

Indirect methods in nuclear astrophysics: recent results from ANC and THM

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Abstract. Nuclear reactions within stars typically occur at energies significantly below 1 MeV. Consequently, the Coulomb barrier exponentially suppresses the cross section, reducing it to values as small as a few nanobarns for charged particles. This challenge in obtaining accurate input data for astrophysics has led to the introduction of indirect methods. Specifically, techniques such as ANC and THM have been employed to derive cross sections for reactions involving photons and charged particles in the exit channel, respectively, eliminating the need for extrapolation. The discussion delves into recent results from the application of these methods. For instance, the ${}^6\text{Li}({}^3\text{He},d){}^7\text{Be}$ measurement is utilized to deduce the ANC's of the ${}^3\text{He}+{}^4\text{He}\rightarrow {}^7\text{Be}$ and $p+{}^6\text{Li}\rightarrow {}^7\text{Be}$ channels, along with their corresponding radiative-capture cross sections. Additionally, the THM measurement of the ${}^{27}\text{Al}(p,\alpha){}^{24}\text{Mg}$ cross section via the ${}^2\text{H}({}^{27}\text{Al},\alpha){}^{24}\text{Mg}n$ reaction is highlighted. In both cases, the cross section at astrophysical energies has been established with unprecedented accuracy.

1 Brief guide to indirect methods

Indirect methods refer to techniques enabling the derivation of the astrophysical factor [1] for a reaction by measuring the cross section of a related process and applying nuclear reaction theory to establish the connection. These methods are crucial when investigating nuclear reactions of astrophysical significance. In astrophysics, particularly during quiescent stellar evolutionary stages, energies of interest are so low that the Coulomb barrier significantly reduces cross sections, rendering direct measurements essentially impossible for charged particles. The energies within the Gamow window [1], typically ranging from a few keV to a few hundred keV, can result in cross sections much smaller than 1 nb, necessitating extrapolation from higher energies. While utilizing the astrophysical factor, a smoothly varying function of energy, aids in minimizing systematic errors introduced by extrapolation, resonant reactions may exhibit significant deviations from this smooth behavior due to unknown or unpredicted resonances. Even in non-resonant cross sections, electron screening introduces uncertainty in extrapolation, as observed in case studies [2]. Electron screening becomes significant when interaction energies are comparable to electron binding energies in atoms, and the presence of atomic electrons cannot be neglected. Electron clouds enhance the astrophysical factor by shielding nuclear charges, making the assessment of the bare-nucleus cross section, crucial

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for astrophysical applications, challenging with traditional beam and solid or gaseous target reactions. Hence, indirect methods prove invaluable.

This work focuses on two indirect techniques. First, the Trojan Horse Method (THM) [3] is discussed. The THM has successfully explored low-energy nuclear reactions induced by charged particles [4, 5], including radioactive ion beams [6] and neutrons [7], using the modified R-matrix theoretical formalism for analyzing multi-resonance reactions [8]. Radiative capture reactions are indirectly studied, among other methods, through the extraction of the Asymptotic Normalization coefficient (ANC), particularly suitable for pure external direct capture processes [9].

1.1 A sketch of the THM method

Within the general framework of the THM, the cross section of the $A(x, b)B$ reaction is derived through the $A(a, bB)s$ reaction conducted at energies significantly higher than astrophysical levels (several tens of MeV). This ensures that neither Coulomb nor centrifugal barriers in the entrance channel impede the cross section, and electron screening does not impact the astrophysical factor at astrophysical energies. To link the $A(a, bB)s$ process to the $A(x, b)B$ astrophysical reaction, the quasi-free (QF) is enforced by selecting the phase space region where particles x and s , making up the TH nucleus a , have distances larger than the nuclear interaction radius. This condition corresponds to small $x-s$ relative momenta, lower than about $(2\mu_{xs}B_{xs})^{1/2}$ (where μ_{xs} and B_{xs} are the $x-s$ system reduced mass and binding energy, respectively), and implies that the $A-x$ interaction is independent of the presence of s (especially if x or s is a neutron, since the Coulomb interaction has a long range). However, since particle x is virtual, the $A(x, b)B$ astrophysical process is half-off-energy-shell (HOES), and it cannot be straightforwardly compared with the corresponding direct one that is on-energy-shell, OES [10]. To handle the complexities introduced by the HOES nature in the multi-resonance scenario, the modified R-matrix approach is employed to extract the astrophysical S-factor of interest from the QF reaction yield [8].

1.2 The ANC approach basic features

The cross section of a $A(a, \gamma)B$ radiative capture reaction can be represented at low energies by the matrix element $M = \langle \psi_B | O(r_{Aa}) | \psi_A \chi_a \rangle$, where $O(r_{Aa})$ is the radial part of the electromagnetic multipole operator dependent on the distance r_{Aa} between the projectile and the target nucleus. The wave functions ψ_A and ψ_B correspond to the initial state of nucleus A and the final state of nucleus B after a -capture, while χ_a represents the wave function of the incident structureless particle a . The matrix element M is thus proportional to the $B = A + a$ overlap function; at an asymptotic distance, it can be described as the product of the ANC $C_{Aa|B, j_B}$ and the Whittaker function [9]. Therefore, in peripheral reactions, where only the outer part of the nuclear radial integrals contributes to the cross section, the $A(a, \gamma)B$ radiative capture cross section is proportional to the square of the ANC $C_{Aa|B, j_B}$.

The key aspect in applying the ANC method is that these ANCs can be determined from transfer reactions such as $A(C, D)B$, where $C = D + a$ is employed to transfer particle a and populate the B system. For this purpose, the $A(C, D)B$ differential cross sections are fitted using, for example, the DWBA approach. From the DWBA differential cross section calculated at the main maximum of the angular distribution, the relevant ANCs can be deduced as outlined in [11]. Similar to the THM case, the deduced cross sections are free from electron screening effects and can extend down to zero energy without requiring extrapolation. However, in general, only the direct capture contribution to the total radiative capture cross section can be established in the ANC framework. Another advantage of the method is that

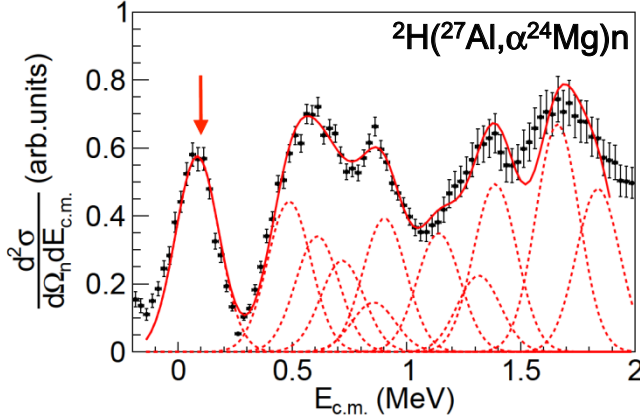


Figure 1. ${}^2\text{H}({}^{27}\text{Al}, \alpha {}^{24}\text{Mg})n$ differential cross section (black circles). The red solid line is the result of the fitting using eq.A5 of [12]. Dashed lines are used to highlight the contribution of the single resonance. The shape of each peak is determined by convoluting the theoretical Breit-Wigner shape with the experimental response function (as described in [13]). The red arrow marks the energy region of astrophysical interest.

ANC provides the absolute normalization of direct capture cross sections. In many instances, theoretical calculations can replicate the trend of the cross sections, making the use of ANC particularly significant.

2 The THM measurement of the ${}^{27}\text{Al}(p, \alpha){}^{24}\text{Mg}$ reaction

The utilization of the THM is especially apt for examining the ${}^{27}\text{Al}(p, \alpha){}^{24}\text{Mg}$ reaction at astrophysical energies. While this reaction impacts various astrophysical scenarios (see [13] and references therein), the extensively adopted rate for astrophysical predictions from [17] at $T_9 = 0.1$ presents a lower, median, and upper value of 1.85×10^{-11} , 4.34×10^{-11} , and $8.51 \times 10^{-11} \text{ cm}^3 \text{ mol}^{-1} \text{ s}^{-1}$, respectively, covering almost an order of magnitude. The uncertainty range expands at lower temperatures. The THM has enabled investigations down to zero energy, shedding light on the energy region of astrophysical interest below about 100 keV. For this purpose, a ${}^{27}\text{Al}$ projectile was accelerated onto a deuteron target nucleus with a simple $p + n$ structure, the $p - n$ motion mainly occurring in s -wave [14]. By selecting small $p - n$ relative momenta corresponding to large $p - n$ relative distances, ${}^{27}\text{Al}$ interacts only with the proton participating in the TH reaction, while the neutron acts as a spectator. ${}^{28}\text{Si}$ excited states were populated, subsequently decaying into the ${}^{24}\text{Mg} + \alpha$ channel, chosen in the offline analysis. Since the beam energy is compensated for by the deuteron binding energy, the ${}^{27}\text{Al} + p$ reaction can occur in the energy region of astrophysical relevance.

Fig. 1 illustrates the result of the THM measurement. As detailed in [13, 15], where the entire data analysis is explained, the ${}^2\text{H}({}^{27}\text{Al}, \alpha {}^{24}\text{Mg})n$ QF cross section exhibits numerous resonances, consistent with expectations from the examination of the ${}^{27}\text{Al}(p, \alpha){}^{24}\text{Mg}$ astrophysical factor. Focusing on the energy region of astrophysical interest around 100 keV, the lowest energy peak was well-fitted with a single resonance centered at 84.3 keV. For this resonance, a strength value was provided for the first time: $1.67 \pm 0.32 \times 10^{-14} \text{ eV}$, while previous works [16] could only set an upper limit $\omega\gamma \leq 2.60 \times 10^{-13} \text{ eV}$. The procedure for extracting resonance strengths and their normalization is outlined in [13, 15]. For other

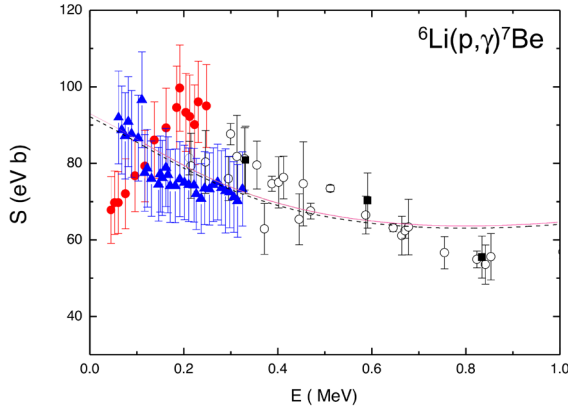


Figure 2. The experimental and calculated astrophysical S factor for the radiative-capture ${}^6\text{Li}(p, \gamma){}^7\text{Be}$ reaction. Solid purple line: S factor obtained by using the ANC values from the near-barrier proton transfer ${}^6\text{Li}({}^3\text{He}, d){}^7\text{Be}$. Black line: S factor calculated from the ANCs deduced from the reanalysis of the data from ref.[22], shown as blue solid triangles. Our result tends to disfavour the resonant trend in ref.[21] (red solid circles), while it is in good agreement with ref.[22] extrapolation, with a much better accuracy (5.9% including systematic errors). Black symbols are used for previous measurements, as discussed in ref.[19].

resonances below about 300 keV, more stringent upper limits than in [16] were set, while resonance strengths of levels above 300 keV aligned well with those in the literature [16], allowing for a further validation test of the method.

Applying the narrow-resonance approximation and the Monte Carlo method using the RatesMC code (see [16] and references therein), we calculated the reaction rate, which turned out to be approximately 3 times lower than in [17] at temperatures where the MgAl cycle is especially crucial. These results suggest that the MgAl cycle would not be closed at such temperatures ($T \geq 0.055$ GK), although further measurements of the ${}^{27}\text{Al}(p, \gamma){}^{28}\text{Si}$ reaction would be necessary. Preliminary astrophysical calculations of AGB star nucleosynthesis indicate that the new ${}^{27}\text{Al}(p, \alpha){}^{24}\text{Mg}$ reaction rate enhances the ${}^{27}\text{Al}$ yields of stars experiencing soft hot bottom burning by up to $\sim 25\%$, for solar metallicity in the $4 - 5 M_{\odot}$ mass domain.

3 The ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ and ${}^6\text{Li}(p, \gamma){}^7\text{Be}$ radiative capture reactions

The ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction plays a crucial role in nuclear astrophysics as the initial step in the 2^{nd} and 3^{rd} $p - p$ chain branches. The uncertainty in its rate significantly influences the predicted flux of ${}^7\text{Be}$ and ${}^8\text{B}$ neutrinos. While solar neutrino detection has become more precise with large detectors, the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction remains critical, with uncertainties around 5-8% compared to the required 3% precision (see [18] for an extensive discussion). The ANC approach proves beneficial for studying the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ astrophysical factor at low energies, where energy trends are well-understood, but absolute values have a large spread.

As discussed in ref.[18, 19], ANCs for the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction were obtained by studying the near-barrier ${}^6\text{Li}({}^3\text{He}, d){}^7\text{Be}$ α particle transfer reaction at backward angles. This process, being a pure external direct capture at stellar energies [20], is less sensitive to nuclear structure details since it proceeds through the tail of the nuclear overlap function. The

obtained ANCs allowed the deduction of the $S_{34}(0)$ factor with high accuracy. The same data set, focusing on angular distributions at forward angles, enabled the deduction of the ${}^6\text{Li}(p, \gamma){}^7\text{Be}$ radiative capture cross section (see [19] for details). This reaction is crucial for understanding the primordial lithium problem, and accurate determination of the ${}^6\text{Li}(p, \gamma){}^7\text{Be}$ astrophysical S-factor is essential. However, available experimental data are contradictory, and the electron screening effect must be considered. Recent experiments [22] challenged the occurrence of a 200 keV resonance claimed in earlier works [21], due to the occurrence of a $J^\pi = (1/2^+, 3/2^+)$ state in ${}^7\text{Be}$.

Using ${}^3\text{He}$ beams from the University of Catania (Italy) and the FN tandem accelerator at Florida State University, the transfer reaction was measured to deduce ANCs for the ${}^3\text{He}+\alpha \rightarrow {}^7\text{Be}$ channel. In the analysis, the DWBA framework was adopted [23] and one-step proton and α -particle transfers were assumed. The resulting square of the ANCs for the ground state and first excited state of ${}^7\text{Be}$ were found to be $C^2=20.84 \pm 1.12$ [0.82; 0.77] fm^{-1} and $C^2=12.86 \pm 0.50$ [0.35; 0.36] fm^{-1} , respectively. The uncertainties include both experimental ones on the $d\sigma^{\text{exp}}/d\Omega$ and on the ANC of the $d+{}^4\text{He} \rightarrow {}^6\text{Li}$ channel (first term in square parentheses), as well as the model uncertainties (second term in square parentheses). Subsequently, the contribution of direct capture to the ${}^3\text{He}({}^4\text{He}, \gamma){}^7\text{Be}$ astrophysical factor was determined within the Modified Two-Body Potential Model (MTBPM) [18, 19]. The resulting S factors, $S_{34}(0)$ and $S_{34}(23 \text{ keV})$ (23 keV being the most relevant energy for Solar fusion), were determined as $S_{34}(0) = 0.534 \pm 0.025$ [0.015; 0.019] keVb and $S_{34}(23 \text{ keV}) = 0.525 \pm 0.022$ [0.016; 0.016] keVb. A comparison with literature values demonstrates enhanced accuracy compared to the current recommended value in [20]. However, the uncertainty remains higher than the desired one, prompting further efforts for reduction.

For the ${}^6\text{Li}(p, \gamma){}^7\text{Be}$ system, squared ANCs (C_p^2) and their uncertainties were determined, being 4.51 ± 0.21 [0.15; 0.15] fm^{-1} for the ground state and 4.37 ± 0.44 [0.31; 0.31] fm^{-1} for the first excited state of ${}^7\text{Be}$. The experimental error encompasses statistical and normalization errors, while the theoretical uncertainty accounts for variations in the adopted potential parameters. The ${}^6\text{Li}(p, \gamma){}^7\text{Be}$ S-factor, mainly influenced by $E1$ transitions, was derived within the modified two-body potential model, resulting in improved accuracy with respect to direct measurements [21, 22]. The ANC astrophysical factor is shown in fig. 2 as a green line, along with the available experimental data. An additional calculation, shown as a black line, shows the astrophysical factor calculated by deducing the ANCs from the ref.[22] data, to test the validity of our approach. Quantitatively, the indirect $S_{61}(E)$ equals $90.4 \pm 2.4 \text{ eV}\cdot\text{b}$ for $E=0$ and $89.2 \pm 2.3 \text{ eV}\cdot\text{b}$ for $E= 15.1 \text{ keV}$ (the Gamow peak energy in the Sun), in excellent agreement with the extrapolated S-factor to zero energy ($S(0) = 95 \pm 9 \text{ eV}\cdot\text{b}$) of ref.[22], with a much better accuracy. The results tend to rule out the claimed 200 keV resonance [21] and support a smooth S-factor increase towards zero energy, as it is clearly shown in fig. 2.

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