Current-induced Hall effect

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Abstract. The properties of the current-induced Hall effect are analysed, for which the static magnetic field, supplied by an external source in the traditional experiment, is created by the current itself. The special experimental setup, needed for its observation, is described. It is shown how, combined with the skin effect, it could give access to the concentration of conduction electrons in superconductors. Besides this experiment might permit to dodge shortcomings, ensueing from the Meissner effect and limiting severely the sensitivity of the conventional Hall voltage measurement.

keywords : Hall effect, Meissner effect, skin effect

1 Introduction

When a superconducting material of type I is cooled in a static magnetic field H, starting from its normal state, H is screened[1] inside bulk material by eddy currents, while crossing the critical temperature T_c, at which superconductivity sets in. Moreover Newton's law requires a net force accelerating the conduction electrons for the current to grow from 0 up to its final value. However, this manifestation of the Meissner effect has kindled a lasting controversy [2, 3] over the very nature of this force, because H remains *unaltered* at T_c. In particular, quantum[4] and classical[5–7] origins have been called forward. Nevertheless, this debate has remained so far inconclusive, because the original experimental setup[8] does not enable one to sort out the relevant interpretation. Fortunately, a new kind of experiment has been proposed recently[4], a slightly modified version of which might offer such a possibility by taking advantage of the *current induced* Hall effect[7], as shown in this article.

The outline is as follows : the experimental setup, used to measure the Hall voltage, is described in section II, whereas the quantum and classical explanations are discussed in sections III and IV, respectively; an experiment, based on the skin effect, enabling one to distinguish between both views, is discussed in section V.

2 Experimental setup

As seen in Fig.1, the superconducting sample consists in a hollow cylinder, characterized by its symmetry axis z and inner and outer radii r_i, r_o. It is located in a cylindrical frame with coordinates r, θ, z ($\Rightarrow r_i < r < r_o$). Its length $2b$ ($\Rightarrow -b < z < b$) is taken $>> r_o$, in order to get rid of any end effect. The material contains conduction electrons

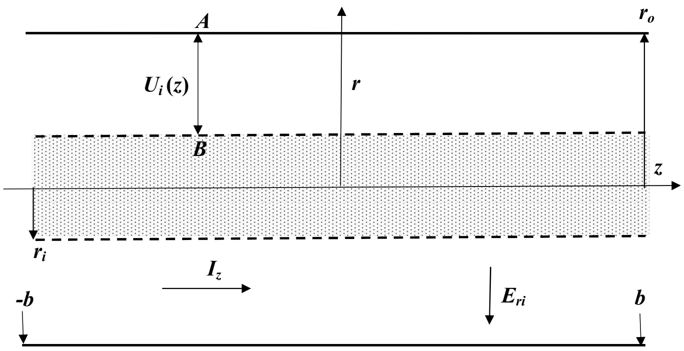

Figure 1. Cross-section of the sample; the current I_z and the electric field E_{ri} ($i = 0, 1, 2$), aroused by the Hall effect, are respectively parallel to the z, r axes, whereas the magnetic field H_θ, induced by I_z, is along the azimuthal axis, that is normal to the z, r axes; $i = 0$ refers to the case of the normal metal, whereas $i = 1, 2$ correspond respectively to the quantum and classical analyses for a superconductor; the matter (dotted area), making up the cylinder of radius r_i, delineated by the dashed lines, is to be carved out to measure the Hall voltage $U_{i=0,1,2}(z)$ between A, B

of charge e, effective mass m and concentration c_0. The current I_z, flowing along the z axis, will be kept constant. Besides, the corresponding current density j_{z0} is homogeneous in the normal state ($T > T_c$). It then reads

$$j_{z0} = \frac{I_z}{\pi\left(r_o^2 - r_i^2\right)} \quad . \tag{1}$$

There is also[9] $j_{z0} = c_0 e v_{z0}$ with v_{z0} standing for the electron mass center velocity. Thanks to the Ampère-Maxwell law, it gives rise, to an azimuthal magnetic field $H_\theta(r)$, which in turn exerts a radial Lorentz force $F_r(r)$ on each electron

$$\begin{aligned} H_\theta(r) &= \tfrac{r^2 - r_i^2}{2r} j_{z0} \Rightarrow \\ F_r &= -\mu_0 e v_{z0} H_\theta(r) = -\tfrac{\mu_0}{2c_0}\tfrac{r^2-r_i^2}{r} j_{z0}^2 \end{aligned} \quad , \tag{2}$$

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with μ_0 being the permeability of vacuum. Then $F_r(r)$ displaces the conduction electrons, which gives rise to a space charge density $Q(r)$ and thence to a radial electric field $E_{r0}(r)$, such that $Q(r) = \mathrm{div}\,(\epsilon_0 E_{r0}(r))$ (ϵ_0 stands for the electrical permittivity of vacuum). Finally, in the steady regime, $F_r(r)$ is offset by $eE_{r0}(r)$, which results into the Hall voltage U_0, measured between points A, B in Fig.1. Thus E_{r0}, U_0 read in the normal state

$$\begin{aligned} E_{r0} = -\frac{F_r}{e} \Rightarrow U_0 &= \int_{r_i}^{r_o} E_{r0}(r)dr \\ &= \frac{\mu_0}{2\pi^2 e c_0 \left(r_o^2 - r_i^2\right)} \left(\frac{1}{2} - \frac{r_i^2}{r_o^2 - r_i^2} \log \frac{r_o}{r_i}\right) I_z^2 \end{aligned} \quad . \qquad (3)$$

Hence measuring the Hall voltage U_0 gives access to the electron concentration c_0. At last, several remarks are in order

- due to cylindrical symmetry, U_0 is independent from θ and in addition proportional to I_z^2, unlike the usual Hall voltage proportional to I_z, because the field H_θ *itself* is proportional to the current I_z;
- since the sign of U_0 cannot be predicted[9] in general, the experiment sketched above should be done first in the normal state, i.e. at $T > T_c$, for calibration purposes, because c_0 is well known;
- due to different sample shape, i.e. parallelepiped versus cylinder, the superficial charge, giving rise to the electric field, lies at the outer edges of the sample in the traditional measurement, whereas it is a space charge, distributed over the whole bulk matter, in the experiment discussed here.

3 Quantum approach

Let the sample, depicted in Fig.1, be cooled down with I_z kept constant for time $t > 0$. Actually, it does not become superconducting until $t = t_f$, at which $T(t_f) < T_c$ is *low*[10] enough, so as to fulfil $j_{z0} < j_c(T(t_f))$ with $j_c(T)$ referring to the critical current density. Note that, due to *irreversible* Joule dissipation resulting from the *finite ac resistivity* in the superconducting state[7, 11], there is no one to one correspondence between H_θ and j_c, so that, contrary to an ubiquitous misconception[1, 15], the phrase "*critical magnetic field*" is meaningless. Conversely, the critical current is well-defined, because the *current density* together with *temperature* T do characterise[10] the thermodynamical state of every superconductor, as recalled below in section V.

It is then argued[4] that, once the sample becomes superconducting, the Lorentz force, combined with some undefined *quantum force*[4], push the electrons outward along the radial axis, which causes the current density j_{z1} to become r dependent but to remain independent from z for $|z| << b$ in the steady regime, defined by $T(t > t_f) = T(t_f)$. Then j_{z1} reads[4]

$$j_{z1}(r) = j_{z1}(r_o) e^{\frac{r - r_o}{\lambda_L}} \quad , \quad \lambda_L = \sqrt{\frac{m}{\mu_0 c_1 e^2}} \quad , \qquad (4)$$

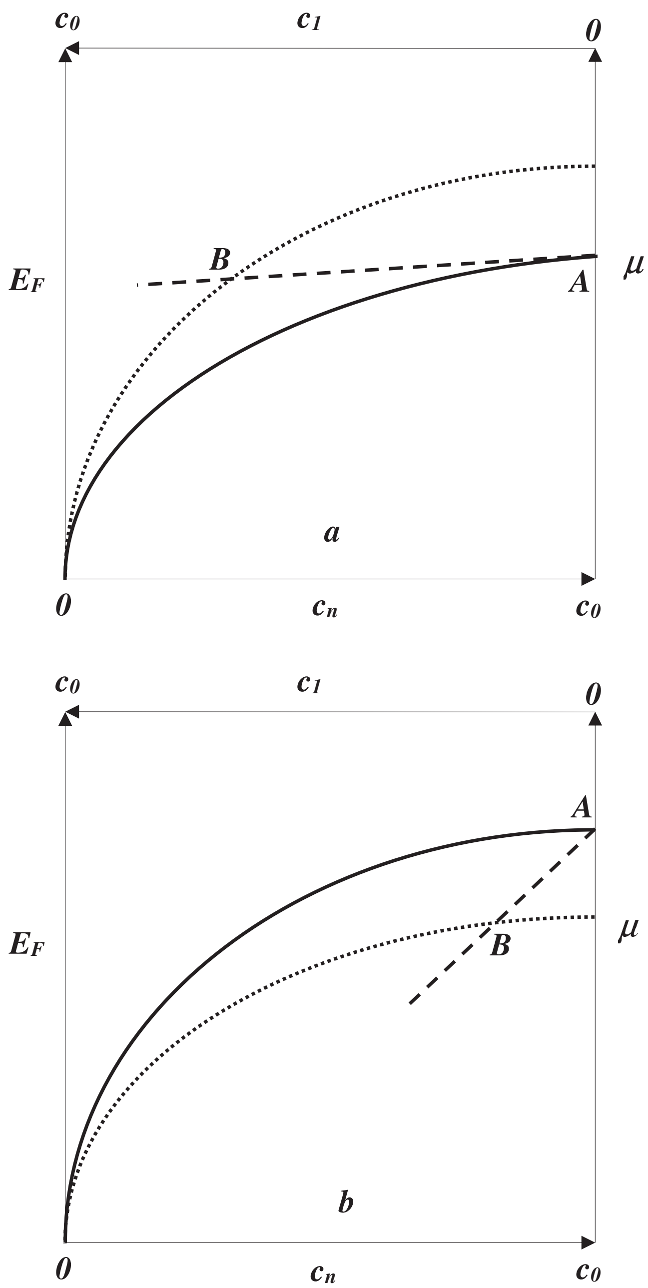

Figure 2. schematic plots of $E_F(T_c, c_n)$, $E_F(0, c_n)$ and $\mu(c_1)$ as solid, dotted and dashed lines, respectively; cases a, b correspond to $\rho'(E_F) > 0$ and $\rho'(E_F) < 0$, respectively; $\frac{\partial \mu}{\partial c_s}$ has been taken to be constant for simplicity; the origin $E_F = \mu = 0$ is set at the bottom of the conduction band; the crossing points A, B of $E_F(T_c, c_n), E_F(0, c_n)$ with $\mu(c_1)$ in Fig.2*a,b* exemplify stable and unstable solutions of Eq.8, respectively; the tiny differences $E_F(T_c, c_n) - \mu(c_0 - c_n)$ and $E_F(0, c_n) - \mu(c_0 - c_n)$ have been hugely magnified for the reader's convenience

for which $\lambda_L << r_i, c_1$ refer to London's length[1, 2] and the concentration of superconducting electrons, respectively. In addition, $j_{z1}(r_o)$ can be assigned to its value by requiring

$$I_z = 2\pi \int_{r_i}^{r_o} j_{z1}(r) r dr \Rightarrow j_{z1}(r_o) = \frac{I_z}{2\pi \lambda_L r_o} \quad . \qquad (5)$$

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By proceeding furthermore as in the previous section, the magnetic field H_θ, the Lorentz force F_r, the electric field E_{r1} and the resulting Hall voltage U_1 are worked out to read

$$\begin{array}{c} H_\theta(r) = \int_{r_i}^{r} j_{z1}(u)\frac{u}{r}du = \frac{e^{\frac{r-r_o}{\lambda_L}}}{2\pi r_o}\left(1 - \frac{r_i}{r}e^{\frac{r_i-r}{\lambda_L}}\right)I_z \\ \Rightarrow F_r(r) = -\frac{\mu_0}{c_1}H_\theta(r)j_{z1}(r) \Rightarrow E_{r1}(r) = -\frac{F_r(r)}{e} \\ \Rightarrow U_1 = \int_{r_i}^{r_o} E_{r1}(r)dr = \frac{\mu_0}{8ec_1(\pi r_o)^2}I_z^2 \end{array} \quad . \tag{6}$$

Hence c_1 can be known by measuring U_1 owing to Eq.(6).

However several issues appear questionable in this analysis[4]:

- it relies upon London's equation[2]

$$B_z(r) + \mu_0\lambda_L^2 \text{curl}(j_\theta(r)) = 0 \quad , \tag{7}$$

  with B_z, j_θ being the magnetic induction, parallel to the z axis, and the persistent current density, parallel to the azimuthal axis. Actually, the validity of Eq.(7) has been ascertained[7], but for *infinite* electrical conductivity, whereas the *ac* conductivity has been proved[11] to be *finite* on the basis of low-frequency susceptibility data[12, 13], which thereby invalidates Eq.(7). As another consequence of finite *ac* conductivity, the thermodynamical state, characterising the superconducting sample in the steady regime $t > t_f$, *depends*[14] upon $T(t \in [0, t_f])$. At last, though Eq.(7) has been worked out[2, 7] for a *magnetic field parallel* to the z axis, and *azimuthal current*, it is applied[4], with no justification, to a physical case, characterised by an *azimuthal field* and *current* flowing along the z axis. Accordingly, since Eq.(7) ensues[7] from solving Maxwell's equations for a transient regime ($\Rightarrow \frac{\partial B_z}{\partial t}(t \in [0, t_f]) \neq 0, \frac{\partial j_\theta}{\partial t}(t \in [0, t_f]) \neq 0$), the claim, that it applies as well to a steady regime[4] ($\Rightarrow \frac{\partial B_\theta}{\partial t}(t > t_f) = \frac{\partial j_z}{\partial t}(t > t_f) = 0$) with undefined $B_\theta(t < t_f)$, $j_z(t < t_f)$, is groundless;

- the origin of the *quantum force* is self-admittedly[4] *unknown*, as is the expression of the *potential*, which it is supposed to derive from, so that this very phrase is actually misleading. In addition, the matching conditions[4] at $z = \pm b$ require a *persistent* radial current j_r to flow in the steady regime $t > t_f$. Then the azimuthal magnetic field H_θ, acting on j_r, will give rise to a Lorentz force, parallel to the z axis, which will cause eventually I_z to grow till the critical value is reached and the sample goes thereby normal, which contradicts the assumption[4] of a stable, steady regime;

- a key assumption[4] says that the current should be carried by *holes* in the normal metal. This entails that the Fermi energy E_F ought to be close to the Van Hove singularity[9], located at the upper edge of the one-electron band ϵ_u. Consequently, there is $\rho(E_F) \propto \sqrt{\epsilon_u - E_F}$ for a three-dimensional sample[9], with $\rho(\epsilon)$, ϵ standing for the one-electron density of states and energy, respectively. Then it implies finally $\rho'(E_F) = \frac{d\rho}{d\epsilon}(E_F) < 0$. However such a condition will be proved below to be at loggerheads with a *thermodynamical* rationale, showing the opposite, namely a *stable* superconducting phase *requires* $\rho'(E_F) > 0$.

As shown elsewhere[16], there is

$$\begin{array}{c} \frac{dE_F}{dT}(T) = -\frac{(\pi k_B)^2}{3}\frac{\rho'(E_F)}{\rho(E_F)}T \\ \Rightarrow \rho'(E_F)\frac{dE_F}{dT}(T) < 0 \end{array} \quad ,$$

wherein k_B is Boltzmann's constant and $\frac{dE_F}{dT}$ has been reckoned at fixed normal electron concentration c_n. Hence $E_F(T, c_n)$ at fixed T looks as plotted in Fig.2*a*, Fig.2*b*, for $\rho'(E_F) > 0$ and $\rho'(E_F) < 0$, respectively. The infinite slope $\frac{\partial E_F}{\partial c_n}(c_n \to 0) \to \infty$ is typical of a 3 dimensional Van Hove singularity[9], associated with the bottom of the conduction band. Note that Fig.2 displays $\frac{\partial \mu}{\partial c_1} < 0$ ($\mu(c_1)$ refers to the chemical potential of superconducting electrons[10, 17]), which has been shown to be a prerequisite for persistent currents[17], thermal equilibrium[10], occurrence of superconductivity[16, 18] and the Josephson effect[19]. The two-fluid system, comprising normal and superconducting electrons in respective concentration c_n, c_1 (charge conservation requires $c_n + c_1 = c_0$), is at thermal equilibrium at $T \leq T_c$, provided the following equation is fulfilled[16–18]

$$E_F(T, c_n) = \mu(c_1) \quad . \tag{8}$$

Eq.(8) means that the free energy of the whole electron system is stationary at fixed T. However, as shown elsewhere[17], Fig.2*a* and Fig.2*b* depict, respectively, the case of *stable* ($\rho'(E_F) > 0 \Rightarrow \frac{\partial E_F}{\partial c_n} + \frac{\partial \mu}{\partial c_1} > 0$) and *unstable* ($\rho'(E_F) < 0 \Rightarrow \frac{\partial E_F}{\partial c_n} + \frac{\partial \mu}{\partial c_1} < 0$) equilibrium. This statement of fact completes the proof that the hole-driven superconductivity[4] *cannot* be observed because it implies $\rho'(E_F) < 0$. Likewise, for the reader's convenience, we recall[16, 18] the necessary conditions for a second order transition to occur at T_c

$$\begin{array}{c} E_F(T_c, c_0) = \mu(0), \quad \rho'(E_F(T_c, c_0)) > 0 \\ \frac{\partial E_F}{\partial c_n}(T_c, c_0) = -\frac{\partial \mu}{\partial c_1}(0) \end{array} \quad .$$

4 Classical approach

The magnetic susceptibility not being continuous at T_c has been shown[7] to be responsible for the Meissner effect taking place in a superconductor, cooled in a static magnetic field. Since no paramagnetic contribution has ever been observed in the superconducting state[1, 9], the latter is deemed to be in a macroscopic singlet spin state, so that its susceptibility χ_s is determined *entirely* by Lenz's law[7] $\Rightarrow \chi_s < 0$. Meanwhile the magnetic susceptibility of normal electrons χ_n comprises[9] two components, respectively paramagnetic (Pauli) and diamagnetic (Landau), and is in general > 0. Thus the local, magnetic inductions $B_s(r), B_n(r)$, both being parallel to the azimuthal axis, read, respectively, in the superconducting and normal states

$$B_s(r) = \mu_0(1 + \chi_s)H_\theta(r), \quad B_n(r) = \mu_0(1 + \chi_n)H_\theta(r).$$

Due to $\chi_s \neq \chi_n$, the magnetic induction undergoes a finite step at T_c

$$\frac{\delta B(r)}{\delta t} = \frac{B_s(r) - B_n(r)}{\delta t} = \mu_0\frac{\chi_s - \chi_n}{\delta t}H_\theta(r) \quad ,$$

where δt refers to the time needed in the experimental procedure for T to cross T_c. Owing to the Faraday-Maxwell equation, $\delta B/\delta t$ may induce two transient, electric fields $E_r(z), E_z(r)$, respectively parallel to the r, z axes, such that

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -\frac{\delta B}{\delta t} \quad .$$

Let us begin with showing $E_z(r) = 0$. The proof is by contradiction. As a matter of fact, $E_z(r) \neq 0$ implies that the voltage drop

$$\delta U = U(z = b) - U(z = -b) = -\int_{-b}^{b} E_z(r)dz$$

is *r-dependent* too. But this is inconsistent with δU being *r-independent* at $z = \pm b$, i.e. at the interface with the leads, straddling the superconducting sample, because they are made out of *normal* metals, which entails $U(z)$ being *r-independent* for any z. *Q.E.D.*

Meanwhile, $E_r(z) = -\frac{\delta B}{\delta t} z$ will give rise to a transient *z-dependent* voltage

$$\delta U(z) = E_r(z)\,(r_o - r_i) \quad .$$

Observing $\delta U(z)$ could buttress the validity of this analysis.

Hence, due to the current density remaining *r*-independent in the steady regime $t > t_f$, the calculation of E_{r2}, U_2 proceeds as that of E_{r0}, U_0 in Eq.(3), except for the concentration of superconducting electrons c_2 showing up instead of c_0

$$\begin{array}{c} U_2 = \int_{r_i}^{r_o} E_{r2}(r)dr = \\ \frac{\mu_0}{2\pi^2 e c_2 (r_o^2 - r_i^2)} \left(\frac{1}{2} - \frac{r_i^2}{r_o^2 - r_i^2} \log \frac{r_o}{r_i} \right) I_z^2 \end{array} \quad . \tag{9}$$

Comparing Eq.(6) with Eq.(9) leads to $c_1 \neq c_2$. Therefore an independent determination of the concentration of superconducting electrons is needed to assess the respective validity of the quantum and classical analyses. This is the purview of the next section.

5 Skin effect

An electromagnetic field of frequency ω, shone on a metal, remains confined[20] within a thin layer of thickness $\delta(\omega)$, called the skin depth, and located at the outer edge of the conductor, for $\omega < \omega_p$, where $\omega_p \approx 10^{16}$Hz stands for the plasma frequency. This stems from the real part of the dielectric constant being negative for $\omega < \omega_p$. Drude's conductivity σ and δ have been shown to read[7, 21]

$$\begin{array}{c} \sigma = \frac{c_s e^2 \tau}{m} \\ \delta(\omega << \omega_c) = \frac{\lambda_L}{\sqrt{2\omega\tau}} \quad , \quad \delta(\omega >> \omega_c) = \frac{\lambda_L}{\sqrt{2}} \end{array} \quad .$$

c_s, τ are the concentration of superconducting electrons and their associated scattering time[9, 20], respectively, and $\omega_c = \frac{1}{\tau}$. Note that the normal electrons play no role, because their conductivity is $<< \sigma$. Then, thanks to $\lambda_L = \sqrt{\frac{m}{\mu_0 c_s e^2}}$, measuring $\delta(\omega << \omega_c), \delta(\omega >> \omega_c)$ would give access to c_s, τ, provided m is known. Nevertheless several remarks are in order, concerning the measurement :

- there being a *one to one* correspondence for conductivity $\sigma \leftrightarrow c_s$ and current density[10] $j \leftrightarrow c_s$, illustrated by

$$\begin{array}{c} \sigma(c_s) = \frac{c_s e^2 \tau}{m} \\ j_s(c_s) = c_s e \sqrt{\frac{2}{m} \left(E_F(T, c_0 - c_s) - \mu(c_s) \right)} \end{array} \quad ,$$

  entails that σ *depends* on j, so that measurements of the skin depth and the Hall voltage should be performed with the *same* current I_z. Besides, as for the quantum case[4], the r-dependent $j_{z1}(r)$ leads to a r-dependent concentration $c_1(r)$, which complicates further the interpretation of the Hall effect experiment;

- in pure materials, ω_c is likely to lie in the microwave range ($\omega_c < 10^{12}$Hz), while it might rather belong in the IR one ($\omega_c > 10^{12}$Hz) for poor conductors such as high-T_c compounds.

6 Conclusion

The properties of the current-induced Hall effect have been presented. It has been argued that it might shed light on the very nature of the force at work in the Meissner effect, i.e. either quantum or classical. The quantum explanation fails seemingly to account for the force giving rise to the eddy currents at T_c, whereas the electrons, making up the eddy currents, are driven merely by Faraday's electric field, induced by the magnetic susceptibility varying at T_c within the classical interpretation.

Experiments, combining the current-induced Hall effect and the skin effect, have been discussed, which might enable one to assess the respective validity of each view. At last the skin depth measurement might help to chart the so far *unknown* $c_s(j)$, which is of great significance for the superconducting to normal transition[10].

Since the Hall voltage grows like the inverse of the electron concentration (see Eqs.(3,6,9)), it is in general much weaker in metals than in semiconductors. This hurdle is even more daunting in type I superconductors [22], because the Meissner effect prevents the magnetic field from entering the sample. Thus the corresponding Hall voltage, equal to the circulation of the Hall electric field along the tiny London length, is likely to lie under the detection threshold. Conversely the experiment, discussed in section IV, does not suffer from this drawback, because the current induced magnetic field penetrates indeed into the whole bulk matter. Consequently, it could be performed even in high-T_c compounds at relatively high currents densities, yet giving rise to a magnetic field $< H_{c_1}$ in the *whole bulk matter* (to avoid vortices building up) and thereby ensuring good sensitivity.

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