New Interacting Boson Fermion-Fermion Model results

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Abstract. We present a new application of the Interacting Boson Model (IBM) and its extensions for describing double charge exchange reactions. The study of double charge exchange reactions induced by heavy ions involving candidate nuclei for neutrinoless double beta decay is a complex task carried out by the NUMEN collaboration. This investigation faces the intricacies of complex odd-odd intermediate nuclei in sequential double charge exchange processes. We will offer a comprehensive description of heavy odd-odd nuclei using the Interacting Boson Fermion-Fermion Model (IBFFM). Additionally, we will outline the methodology for describing transfer operators within this framework. Finally, we will explore the potential applications of our results for describing double charge exchange reactions.

1 Introduction

This contribution aims to investigate the transition from an even-even nucleus to an odd-odd nucleus within the framework of the Interacting Boson Fermion-Fermion Model (IBFFM) [1]. The IBFFM extends the Interacting Boson Model (IBM) to describe odd-odd nuclei. In this model, the two fermions, one neutron and one proton, are coupled to the even-even core, which is described by the IBM framework [2]. These transitions are of significant interest in nuclear physics, as they provide insights into nuclear structure and properties. Understanding such transitions is essential for describing the complex behavior of heavy nuclei.

Moreover, we introduce a new application of spectroscopic amplitudes (SAs) within the IBFFM. These SAs play a crucial role in planned double charge exchange (DCE) reactions induced by heavy ions, conducted as part of the NUMEN (NUclear Matrix Elements for Neutrinoless double beta decay) Collaboration efforts in the study of the neutrinoless double beta decay nuclear matrix elements ($0\nu\beta\beta$ decay NME) [3, 4]. Specifically, the DCE reactions can be studied as two single charge exchange reactions passing through the odd-odd nucleus. It is necessary to describe the process from an even-even $0\nu\beta\beta$ decay candidate to an odd-odd nucleus, and from the odd-odd nucleus to the final even-even $0\nu\beta\beta$ daughter nucleus.

2 IBM and its extensions

We use the Interacting Boson Model 2 (IBM-2) and its extensions (IBFM-2 and IBFFM-2) [2, 5, 6], where we distinguish between protons and neutron degrees of freedom. The

Hamiltonian in IBM-2 is given as follows:

$$
\begin{aligned}
H^{\mathrm{B}} &= \epsilon_d\,(n_{d_\nu} + n_{d_\pi}) + \kappa\,(Q^{\mathrm{B}}_\nu \cdot Q^{\mathrm{B}}_\pi) \\
&+ \frac{1}{2}\xi_2\,((d^\dagger_\nu s^\dagger_\pi - d^\dagger_\pi s^\dagger_\nu)\cdot(\tilde{d}_\nu s_\pi - \tilde{d}_\pi s_\nu)) \\
&+ \sum_{K=1,3} \xi_K\,([d^\dagger_\nu \times d^\dagger_\pi]^{(K)} \cdot [\tilde{d}_\pi \times \tilde{d}_\nu]^{(K)}) \\
&+ \frac{1}{2}\sum_{K=0,2,4} c^{(K)}_\nu([d^\dagger_\nu \times d^\dagger_\nu]^{(K)} \cdot [\tilde{d}_\nu \times \tilde{d}_\nu]^{(K)}),
\end{aligned} \tag{1}
$$

where

$$Q^{\mathrm{B}}_\nu = d^\dagger_\nu s_\nu + s^\dagger_\nu \tilde{d}_\nu + \chi_\nu\,[d^\dagger_\nu \times \tilde{d}_\nu]^{(2)}, \tag{2}$$

$$Q^{\mathrm{B}}_\pi = d^\dagger_\pi s_\pi + s^\dagger_\pi \tilde{d}_\pi + \chi_\pi\,[d^\dagger_\pi \times \tilde{d}_\pi]^{(2)}. \tag{3}$$

Here, the subscripts π and ν refer to the proton and neutron, respectively.

2.1 IBFM-2

For describing odd-even nuclei, we utilized the IBFM-2. The Hamiltonian for odd-even nuclei (IBFM-2) is given by:

$$H = H^{\mathrm{B}} + H^{\mathrm{F}}_\rho + V^{\mathrm{BF}}_\rho. \tag{4}$$

Here, the boson Hamiltonian H^{B} represents the core Hamiltonian. The symbol ρ indicates the nucleon type, e.g., $\rho = \nu$ when is neutron and $\rho = \pi$ when is proton. V^{BF}_π or V^{BF}_ν describe the core-particle coupling of the odd proton or odd neutron in the IBFM-2 model [7–9] as the sum of a quadrupole term (Γ_ρ), an exchange term (Λ_ρ), and a monopole term (A_ρ):

$$V^{BF}_\rho = \Gamma_\rho Q^{(2)}_{\rho'} \cdot q^{(2)}_\rho + \Lambda_\rho F_{\rho'\rho} + A_\rho \hat{n}_{d_{\rho'}} \cdot \hat{n}_\rho \tag{5}$$

where $\rho, \rho' = \nu, \pi$ but $\rho' \neq \rho$. The first term in Eq. 5 is a quadrupole-quadrupole interaction with:

$$
\begin{aligned}
q^{(2)}_\rho &= \sum_{j_\rho, j'_\rho}(u_{j_\rho}u_{j'_\rho} - v_{j_\rho}v_{j'_\rho})Q_{j_\rho j'_\rho}(a^\dagger_{j_\rho} \times \tilde{a}_{j'_\rho})^{(2)} \\
Q^{(2)}_\rho &= (s^\dagger_\rho \times \tilde{d}_\rho + d^\dagger_\rho \times \tilde{s}_\rho)^{(2)} + \chi_\rho(d^\dagger_\rho \times \tilde{d}_\rho)^{(2)}.
\end{aligned} \tag{6}
$$

The second term is the exchange interaction:

$$
\begin{aligned}
F_{\rho,\rho'} &= -\sum_{j_\rho j'_\rho j''_\rho} \beta_{j_\rho j'_\rho}\beta_{j''_\rho j_\rho}\sqrt{\frac{10}{N_\rho(2j_\rho+1)}} \\
&\times\; Q^{(2)}_{\rho'}\cdot :[(d_\rho \times \tilde{a}_{j''_\rho})^{(j_\rho)} \times (a^\dagger_{j'_\rho} \times \tilde{s}_\rho)^{(j'_\rho)}]^{(2)} : +h.c.
\end{aligned} \tag{7}
$$

The coefficients $\beta_{j_\rho j'_\rho}$ are related to the single-particle matrix elements of the quadrupole operator $Q_{j_\rho j'_\rho}$ by

$$
\begin{aligned}
\beta_{j_\rho j'_\rho} &= (u_{j_\rho}v_{j'_\rho} + v_{j_\rho}u_{j'_\rho})Q_{j_\rho j'_\rho} \\
Q_{j_\rho j'_\rho} &= \langle j_\rho \| Y^{(2)} \| j'_\rho \rangle.
\end{aligned} \tag{8}
$$

The last term is the monopole-monopole interaction with:

$$\begin{aligned} n_{d_\rho} &= \sum_m d^\dagger_{\rho,m} d_{\rho,m} \\ \hat{n}_\rho &= \sum_{j_\rho} \hat{n}_{j_\rho} = \sum_{j_\rho,m} a^\dagger_{j_\rho,m} a_{j_\rho,m} \end{aligned} \tag{9}$$

2.2 IBFFM-2

We describe the odd-odd nucleus using the interacting boson-fermion-fermion model (IBFFM-2) [1, 6]. In this extension, the Hamiltonian is

$$H = H^{\mathrm{B}} + H^{\mathrm{F}}_\pi + V^{\mathrm{BF}}_\pi + H^{\mathrm{F}}_\nu + V^{\mathrm{BF}}_\nu + V_{\mathrm{RES}}, \tag{10}$$

where V_{RES} is the residual interaction between the odd proton and the odd neutron given as [1].

$$\begin{aligned} V_{\mathrm{RES}} &= 4\pi\, V_\delta\, \delta(\boldsymbol{r}_\pi - \boldsymbol{r}_\nu)\, \delta(r_\pi - R_0)\, \delta(r_\nu - R_0) \\ &\quad - \sqrt{3}\, V_{\sigma\sigma}\, (\boldsymbol{\sigma}_\pi \cdot \boldsymbol{\sigma}_\nu) \\ &\quad + 4\pi\, V_{\sigma\sigma\delta}\, (\boldsymbol{\sigma}_\pi \cdot \boldsymbol{\sigma}_\nu) \delta(\boldsymbol{r}_\pi - \boldsymbol{r}_\nu)\, \delta(r_\pi - R_0)\, \delta(r_\nu - R_0) \\ &\quad + V_T \left(3\, \frac{(\boldsymbol{\sigma}_\pi \cdot \boldsymbol{r}_{\pi\nu})(\boldsymbol{\sigma}_\nu \cdot \boldsymbol{r}_{\pi\nu})}{r^2_{\pi\nu}} - (\boldsymbol{\sigma}_\pi \cdot \boldsymbol{\sigma}_\nu) \right). \end{aligned} \tag{11}$$

The boson and the fermion Hamiltonian parameters are given in the previous sections.

The matrix elements of the residual interaction are calculated in the quasi-particle basis which is related to the particle basis by [5]

$$\begin{aligned} &\langle j'_\nu j'_\pi; J | V_{\mathrm{REF}} | j_\nu j_\pi; J \rangle_{\text{quasi-particle}} \\ &= (u_{j'_\nu} u_{j'_\pi} u_{j_\nu} u_{j_\nu} + v_{j'_\nu} v_{j'_\pi} v_{j_\nu} v_{j_\nu}) \langle j'_\nu j'_\pi; J | V_{\mathrm{RES}} | j_\nu j_\pi; J \rangle \\ &\quad - (u_{j'_\nu} v_{j'_\pi} u_{j_\nu} v_{j_\pi} + v_{j'_\nu} u_{j'_\pi} v_{j_\nu} u_{j_\pi}) \\ &\qquad \times \sum_{J'} (2J' + 1) \begin{Bmatrix} j_{\nu'} & j_\pi & J' \\ j_\nu & j_{\pi'} & J \end{Bmatrix} \langle j'_\nu j_\pi; J' | V_{\mathrm{RES}} | j_\nu j'_\pi; J' \rangle. \end{aligned} \tag{12}$$

The strengths of the delta interaction (V_δ), the spin-spin interaction ($V_{\sigma\sigma}$), the spin-spin-delta interaction ($V_{\sigma\sigma\delta}$) and the tensor interaction (V_T) are determined from a fit to the odd-odd experimental levels.

3 Transfer operators

In the IBFM-2 and IBFFM-2 models, we have to consider both boson and fermion degrees of freedom, thus we have to map the single particle operator to the IBFM-2 and IBFFM-2 spaces.

3.1 Operator in the IBFM-2

In the transitions between the even-even nucleus and odd-even nucleus, we have to distinguish the cases where we have the same core in the initial and final states and the cases where the number of bosons changes between the cores, thus we have two kinds of operators.

The operator for one nucleon transfer in which the number of bosons is conserved between cores is given by

$$P^{\dagger}_{j_\rho} = \xi_{j_\rho} a^{\dagger}_{j_\rho} + \sum_{j'_\rho} \xi_{j_\rho j'_\rho} [[s^{\dagger} \times \tilde{d}_{j_\rho}]^{(2)} \times a^{\dagger}_{j'_\rho}]^{(j_\rho)}, \tag{13}$$

where $s^{\dagger}$ is the creation operator and $s = \tilde{s}$ is the s-boson annihilation operator respectively, and $\tilde{d}$ is related to the d-boson annihilation operator by $\tilde{d}_\mu = (-1)^\mu d_{-\mu}$.

In case the boson number is changed by one unit, we have

$$Q^{\dagger}_{j_\rho} = \theta_{j_\rho} (s^{\dagger} \times \tilde{a}_{j_\rho})^{(j_\rho)} + \sum_{j'_\rho} \theta_{j_\rho j'_\rho} [d^{\dagger} \times \tilde{a}_{j'_\rho}]^{(j_\rho)}, \tag{14}$$

where the subindex refers to a proton or a neutron occupying the orbitals j_π and j_ν, respectively. The coefficients ξ_{j_ρ}, $\xi_{j_\rho j'_\rho}$ and θ_{j_ρ}, $\theta_{j_\rho j'_\rho}$ are the particle- or hole-coupling coefficients defined in Ref. [2, 9].

3.2 Operator in the IBFFM-2

In the transitions between the even-even nucleus and odd-odd nucleus, we follow the constructions of the operator in Ref. [6], where we have to couple two operators of the previous section. We get

$$[a^{\dagger}_{\rho} \times \tilde{a}_{\rho'}]^{(\lambda)} \to [P^{\dagger}_{j_\rho} \times \tilde{Q}_{j'_\rho}]^{(\lambda)} \equiv T^{(\lambda)}_{j_\rho j'_\rho}, \tag{15}$$

where $a^{\dagger}_{j_\rho}$ and $\tilde{a}_{j_{\rho'}}$ are the fermion creation and annihilation operators, while $P^{\dagger}_{j_\rho}$ and $\tilde{Q}_{j'_\rho}$ are the single nucleon transfer operator in the IBFM-2 scheme [9].

4 One nuclen transfer reaction

The competitive processes of nucleon transfers, including one nucleon and two nucleon transfers, have been extensively investigated within the theoretical frameworks of the IBM-2 and IBFM-2 schemes [10, 11]. Recently, our focus has been on the ^{76}Se (^{18}O,^{19}F) ^{75}As reaction, specifically studying its one-proton pick-up mechanism as part of the NUMEN [3, 4] and NURE (NUclear REactions for neutrinoless double beta decay) [12] projects. These investigations have underscored the crucial role of experimental data from NUMEN in refining and constraining nuclear models.

5 IBFFM spectroscopic amplitudes applications

The SAs between even-even and odd-odd nuclei in the IBFFM serve as crucial elements for upcoming experiments involving DCE reactions, particularly for the collaborative efforts of the NUMEN project and the study of $0\nu\beta\beta$ decay. These spectroscopic amplitudes, whether dressed as charge exchange operators or simple transition amplitudes, are essential for the theoretical framework of the NUMEN project, as outlined in Ref. [3, 4], when they are inserted into reaction codes. In a recent calculation, the SAs within the IBFFM were used to calculate the $0\nu\beta\beta$ nuclear matrix elements [6].

6 Conclusions

In conclusion, the recent experimental data provided by the NUMEN collaboration plays a crucial role in constraining nuclear models. This is particularly significant given the complex nature of nuclear reactions, where precise theoretical models are essential for accurate predictions. We use the IBFFM for describing odd-odd nuclei, which are intermediate states in a sequential double charge exchange processes. We compute the SAs within IBFFM to be utilized as inputs in reaction codes. Therefore, the synergy between experimental data from NUMEN and theoretical calculations will enhance our understanding of the candidates for $0\nu\beta\beta$ decay.

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