Anisotropy of the Wake behind Circular Cylinder in Cross-Flow

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Abstract. The turbulence anisotropy is studied experimentally in the wake behind a circular cylinder in crossflow. The circular cylinder of aspect ratio 12 was placed in low turbulence flow, Reynolds number was 5k. For the 3-component velocity vector evaluation, the stereo Particle Image Velocimetry method was used. The Lumley triangle method was applied to quantify the turbulence anisotropy.

1 Introduction

Circular cylinder in crossflow is a typical engineering problem, appearing in many practical applications very often in various forms. In fluid mechanics, this case is considered to be a typical canonical case, with relatively simple and straightforward boundary conditions, but complex structure of the flow.

The wake behind the circular cylinder was studied extensively. There are numerous theoretical, numerical and experimental studies. To mention the most famous classical experimental studies, we could cite the von Karman’s work \cite{1}, where he described periodical nature of the wake, today known as the “Karman vortex street”. This phenomenon appears for Reynolds numbers (Re, defined using incoming velocity and cylinder diameter) from 44. However up-to $\text{Re} = 150$ the wake remains laminar, turning gradually into turbulent structure with all its attributes, as 3D fractal structure, dissipation, tendency to homogeneity and isotropy, etc. The other literal resources on the subject considered as classical are e.g. \cite{3,4}.

In the presented study we will address structure of the turbulence in the wake behind a circular cylinder, turbulence anisotropy features in particular. The chosen Reynolds number 5k indicates fully turbulent wake.

2 Experimental setup

For experiments the wind tunnel in IT ASCR was used. In the closed test section 250 x 250 x 1000 mm\textsuperscript{3} the model of circular cylinder with diameter $D = 15$ mm was placed. The low turbulent incoming flow with velocity $U_i = 5$ m/s defined the Reynolds number around 5 thousand. The cylinder is sufficiently long (about 250 mm) to consider 2D boundary condition, except close to its ends. The incoming flow is low-turbulent, intensity of turbulence around 0.1 \% and regular, with departures outside the boundary layers up to 1 \%. The details on the used experimental facility could be found e.g. in \cite{4}.

The Particle Image Velocimetry (hereinafter PIV) measuring technique was used in classical and stereo versions. In experiments the 2 types of Planes of Measurement (hereinafter PoM) are chosen to demonstrate the wake structure qualitatively. The situation is shown in Fig. 1.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Schema of experimental setup and Planes of Measurement.}
\end{figure}

The PoM 1 is streamwise oriented, perpendicular to the cylinder axis in green and the PoM 2 in red is spanwise oriented, parallel to the cylinder axis in a given distance downstream and perpendicular to the flow.

3 Instrumentation and analysis methods

Instrumentation to acquire velocity data and data analysis methods are to be described.

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3.1 Particle Image Velocimetry instrumentation

The PIV measuring method was used for all experiments. In the PoM 1 the classical version with use of a single camera, while in the PoM 2 its stereo version was applied. This implementation allows for evaluation of all 3 instantaneous velocity components in every point of the PoM 2. Those data were used for all statistical quantities evaluation, presented in this study.

The measuring apparatus consists of laser and 2 CMOS cameras by Dantec Company. The laser is New Wave Pegasus, Nd:YLF double head with wavelength of 527 nm, with maximal frequency 10 kHz and shot energy of 10 mJ (for 1 kHz), thus the corresponding power is 10 W per one head. The 2 cameras VEO 410 with resolution of 1280 x 800 pixels are able to acquire double-snaps with frequency up to 2 kHz (full resolution) and they use internal memory 16 GB each. The Scheinpflug mounting were used for the cameras lenses to get the focus planes identical with the laser-sheet plane. The particle generator SAFEX was used. To acquire the relevant data sets for statistical analysis, 1000 measurements with acquisition frequency 100 Hz were performed for each measurement.

More details about the measuring technique could be found e.g. in [4].

3.2 Turbulence anisotropy evaluation

For evaluation of turbulence anisotropy features, the classical method was used, it will be specified below.

The Reynolds stress is the component of the total stress tensor in a turbulent fluid flow obtained from the averaging operation over the Navier-Stokes equations to account for turbulent fluctuations in fluid momentum. The Reynolds stress tensor is defined as

\[ \tau_{ij} = -\rho \frac{\partial u_i}{\partial x_j}, \]

where \( \rho \) stands for fluid density and \( u_i \) is the \( i \)-th velocity component fluctuation, the operation of ensemble averaging is denoted \( \overline{\cdot} \). From the mathematical point of view, the Reynolds stress tensor is a symmetrical matrix.

From linear algebra it is known that any symmetrical matrix could be decomposed into isotropic and anisotropic parts, respectively. The dimensionless anisotropic part of Reynolds stress tensor could be characterized by a set of the eigenvalues and the related eigenvectors. The eigenvalues, called the principal stress, are defined using Cayley-Hamilton theorem in the form of the characteristic equation. This theory is presented in details e.g. in [5]. Then, the three invariants could be defined as follows:

\[ I = b_{xx}, \]

\[ II = -b_{xy}b_{yx}/3, \]

\[ III = b_{yy}b_{xx}b_{yy}/3 = \text{det}(b_{ij}), \]

where

\[ b_{ij} = \frac{\tau_{ij}}{\tau_{xx}} \frac{\delta_{ij}}{3} \]

The invariant \( I \) vanishes identically, the other two invariants, \( II \) and \( III \), define the space of anisotropy cases within so called Anisotropy Invariant Map (AIM). As the mapping in this space is highly nonlinear, the modified dimensionless invariants \( \xi \) and \( \eta \) are defined by (see [6])

\[ \xi^3 = \frac{III}{2}, \quad \eta^2 = -\frac{II}{3}. \]

The new definition suggests much more regular shape of the space borders, forming so called “Lumley triangle”, in which all the possible states of turbulence must lie. The borders are depicted in Fig. 2.

![Fig. 2. Anisotropy Invariant Map, Lumley triangle.](image)

In Fig. 2 the straight lines define axisymmetric cases, left in green and right in blue. The 2D case is characterized by nonlinear expression in black. The origin in red represents isotropic turbulence. The 1D turbulence with 2 velocity fluctuation components vanishing is the point on the top-right in purple.

To explain the possible situations, we could characterize the limiting states in Fig. 2 in geometrical form of the stress tensor. In general, the turbulent Reynolds stress tensor could be shown in the form of energy ellipsoid. For the isotropic turbulence, represented in Fig. 2 by the red point in origin, the energy ellipsoid has a spherical form. The axisymmetric expansion is characterized by the energy ellipsoid as an oblate spheroid, i.e. a pancake shape. The axisymmetric contraction results in the energy ellipsoid in a form of a prolate spheroid, i.e. a cigar shape. In the 2D turbulence, the energy ellipsoid is in the form of a disk and for the 1D turbulence, the energy ellipsoid is in a form of a line. More details could be find e.g. in [7].

To quantify the 3D isotropy of the turbulence easily, Choi and Lumley [6] has introduced the anisotropic factor \( F \):

\[ F = 1 + 27 III + 9 II \]

The \( F \) vanishes whenever turbulence becomes 2D, and it becomes unity when turbulence enters a 3D isotropic state. So, in fact it quantifies “degree of anisotropy”.

Please note, that there are some other possibilities to study the turbulence anisotropy if we have not the full Reynolds stress tensor available. Example of the analysis of such data obtained from hot-wire measurements could be find e.g. in [8].
4 Results

The results are to be shown in graphical form.

All results are presented in dimensionless form, distances (coordinates) are expressed as multiples of the cylinder diameter $D$, while velocities in multiples of the incoming velocity $U_i$. The anisotropy quantities $\xi$, $\eta$ and $F$ are dimensionless from definition.

4.1 Statistics

First, distributions of statistical characteristics are to be presented, to characterize the flow in the wake. Statistical results from the PoM 1, perpendicular to the cylinder axis, are shown in Figs. 3 and 4.

Fig. 3. Mean velocity distribution, vectors and streamwise velocity component (colour).

In Fig. 3 distribution of mean velocity vectors in the PoM 1 are shown, the colour denotes the mean streamwise velocity component, white line delimits the positive (forward) and negative (backward) mean velocity orientations.

Fig. 4. Turbulence Kinetic Energy distribution.

Fig. 4 shows distribution of the Turbulence Kinetic Energy within the wake. Please note, that TKE is defined using the two velocity components only and it is dimensionless, see above.

The Figs. 3 and 4 define the wake width, see the method presented in [9].

4.2 Anisotropy invariant maps

To evaluate invariants of anisotropy tensor, see ch.3.2, we need 3 velocity components measurements in each point. We applied Stereo PIV measurements in PoM2 in various distances in $x$ direction, only data for $z = 0$ was used, $y$ varying.

In Fig. 5 the AIM is shown, the points of a single colour connected by lines represent the profile in $y$ direction located in a given $x$ distance: 2, 2.67, 4, 6.67, 10 and 13.33 $D$, respectively. The full dot denotes position on the axis, $y = 0$.

The position $x = 0$ means situation with no cylinder present, the corresponding results are located in the graph close to the position [-0.09;0.09], which corresponds to the factor $F \approx 0.8$.

We could observe a different behaviour of the “near wake” characterized by $x = 2-4$ and the “far wake” for $x = 6.67$ and more.

The following tendencies are to be observed:

- Higher $x$ generated the turbulence closer to isotropic case – position closer to origin in the AIM graph.
- The turbulence in points located on the streamwise axis (full dots) could be characterized as axisymmetric contraction.
- For the near wake, moving away from the axis to the sides, turbulence tends to isotropy and to the axisymmetric expansion.
- For the far wake, moving away from the axis to the sides, turbulence tends to isotropy again, but it remains in axisymmetric contraction situation.

The situation of axisymmetric contraction corresponds to equal energy ellipsoid dimensions in...
streamwise and spanwise directions $x$ and $z$, while in spanwise direction $y$ the dimension is extended. The same holds for the axisymmetric expansion situation, but the dimension in $x$ direction is reduced.

If we are interested in the level of anisotropy only, we could use the anisotropic factor $F$ as a relevant parameter. In Fig. 6 there is the distribution of the factor $F$ in the plane $xy$ averaged in the $z$ direction in the range $(-2;2)$ denoted as $F_{\text{avg}}$.

![Fig. 6. Averaged anisotropic factor $F_{\text{avg}}$ distribution.](image)

In Fig. 6 we recognize clearly background turbulence $x = 0$, near wake $x$ in the range $(2;4.5)$ and the far wake for $x$ in the range $(4.5;13.33)$.

The background turbulence is characterized by $F \approx 0.8$, independent on $y$, the turbulence is close to isotropy. However, note that the TKE is very low, intensity of turbulence is approximately 0.1 %, see Fig. 4.

In the near wake the $y$ profiles of $F$ are similar to each other with values about 0.1 in the centre, indicating highly anisotropic turbulence, and approaching 0.4 on the sides $y = \pm 2$.

In the far wake the shape of the profiles defined in the near wake is kept, however they are shifted to higher values $F$ for higher $x$. The $F$ value grows on the axis with $x$ position, $x = 6.67$ corresponds to $F \approx 0.3$, $x = 10$ corresponds to $F \approx 0.45$ and $x = 13.33$ corresponds to $F \approx 0.6$. The situation far on sides from the axis approaches the free stream turbulence.

## 5 Conclusions

The turbulence structure in the wake behind the circular cylinder was examined with regards for its anisotropy features. The Lumley triangle method in Anisotropy Invariant Map was used.

The near wake in distance from 2 up to 4.5 cylinder diameters behind the cylinder axis exhibits highly anisotropic structure with dominant spanwise velocity fluctuations perpendicular to the cylinder axis in the wake central part. The turbulence structure tends to isotropy towards the wake edges.

In the far wake, for the distance greater than 4.5 cylinder diameter, the turbulence tends gradually to isotropy, however the anisotropic wake centre remains.

### List of symbols and abbreviations

- $b$: Anisotropic part of Reynolds stress tensor [1]
- $D$: Cylinder diameter [m]
- $F$: Anisotropic factor [1]
- $I$, $II$, $III$: 1st, 2nd and 3rd invariants [1]
- $Re$: Reynolds number based on $D$ [1]
- $U_i$: Incoming flow velocity [m/s]
- $U$: Streamwise mean velocity [1]
- $x,y,z$: Cartesian coordinates [1]
- $\xi, \eta$: Modified dimensionless invariants [1]
- $\rho$: Fluid density [kg/m$^3$]
- $\tau$: Reynolds stress [kg/(m.s$^2$)]

AIM: Anisotropy Invariant Map

PIV: Particle Image Velocimetry

PoM: Planes of Measurement

TKE: Turbulence Kinetic Energy

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References