Nuclear Structure Aspects of DCX Reactions and $\beta\beta$ Decay Transitions

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Abstract.
The theory of Double Charge Exchange (DCX) reactions between nuclei induces both the description of the reaction mechanism, which consists of the exchange of nucleons between the target and the projectile, as well as the structure of the matrix elements of the operators which mediate the transitions. From the point of view of the nuclear structure theory it requires the use of microscopic models for light and heavy mass nuclei. The reaction sector requires the use of momentum-dependent operators. It has been argued that these matrix elements could be associated to decay channels like the neutrinoless nuclear double beta decay ($0\nu\beta\beta$). The motivation related to this claim is the need to determine, as accurately as possible, the values of the matrix elements of operators involved in the decay. The detected mode (e.g. the two neutrino mode) does not allow for a determination of the effective electron neutrino mass. It is a decay where a mother nucleus with $A,N,Z$ components decay into a daughter nucleus with $A,N-2,Z+2$ components accompanied by the emission of two electrons and two antineutrinos.

Keywords: double charge exchange reactions, nuclear models, double beta decay modes

INTRODUCTION

The standard model of electroweak interactions assumes the maximal breaking of symmetry, only left handed currents resulting from the SU(2) left symmetry representation with three massive bosons mediating the decays between charged and neutral particles. It also assumes left handed doublets and right handed singlets for the leptonic sector of the theory. Neutrinos are represented as massless, an assumption which is at odds with the existence of neutrino oscillations. Then, the search for processes where the neutrinos may reveal their massive nature has become mandatory and several long range experimental efforts are devoted to this search. This is the case of the measurements of nuclear double beta decay transitions. The detected mode (e.g. the two neutrino mode) does not allow for a determination of the effective electron neutrino mass. It is a decay where a mother nucleus with $(A,N,Z)$ components decay into a daughter nucleus with $(A,N-2,Z+2)$ components accompanied by the emission of two electrons and two antineutrinos. The spectrum of the emitted electrons is a continuous one up to the Q-value of the decay. The half-life of it is of the order of $10^{24}$ years. Another decay mode with goes by the same initial and final nuclei is the neutrinoless double beta decay, where two electrons are emitted but not neutrinos, since they are emitted and absorbed as mediators of the decay. This mode, if observed as a single signal of two electrons with an energy-sum equal to the Q-value and opposite momenta, could determine the value of the effective neutrino mass provided the nuclear matrix elements of the operators involved in the decay are known. It will also be the experimental confirmation of a process which violates lepton number conservation. The actual limits on the non-observation of the decay are compatible with half-life of the order of (or larger than) $10^{24}$ years.

A Double Charge Exchange (DCX) reaction is a process induced by a projectile $a$ on a target $A$, in which two protons (neutrons) of the target are converted in two neutrons (protons), $\Delta A = \pm 2$, $\Delta N_A = \mp 2$, being the mass number $A$ unchanged, with opposite transitions simultaneously taken place in the projectile, $\Delta Z_a = \mp 2$, $\Delta N_a = \pm 2$. In the isospin representation, DCX reactions probe the double isovector excitations generated, at four-body level, by the $\tau^+_A \tau^-_a \tau^+_a \tau^-_A$ combination of the isospin rising and lowering operators acting on two nucleons in the projectile $a$ and the target $A$, respectively. There are two possible mechanisms to consider at the time of describing DCX reactions [6], namely:

- i) Two single charge exchange processes: $(A,N,Z) \to (A,N-1,Z+1) \to (A,N-2,Z+2)$, for the heavy nuclei (the target) and $(a,n,z) \to (a,n+1,z-1) \to (a,n+2,z-2)$ for the light projectile
- ii) A collective two proton-two neutron exchange process

Since in DCX and $0\nu\beta\beta$ the same nuclear systems participate, it was expected that the measurement of DCX reactions will fix the values of the decay channels of the lepton-number violating electroweak transitions, but as we are going to show in this note it is possibly not the case because of strong interference effects between the channels participant in the reactions due to their dependence upon the momentum transferred between nuclei.
THE FORMALISM

The cross section of DCX reactions are given by the equations:

\[
\frac{d\sigma}{dΩ} = \frac{k'}{k} \left( \frac{\mu}{4\pi\hbar^2} \right)^2 |T_\ell|^2
\]  

where \( \mu \) is the reduced mass of the target-projectile system, \( k \) and \( k' \) are the incoming and outgoing momentum and \( T_\ell \) is the T-matrix of the reaction.

\[
T_\ell = \langle \Psi^-_E\Phi_i \rvert V \rvert \Psi^+_k\Phi_i \rangle
\]

The transition amplitude \( M_\ell \), between the intrinsic nuclear states, is given by the expression:

\[
M_\ell = \langle \Phi_i \rvert \tilde{V}(\vec{r}) \rvert \Phi_i \rangle \ .
\]  

In these expressions \( \Psi^-_E \) and \( \Phi_i \) are the scattering waves and the intrinsic wave functions of the nuclei before and after the interaction, which can be written as the product of projectile(target) and nucleon wave function.

Each channel is described by the isospin and angular momenta of the projectile and target nuclei, \( V(\vec{r}) \) is the charge exchange potential, and \( \vec{r} \) is the relative distance of the involved nucleons in the target and projectile, defined as \( \vec{r} = \vec{R} + \vec{r}_T - \vec{r}_P \).

The momentum dependence of the amplitude \( M_\ell(\vec{q}) \) is given by

\[
M_\ell(\vec{q}) = \langle \Phi_i \rvert \Phi_f \rangle \ e^{-i\vec{q}\cdot\vec{r}_P} V_{CE}(\vec{q}) e^{i\vec{q}\cdot\vec{r}_T} \langle \Phi_f \rvert \Phi_i \rangle
\]  

where \( V_{CE}(\vec{q}) \) is the nucleon-nucleon charge-exchange effective potential. The expression of Eq.(3) in terms of the potential may be written as

\[
M_\ell(\vec{q}) = \frac{4\pi}{3} w(q) \sum_{\mu,\lambda} \sum_{\lambda'} Y_{1,\mu}(\hat{q}) Y_{1,-\mu}(\hat{q}) \langle \phi_{jlm} | \sigma_\mu \tau_\lambda e^{i\vec{q}\cdot\vec{r}_T} | \phi_{jlm} \rangle \langle \phi_{jlm} | \sigma_\mu \tau_\lambda e^{-i\vec{q}\cdot\vec{r}_P} | \phi_{jlm} \rangle
\]

where

\[
w(q) = -\left( \frac{f_\pi^2}{m_N^2} \right) \frac{q^2}{m_N^2 + q^2}
\]

and

\[
\langle \phi_{j'lm'} | \sigma_\mu \tau_\lambda e^{i\vec{q}\cdot\vec{r}} | \phi_{jlm} \rangle = 4\pi \sum_{LM} j_L j'_{LM}(\hat{q}) \langle \phi_{j'lm'} | \sigma_\mu \tau_\lambda Y_{LM}(\hat{r}) j_l(qr) | \phi_{jlm} \rangle ,
\]  

and

\[
\langle \phi_{j'lm'} | \tau_\lambda e^{i\vec{q}\cdot\vec{r}} | \phi_{jlm} \rangle = 4\pi \sum_{LM} j_L j'_{LM}(\hat{q}) \langle \phi_{j'lm'} | \tau_\lambda Y_{LM}(\hat{r}) j_l(qr) | \phi_{jlm} \rangle ,
\]  

are the nuclear matrix elements of the multipole, momentum dependent operators between nuclear states |\( \phi_{jlm} \rangle \).
NUCLEAR STRUCTURE ASPECTS OF THE CHARGE EXCHANGE REACTIONS AND DOUBLE BETA DECAY TRANSITIONS

As mentioned in the previous section, one may think of a sequence of two successive single charge exchange. Then, the reaction $a_i \times A_i \rightarrow a_n \times A_n \rightarrow a_f \times A_f$ implies the activation of the intermediate channels represented by the intermediate light and heavy nuclei $a_n$ and $A_n$ respectively. In terms of the operators which are acting on the nuclei we write, for each step of the reaction, starting with

$$\langle a_n, f'l'm'|\sigma_\mu \tau_2 e^{i\hat{q}\cdot\hat{r}}|A_i, jlm\rangle = 4\pi \sum_{IM} \int d^3r Y_{IM}^\ast (\hat{q}) \langle a_n, f'l'm'|\sigma_\mu \tau_2 Y_{IM}(\hat{r}) jj(qr)|A_i, jlm\rangle$$

$$\langle a_n, f'l'm'|\tau_2 e^{i\hat{q}\cdot\hat{r}}|A_i, jlm\rangle = 4\pi \sum_{IM} \int d^3r Y_{IM}^\ast (\hat{q}) \langle a_n, f'l'm'|\tau_2 Y_{IM}(\hat{r}) jj(qr)|a_i, jlm\rangle$$

and followed by the analogous process leading to the final state, with the amplitude:

$$\langle A_f, f'l'm'|\sigma_{-\mu} \tau_2 e^{i\hat{q}\cdot\hat{r}}|A_n, jlm\rangle = 4\pi \sum_{IM} \int d^3r Y_{IM}^\ast (\hat{q}) \langle A_f, f'l'm'|\sigma_{-\mu} \tau_2 Y_{IM}(\hat{r}) jj(qr)|A_n, jlm\rangle$$

$$\langle a_f, f'l'm'|\tau_2 e^{i\hat{q}\cdot\hat{r}}|A_n, jlm\rangle = 4\pi \sum_{IM} \int d^3r Y_{IM}^\ast (\hat{q}) \langle a_f, f'l'm'|\tau_2 Y_{IM}(\hat{r}) jj(qr)|a_n, jlm\rangle$$

Making use of these matrix elements, the transition amplitude $T_{if}$ is written

$$T_{if} = \sum_{A_i, a_i, q} \int d\hat{q}_1 d\hat{q}_2 d\hat{R} \langle A_f a_f|V_{CE}(\hat{R})|A_i a_i\rangle \left( \frac{\exp[i\hat{q}_1 \cdot \hat{R} + i\chi(b)]}{(E_{A_i}^n - E_{A_i}^f)^2 + (E_{E_i}^n - E_{E_i}^f)^2 + i\varepsilon} \right) \langle A_i a_i|V_{CE}(\hat{R})|A_n a_n\rangle$$

where with capital ($A_i$) and lower case ($a_i$) characters we have denoted the states of the heavy and light nuclei, respectively. By Fourier transforming $V_{CE}(\hat{R})$ we can rewrite $T_{if}$ as

$$T_{if} = \int d\hat{q}_1 d\hat{q}_2 d\hat{R} M(\hat{q}_1, \hat{q}_2) \exp \left[ i(\hat{q}_1 \cdot \hat{R} + \chi(b)) + i(\hat{q}_2 \cdot \hat{R}) \right]$$

Then, we further write the momentum dependent matrix elements as

$$M(\hat{q}_1, \hat{q}_2) = \int \frac{d^3r}{(E_{A_i}^n - E_{A_i}^f)^2 + (E_{E_i}^n - E_{E_i}^f)^2 + i\varepsilon} \langle A_i a_i|\langle \hat{q}_1 \cdot \hat{r} \rangle e^{-i(\hat{q}_2 \cdot \hat{r})}|A_n a_n\rangle$$

where

$$\langle A'|a'|V_{CE}(\hat{q})e^{-i(\hat{q}_2 \cdot \hat{r})}|Aa\rangle = \frac{4\pi}{3} n(a) \sum_{\mu, \lambda, \lambda'} Y_{1, \mu}(\hat{q}) Y_{1, \mu'}(\hat{q}) \langle \phi_{\lambda m}\gamma_\mu |\sigma_\mu \tau_2 e^{i\hat{q}\cdot\hat{r}}|\phi_{\lambda m}\gamma_{\mu'}\rangle$$

$$+ C_{GT} \sum_{\mu, \lambda} \langle \phi_{\lambda m}\gamma_{\mu} |\sigma_\mu \tau_2 e^{i\hat{q}\cdot\hat{r}}|\phi_{\lambda m}\gamma_{\mu'}\rangle$$

The differential cross-section is obtained by an average of the initial spins and the sum over the final spins of the final target and projectile

$$\frac{d\sigma}{d\Omega} = \frac{k'}{k} \left( \frac{2\pi n}{h} \right)^2 \frac{1}{(2j_f + 1)(2j_f + 1)} \sum_{\text{spins}} |T_{if}|^2$$
Let us take, as an example, the wave functions of the participant nuclei, when the heavy nuclei sector is represented by two-particle states with amplitudes calculated by means of collective multipole excitations on top of the ground state of the initial nuclei, we write, for the two-neutron, proton-neutron, and two-proton configurations

\[
| A_i(J\pi) \rangle = \frac{A(n_1, n_2, J\pi)}{\sqrt{1 + \delta(n_1, n_2)}} | (n_1 a_1)(n_2 a_1) \rangle_{J\pi}|0\rangle
\]

\[
| A_n(J\pi) \rangle = \sum_{p,n} A(p, n, J\pi) | a_1^\dagger(p)a_1(n) \rangle_{J\pi}|0\rangle
\]

\[
| A_f(J\pi) \rangle = \sum_{p_1, p_2} \frac{A(p_1, p_2, J\pi)}{\sqrt{1 + \delta(p_1, p_2)}} | a_1^\dagger(p_1) a_1^\dagger(p_2) \rangle_{J\pi}|0\rangle
\]

respectively. As usual, the wave functions and energy levels of the participant light nuclei are calculated by applying shell model techniques. In the shell model’s notation we write

\[
|JM\rangle = \sum_{n_i, p_i, J_{p_i}} A(J_n, J_p; J) \Pi_{\text{IM}} \left( \begin{matrix} I_{M_n}^a & I_{M_p}^a \\ I_{M_n}^c & I_{M_p}^c \end{matrix} \right) | \text{core}\rangle
\]

With these wave functions, the matrix element \(M(\kappa, \gamma, \lambda, q; J_i \rightarrow J_f)\) of the light nuclear sector is written

\[
M(\kappa, \gamma, \lambda, q; J_i \rightarrow J_f) = \frac{|J_f|}{|J_i|} \sum_{J_n, J_p} A(J_n, J_p; J) A(J_n, J_p; J_f) \sqrt{|J_p| |J_n| |J_{f_n}| |J_{f_p}| F_\kappa(q, pn)} \langle J_n || J^\kappa Y_\kappa || J_n \rangle (1/2 \| \sigma_\gamma \| 1/2) \left\{ \begin{array}{c} J_n & J_p & J_f \\ J_{f_n} & J_{f_p} & J_f \end{array} \right\} \left\{ \begin{array}{c} l_n & 1/2 & J_n \\ J_{f_n} & J_{f_p} & J_f \end{array} \right\} \left\{ \begin{array}{c} \bar{\kappa} & \gamma & \bar{\lambda} \\ J_{f_n} & J_{f_p} & J_f \end{array} \right\} \right\}
\]

(12)

For the heavy nuclei we have, for each operator \(\hat{O}_{\lambda,\mu}\) acting on them, the expression

\[
\sum \frac{1}{\sqrt{(1 + \delta_{(n_1, n_2)}) (1 + \delta_{(p_1, p_2)})}} \langle f \mid \hat{O}_{\lambda,\mu} \mid IM \rangle \langle IM \mid \hat{O}_{\lambda,\mu} \mid i \rangle = (-1)^{J_1 + J_2 + \lambda_1 + \lambda_2} \langle J_1 || \hat{O}_{\lambda_2,\mu_2} || J_2 \rangle \langle J_1, J_2, \lambda_1, \mu_1 || IM \rangle
\]

\[
\sum \frac{1}{\sqrt{(1 + \delta_{(n_1, n_2)}) (1 + \delta_{(p_1, p_2)})}} \langle f \mid \hat{O}_{\lambda,\mu} \mid IM \rangle \langle IM \mid \hat{O}_{\lambda,\mu} \mid i \rangle = (-1)^{J_1 + J_2 + \lambda_1 + \lambda_2} \langle J_1 || \hat{O}_{\lambda_2,\mu_2} || J_2 \rangle \langle J_1, J_2, \lambda_1, \mu_1 || IM \rangle
\]

(13)

The expression of the matrix elements, associated to the double beta decay transitions, depends on the mode of decay, then for the double beta decay with the emission of two neutrinos we write

\[
M^{(2\nu)}(J) = \sum_{k_1 k_2} \frac{M^2_k (1^+_{k_2}) (1^+_{k_1}) M_{k_2} (1^+_{k_1})}{(\frac{1}{2} \Delta + \frac{1}{2}(E(1^+_{k_1}) + E(1^+_{k_2})) - M c^2)/mc^2}.
\]

\[
\langle J^\pi \mid f_{k_1} \rangle = \sum_{p_m} \left[ X^f_{p_m} Y^{f_{k_1}}_{p_m} - Y^f_{p_m} X^{f_{k_1}}_{p_m} \right].
\]

where \(X\) and \(Y\) are the forward and backward going amplitudes of the intermediate \(f^\pi k_1\) states, obtained by the solutions of the Quasiparticle (or Particle-Hole) Random Phase Approximation [2]. It is a second order term, the intermediate states may be different for each side of the virtual transitions.
With these matrix elements, the transition densities are written

\[ M_k(1^n_k) = (1^n_k \| \sum_n t_n \sigma_n |0^+_n) , \quad M_k^f(1^n_k) = (J^+_k \| \sum_n t_n \sigma_n |1^n_k) \]

\[ M_k(1^n_k) = \frac{1}{\sqrt{3}} \sum_{pm} (p|\sigma|n) (1^n_k \| [c^+_p \tilde{c}_n]|0^+_n) , \]

\[ M_k^f(1^n_k) = \frac{1}{\sqrt{3}} \sum_{pm} (p|\sigma|n) (J^+_k \| [c^+_p \tilde{c}_n]|1^n_k) . \]

The use of the isospin ladder operators, \( t^\pm \), may not be correct for open shell systems, due to induced isospin violations. For allowed Gamow-Teller transitions the strength reads:

\[ GT_k^- = |(1^n_x \| \sum_n t_n \sigma_n |0^+_n)|^2 \]

\[ GT_k^+ = |(1^n_x \| \sum_n t_n \sigma_n |0^+_n)|^2 , \]

Other correlations, like IVSM modes, may add (subtract) to the strength due to couplings with the GT correlations. By the other hand, the half-life of \( 0\nu\beta\beta \) transitions reads

\[ t_{1/2}^{0\nu} = \frac{2}{\pi R_A} \int dq \frac{q h_K(q^2)}{q + \varepsilon_k (E_k + E_\lambda)/2} J_0(q r_{mn}) \]

and the matrix elements

\[ M_K^{(0\nu)} = \sum_{J,F,J',F'} \sum_{p,p'} (-1)^J \epsilon^{J,F,J'} \sqrt{2J+1} \left\{ \begin{array}{ccc} J & J & J' \\ J' & J & J' \end{array} \right\} (pp') : J' \| \sigma_k |mn' : J' \times (0^+_j \| [c^+_p \tilde{c}_n]|J_j) (J^+_k \| J^{p_j}_{J_j}) (J^+_k \| [c^+_p \tilde{c}_n]|0^+_n) \]

The multipole operators which appear in these expressions are written

\[ \sigma_F = h_F(r,E_k) , \quad \sigma_{GT} = h_{GT}(r,E_k) \sigma_1 \cdot \sigma_2 , \quad r = |r_1 - r_2| , \]

\[ |J^+_F M\rangle = \sum_{pm} (X^F_{pm} \left[ a_{j_1}^+ a_{j_2}^+ \right]_{JM} - Y^F_{pm} \left[ a_{j_1} a_{j_2}^+ \right]_{JM}) (\text{QRPA}) , \]

\[ (0^+_j \| [c^+_p \tilde{c}_n]|J^+_k) = \sqrt{2J+1} \left[ \tilde{c}_p \tilde{a}_n \delta_{J^k L} + \tilde{a}_p \tilde{c}_n \delta_{J^k L} \right] \]

\[ (J^+_k \| [c^+_p \tilde{c}_n]|0^+_n) = \sqrt{2J+1} \left[ a_p \tilde{c}_n \delta_{J^k L} + \tilde{a}_p c_n \delta_{J^k L} \right] . \]

The transition densities for all possible multipoles, are not always amenable to comparison with data. As it is seen from the above expressions and the ones which we have presented before for the double charge exchange reactions it becomes evident that the momentum dependence of both processes is quite different.
CONCLUSIONS

In this contribution we have focus the attention on the possible relation between double charge exchange reactions and double beta decay electroweak transitions. The interest in such a relation is prompted by the development of experimental efforts like NUMEN and others [3]. The main goal of such experimental efforts is the extraction of the matrix elements which determine the decay path by populating the same states via the reaction. This is relevant for the determination of the value of the effective neutrino mass. Here we have derived the explicit expressions of the matrix elements of the operators which participate in DCX reactions. From these expressions it may be concluded that, in contrast to notions advanced in previous studies [3], the extraction of information about nuclear matrix elements participant in double beta decay transitions of the zero neutrino type may not be directly linked to the information coming from analysis of DCX reactions, with the possible exception of the pure Gamow-Teller (σ) channels because of the significant reduction coming from the light nuclei sector, which strongly affect the momentum dependence of the products heavy-light nuclei which appear in the expression of the cross section. Calculations are in progress for DCX reactions on Te-isotopes. [9]

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