, wherein Fermi momenta are disregarded during propagation and exclusively taken into account during collision. The relevant quantities to determine the evolution of angular momentum $\vec{L} = \vec{r} \times \vec{p}$ are the initial angular momentum $\vec{L}_0$ of all nucleons at time $t < 0$ fm, and the final angular momentum $\vec{L}_f$ at a time when all secondary collisions have ceased. This analysis differentiates between remaining angular momentum $\vec{L}_r$ (participants) in the interaction medium and spectators’ contribution $\vec{L}_{sp}$, such that $\vec{L}_{tot} = \vec{L}_r + \vec{L}_{sp}$. In a heavy-ion collision angular momentum generation is predominantly driven by the geometry if the collision setup, shaped by the chosen impact parameter along the x-axis. As we choose nuclei propagation along the z-axis, the only non-zero component of the angular momentum is its y component.

### 3 Results

In this section, we examine the angular momentum dynamics of non-central AuAu collisions across an energy range from $\sqrt{s_{NN}} = 2.41$ GeV to $\sqrt{s_{NN}} = 200$ GeV, where we assume Fermi motion to be turned off as default, unless specified otherwise. Fig. 1 reveals the impact parameter dependence of participants’ angular momentum for three different beam energies. The solid lines illustrate our results without test particles and frozen Fermi motion (upper) or deactivated Fermi motion (lower), respectively. Additional to deactivated Fermi motion, the dashed curve also includes 20 test particles per simulated nucleon. Overall, these curves exhibit a consistent qualitative pattern across various energies, each characterized by a distinct impact parameter $b_{\text{max}}$ at which $L_r$ reaches its maximum. This finding aligns with geometric Glauber model predictions by Becattini et al. [10]. The incorporation of Fermi motion introduces a distinctive contribution to the orbital angular momentum. Despite the initially isotropic sampling of Fermi momenta in the nucleus rest frame, this isotropy is broken upon transitioning to the calculation frame, induced by a Lorentz boost along the z-direction. This boost maintains isotropy within the transverse plane but breaks it along the z-axis, resulting in a noticeable non-zero contribution to $-L_y$ within the interaction region. Across all values of $b$ and calculated beam energies, the pink curve consistently surpasses the blue curve, with the relative difference diminishing for higher beam energies, as anticipated.

From the analysis presented in Fig. 1, we anticipate a specific impact parameter maximizing angular momentum transfer at a given beam energy. Investigating its generality, we extract $b_{\text{max}}$ for various beam energies. The resulting dependence on $\sqrt{s_{NN}}$ is depicted in Fig. 2.
Angular momentum plotted against the impact parameter for AuAu collisions at \( \sqrt{s_{NN}} = 2.41 \), 8.7, and 200 GeV within the fireball. The comparison involves three computations, considering variations with and without Fermi motion. Additionally, the presence of 20 test particles is illustrated by the gray dashed line. From [6].

(left), with solid curves representing \( b_{\text{max}} \) with (upper) and without Fermi motion (lower), while the dashed line additionally includes 20 test particles. Our simulations reveal a weak dependence of \( b_{\text{max}} \) on beam energy over an energy range of two magnitudes. Consistent with observations in Fig. 1, our result reaffirms that Fermi motion shifts the maximum angular momentum transfer towards slightly more peripheral collisions due to a transverse momentum diffusion at the edges of the overlap region as discussed in [6].

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\sqrt{s_{NN}} = 2.41, 8.7, \text{ and } 200 \text{ GeV within the fireball.}
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\text{Contrary to intuitive expectations, our findings indicate a trend towards lower beam energies and more central collisions for a higher relative angular momentum transfer. In heavy-ion collisions, orbital angular momentum from the initial state is dynamically transferred to participants over time through multiple nucleon interactions until secondary collisions cease.}
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Figure 1. Angular momentum plotted against the impact parameter for AuAu collisions at \( \sqrt{s_{NN}} = 2.41 \), 8.7, and 200 GeV within the fireball. The comparison involves three computations, considering variations with and without Fermi motion. Additionally, the presence of 20 test particles is illustrated by the gray dashed line. From [6].

(right), we explore the system size dependence of relative angular momentum transfer at mid-rapidity for varying nucleus sizes from \( A = 16 \) \( (16O) \) to \( A = 208 \) \( (208Pb) \). The solid lines correspond to a beam energy \( \sqrt{s_{NN}} = 8.7 \) GeV with impact parameters \( b = 2.0 \) fm (upper) and \( b = 6.0 \) fm (lower), whereas the dashed lines depict our results for \( \sqrt{s_{NN}} = 200.0 \) GeV, again with \( b = 2.0 \) fm (upper) and \( b = 6.0 \) fm (lower). Contrary to intuitive expectations, our findings indicate a trend towards lower beam energies and more central collisions for a higher relative angular momentum transfer. In heavy-ion collisions, orbital angular momentum from the initial state is dynamically transferred to participants over time through multiple nucleon interactions until secondary collisions cease. Fig. 3 illustrates the time evolution of angular momentum in AuAu collisions with frozen Fermi motion, showcasing the total angular momentum (upper), spectators’ (middle) and participants’ evolution (lower). The dashed lines represent calculations with 100 test particles, addressing locality concerns. We see that broken angular momentum conservation arises from two distinct contributors: geometric collision criterion (breaking Poincaré invariance) and frozen Fermi approach. The test particle method improves conservation, smoothing the initial kink. However, for low beam energies, a residual violation persists, originating from the discontinuity

Figure 2. Left: impact parameter dependence of maximal angular momentum transfer with frozen Fermi motion (upper), without Fermi motion (lower) and with additional 20 test particles (dashed) in AuAu collisions across varying beam energies. Right: ratio of transferred to initial angular momentum as a function of system size and beam energy. From [6].
due to frozen Fermi motion. This assertion is supported in Fig. 3, where angular momentum conservation is fully restored with test particles when Fermi momenta are neglected (off) or considered throughout the evolution (on). Optimal configurations for angular momentum studies involve the test particle method with Fermi motion considered throughout the evolution for low energies and only the test particle method ($N_F = 20$) for medium to high energies, where the contribution from frozen Fermi motion is negligible.

4 Conclusion

The study of angular momentum transfer in heavy-ion collisions, utilizing the SMASH hadronic transport approach, reveals a distinct maximum in the angular momentum of the fireball. This maximum occurs at a unique, beam energy-independent impact parameter, aligning with Glauber model predictions. Analyses of various system sizes suggest a higher angular momentum deposition in more central collisions and at lower beam energies. In general, non-conservation effects from non-local interactions and the frozen Fermi approach have minimal impact in the mid- to high energy regime. However, we demonstrated that the inclusion of test particles and a case dependent treatment of Fermi motion restores angular momentum conservation.

Acknowledgments. This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – Project number 315477589 – TRR 211. Computational resources have been provided by the GreenCube at GSI. H.E. and N. S. acknowledge the support by the State of Hesse within the Research Cluster ELEMENTS (Project ID 500/10.006).

References