EPJ Web of Conferences **301**, 03002 (2024)
XLV Symposium on Nuclear Physics 2024
https://doi.org/10.1051/epjconf/202430103002

The properties of $\phi(2170)$ and its three-body nature

A. Martínez Torres[1,*], *Brenda* B. Malabarba[1,**], and *K. P.* Khemchandani[2,***]

[1]Universidade de Sao Paulo, Instituto de Fisica, C.P. 05389-970, Sao Paulo, Brazil.
[2]Universidade Federal de Sao Paulo, C.P. 01302-907, Sao Paulo, Brazil.

Abstract. In this talk we summarize the results we obtained for the partial decay widths of $\phi(2170)$ into two-body final states formed by a $\bar{K}$ and a Kaonic resonance, like $K(1460)$, $K_1(1270)$, as well as to final states constituted by a ϕ and an η/η' mesons. The results obtained are compared with the values extracted from experimental data on the corresponding branching ratios, which were determined by the BESIII collaboration. A reasonable agreement is found, which together with the previous reproduction of the mass, width and cross section for the process $e^+e^- \to \phi f_0$ strongly indicates the molecular nature of $\phi(2170)$ as a $\phi K\bar{K}$ system.

1 Introduction

Since its discovery in 2006 by the BaBar collaboration, several experimental collaborations have been trying to understand the properties of the $\phi(2170)$ meson [1–7]. Recently, the BESIII collaboration [6, 8, 9] have determined the product between the decay width of $\phi(2170) \to e^+e^-$ and the branching fraction of $\phi(2170) \to \bar{K}K_R$, $\phi\eta$, $\phi\eta'$, with K_R being a Kaonic resonance, from fits to the corresponding data. The results found seem to challenge the theoretical predictions for the partial decay widths of $\phi(2170)$ to the same $\bar{K}K_R$, $\phi\eta$, $\phi\eta'$ channels obtained within a $s\bar{s}$, hybrid or tetraquark picture for its inner structure [6, 8–11]. In Ref. [12], the $\phi K\bar{K}$ system was studied considering interactions in s-wave and the solution of the Faddeev equations was obtained for such system within the approach of Refs. [13–15]. As a result, the three-body T-matrix for the system shows the generation of a state with mass and width compatible with that of $\phi(2170)$ when the $K\bar{K}$ system is forming $f_0(980)$. The $e^+e^- \to \phi f_0(980)$ cross section determined by the BaBar collaboration was also well reproduced with the model of Ref. [12] by implementing the final state interaction in the $e^+e^- \to \phi f_0(980)$ cross section calculated with the approach of Ref. [16], which explained the background of the process, but not the signal observed for $\phi(2170)$.

In view of the recent data obtained by the BESIII collaboration about some partial decay widths of $\phi(2170)$, it would be interesting to know the corresponding values determined with the model of Ref. [12] and check if they are in agreement, or not, with the experimental data. Such a compatibility with the data is by no means trivial, since models considering $\phi(2170)$ as a $s\bar{s}$, hybrid, tetraquark, etc., do not seem to give compatible results for all the known experimental data for $\phi(2170)$, which include the previous mentioned partial decay widths, cross

*e-mail: amartine@if.usp.br
**e-mail: brenda@if.usp.br
***e-mail: kanchan.khemchandani@unifesp.br

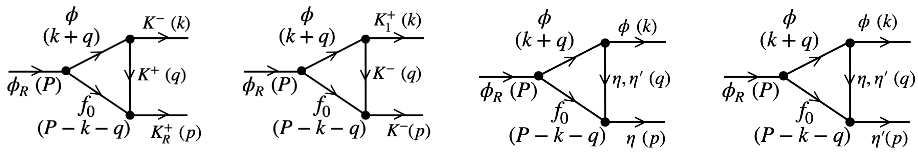

Figure 1. Decay mechanism for $\phi(2170)$ to $\bar{K}K_R$, $K_R \equiv K(1460)$, $\bar{K}K_1$, $K_1 \equiv K_1(1270)$, $K_1(1400)$, and to $\phi\eta$, $\phi\eta'$.

sections obtained from e^+e^- collisions, mass and width. For instance, the BESIII collaboration has found that the decay mode of $\phi(2170)$ to $K^*(892)\bar{K}^*(892)$ is suppressed as compared to other $\bar{K}K_R$ final states. This fact alone does not seem to be understood considering $\phi(2170)$ to be a $s\bar{s}$ or hybrid state.

2 Formalism

The partial decay widths of $\phi(2170)$ to the above mentioned channels not only depend on the nature of $\phi(2170)$, but also on that of the Kaonic resonances present in the final state, like $\mathbb{K} \equiv K(1460)$, $K_1(1270)$, $K_1(1400)$. Having a good description of the properties of these latter states is relevant to have reliable partial decay widths for $\phi(2170)$. In case of $K(1460)$, we consider the model of Ref. [17] in which the state is described from the $KK\bar{K}$ interaction, with a large coupling to the $Kf_0(980)$ configuration. In case of $K_1(1270)$ and $K_1(1400)$ we consider three different approaches: (1) In Ref. [18], the $K\rho$ and pseudoscalar-vector coupled channel dynamics was studied and generation of $K_1(1270)$ was found as a consequence of the superposition of two poles, one at $z_1 = M - i\Gamma/2 = 1195 - i123$ MeV and other at $z_2 = 1284 - i73$ MeV. In this case, no signal for $K_1(1400)$ was obtained. We call this model as A; (2) In Ref. [19], a tensor formalism for the vector mesons was used and $K_1(1270)$ and $K_1(1400)$ were described as states obtained from the mixing of the K_{1A} and K_{1B} states belonging to the nonet of axial resonances. Mixing angles of $29 - 62^\circ$ were shown to be compatible with the experimental data available for these states. We call this model as B; (3) Instead of relying on the results found within the previous two models for, for example, the coupling constants of the K_1 states to pseudoscalar-vector meson channels, we could directly use the data on the radiative decay of $K_1(1270)$ and $K_1(1400)$ available on the particle data book to estimate such couplings. We call this model C.

Having in ming the coupling of $K_1(1270)$ and $K_1(1400)$ to pseudoscalar-vector channels, the molecular nature of $\phi(2170)$ as a $\phi f_0(980)$ state and that of $f_0(980)$ as a state obtained from the $K\bar{K}$ and pseudoscalar-pseudoscalar coupled channel dynamics [20, 21], the decay of $\phi(2170) \to \bar{K}K_R$, $\phi\eta$ and $\phi\eta'$ proceeds as depicted in Fig. 1.

Following Refs. [12, 17, 18, 20], the states $\phi(2170)$, $K(1460)$, $K_1(1270)$, and $f_0(980)$ are generated from the s-wave interactions of three or two hadron systems. Thus, the contribution of the vertices $\phi(2170) \to \phi f_0(980)$, $f_0K^+ \to K^+(1460)$, $\phi \to K_1^+K^-$ present in the decay mechanisms of Fig. 1 can be written as,

$$\begin{aligned} t_{\phi_R} &= g_{\phi_R\to\phi f_0}\epsilon_{\phi_R}\cdot\epsilon_\phi, \quad t_{K_R} = g_{K_R^+\to K^+f_0}, \\ t_{f_0\to\mathcal{P}\mathcal{P}'} &= g_{f_0\to\mathcal{P}\mathcal{P}'}, \quad t_{K_1^+\to\phi K^+} = g_{K_1^+\to\phi K}\epsilon_{K_1^+}\cdot\epsilon_\phi, \end{aligned} \tag{1}$$

where $g_{i\to j}$ is the coupling for the process $i \to j$, $\mathcal{P}$ and $\mathcal{P}'$ represent pseudocalar particles and ϵ_k is the corresponding polarization vector for particle k. To determine the amplitude for the $\phi \to \mathcal{P}_1\mathcal{P}_2$ vertex, we consider the Lagrangian [22]

$$\mathcal{L}_{VVP} = -ig\langle V^{\mu}[\mathbb{P}, \partial_{\mu}\mathbb{P}]\rangle, \tag{2}$$

with V^{μ} and $\mathbb{P}$ being matrices having as elements the vector and pseudoscalar meson octet fields, respectively, $g = M_V/(2f_{\pi})$, $M_V \simeq M_{\rho}$, $f_{\pi} \simeq 93$ MeV, and $\langle\quad\rangle$ indicating the SU(3) trace. The coupling constant $g_{f_0\to\mathcal{P}\mathcal{P}'}$ is obtained from the residue of the two-body t-matrix describing the interaction between two pseudoscalars. This t-matrix is obtained by solving the Bethe-Salpeter equation with a kernel V which is determined from from the lowest-order chiral Lagrangian $\mathcal{L}_{\mathbb{P}\mathbb{P}}$, implementing the $\eta - \eta'$ mixing [23–25],

$$\mathcal{L}_{\mathbb{P}\mathbb{P}} = \frac{1}{12f^2}\langle(\partial_{\mu}\mathbb{P}\mathbb{P} - \mathbb{P}\partial_{\mu}\mathbb{P})^2 + M\mathbb{P}^4\rangle. \tag{3}$$

with

$$\mathbb{P} = \begin{pmatrix} A(\beta)\eta + B(\beta)\eta' + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & A(\beta)\eta + B(\beta)\eta' - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & C(\beta)\eta + D(\beta)\eta' \end{pmatrix}, \tag{4}$$

where

$$\begin{aligned} A(\beta) &= -\frac{\sin\beta}{\sqrt{3}} + \frac{\cos\beta}{\sqrt{6}}, \quad B(\beta) = \frac{\sin\beta}{\sqrt{6}} + \frac{\cos\beta}{\sqrt{3}}, \\ C(\beta) &= -\frac{\sin\beta}{\sqrt{3}} - \sqrt{\frac{2}{3}}\cos\beta, \quad D(\beta) = -\sqrt{\frac{2}{3}}\sin\beta + \frac{\cos\beta}{\sqrt{3}}, \end{aligned} \tag{5}$$

with the mixing angle β being between -15° to -22°, instead of simply considering ideal mixing (i.e., $\sin\beta = -1/3$, thus $\beta \simeq -19.47^{\circ}$) [26], and M is a matrix having as elements

$$M = \begin{pmatrix} m_{\pi}^2 & 0 & 0 \\ 0 & m_{\pi}^2 & 0 \\ 0 & 0 & 2m_K^2 - m_{\pi}^2 \end{pmatrix}, \tag{6}$$

where m_{π}, m_K represent the masses of the pion and of the kaon, respectively. When calculating the coupling of $f_0(980)$ to $\mathcal{P}\bar{\mathcal{P}}'$, two models were considered: (I) We use in Eq. (3) different weak decay constants for the pseudoscalars; (II) We consider a common value $f = f_{\pi} = 93$ MeV.

We refer the reader to Refs. [10, 11] for the values of the coupling constants involved in the vertices depicted in Fig. 1. Using the amplitudes of Eq. (1), we can determine the contribution of the processes depicted in Fig. 1, which depend on different tensor integrals. As a consequence of the four-momenta dependence of the vertices, these tensor integrals can be written as integrals in the loop variable d^4q of a numerator which depend on q_{μ}, $q_{\nu}q_{\mu}$, etc., and a denominator which is a function of q, P and k, with the latter dependence being related to the propagators of the particles in the triangular loops of Fig. 1. Using Lorentz covariance, we can write these tensor integrals in terms of linear combination of k_{μ}, P_{μ} or products of k_{μ} and P_{μ}, depending of the order of the tensor. Such a linear combination introduces several unknown coefficients, which need to be determine.

For example, the amplitude for the process $\phi(2170) \to \phi\mathcal{P}$ depicted in Fig. 1, where $\mathcal{P}$ represents, in this case, an η or η' meson, can be writen as [11]

$$it_{\phi_R\to\phi\mathcal{P}} = \sum_{\mathcal{P}'} 2g_{\phi_R\to\phi f_0}\, g_{f_0\to\mathcal{P}\bar{\mathcal{P}}'}\, g_{\phi\to\phi\mathcal{P}'}\, \epsilon^{\mu\nu\alpha\beta}\epsilon_{\phi_R\nu}(P)k_{\alpha}\epsilon_{\phi\beta}(k)I_{\mu}, \tag{7}$$

where I_μ is the following tensor integral:

$$I_\mu = \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \frac{q_\mu}{[(P-k-q)^2 - m_{f_0}^2 + i\epsilon]} \frac{1}{[(k+q)^2 - m_\phi^2 + i\epsilon][q^2 - m_{\mathcal{P}'}^2 + i\epsilon]}. \tag{8}$$

As a consequence of the Lorentz covariance, we can write I_μ in terms of k_μ and P_μ as

$$I_\mu = a_{\mathcal{P}'} k_\mu + b_{\mathcal{P}'} P_\mu, \tag{9}$$

where $a_{\mathcal{P}'}$ and $b_{\mathcal{P}'}$ are the mentioned unknown coefficients. To calculate them, we proceed as follows: Multiplying Eq. (9) by k^μ and P^μ, respectively, we get two coupled equations which permits to write $a_{\mathcal{P}'}$ and $b_{\mathcal{P}'}$ as

$$a_{\mathcal{P}'} = \frac{P^2(k\cdot I) - (k\cdot P)(P\cdot I)}{k^2P^2 - (k\cdot P)^2}, \quad b_{\mathcal{P}'} = -\frac{(k\cdot P)(k\cdot I) - k^2(P\cdot I)}{k^2P^2 - (k\cdot P)^2}, \tag{10}$$

where we have introduced

$$k\cdot I = \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \frac{k\cdot q}{[(P-k-q)^2 - m_{f_0}^2 + i\epsilon]} \frac{1}{[(k+q)^2 - m_\phi^2 + i\epsilon][q^2 - m_{\mathcal{P}'}^2 + i\epsilon]},$$
$$P\cdot I = \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \frac{P\cdot q}{[(P-k-q)^2 - m_{f_0}^2 + i\epsilon]} \frac{1}{[(k+q)^2 - m_\phi^2 + i\epsilon][q^2 - m_{\mathcal{P}'}^2 + i\epsilon]}. \tag{11}$$

By working in the rest frame of the decaying particle, i.e., $P^\mu = (P^0, \vec{0})$, with $P^0 = m_{\phi_R}$, we can express the previous integrals as

$$\begin{aligned} k\cdot I &= \int_{-\infty}^{\infty} \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dq^0}{(2\pi)} \frac{k^0q^0 - \vec{k}\cdot\vec{q}}{[(P-k-q)^2 - m_{f_0}^2 + i\epsilon]} \frac{1}{[(k+q)^2 - m_\phi^2 + i\epsilon][q^2 - m_{\mathcal{P}'}^2 + i\epsilon]} \\ &\equiv \int_{-\infty}^{\infty} \frac{d^3q}{(2\pi)^3} [k^0 \mathcal{I}_1(m_{f_0}, m_\phi, m_{\mathcal{P}'}) - \vec{k}\cdot\vec{q}\, \mathcal{I}_0(m_{f_0}, m_\phi, m_{\mathcal{P}'})], \\ P\cdot I &= P^0 \int_{-\infty}^{\infty} \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dq^0}{(2\pi)} \frac{q^0}{[(P-k-q)^2 - m_{f_0}^2 + i\epsilon]} \frac{1}{[(k+q)^2 - m_\phi^2 + i\epsilon][q^2 - m_{\mathcal{P}'}^2 + i\epsilon]} \\ &\equiv P^0 \int_{-\infty}^{\infty} \frac{d^3q}{(2\pi)^3} \mathcal{I}_1(m_{f_0}, m_\phi, m_{\mathcal{P}'}), \end{aligned} \tag{12}$$

where we have introduced

$$\mathcal{I}_n(m_1, m_2, m_3) \equiv \int_{-\infty}^{\infty} \frac{dq^0}{(2\pi)} \frac{(q^0)^n}{[(P-k-q)^2 - m_1^2 + i\epsilon]} \frac{1}{[(k+q)^2 - m_2^2 + i\epsilon][q^2 - m_3^2, + i\epsilon]} \tag{13}$$

with $n = 0, 1$. The integral in Eq. (13) can be calculated analytically by using Cauchy's theorem, finding for such the result

$$\mathcal{I}_n(m_1, m_2, m_3) = -i\frac{N_n(m_1, m_2, m_3)}{D(m_1, m_2, m_3)}, \tag{14}$$

The N_n and D in Eq. (14) depend on the energy of the particles involved in the loop and we refer the reader to Refs. [10, 11] for more details. The integral in d^3q in Eq. (12) can be obtained as

$$\int_{-\infty}^{\infty} \frac{d^3q}{(2\pi)^3}(\cdots) \to \int_0^{\infty} \frac{d|\vec{q}\,||\vec{q}\,|^2}{(2\pi)^2} \int_{-1}^{1} \mathrm{d}\cos\theta F(\Lambda, |\vec{k}+\vec{q}|)F(\bar{\Lambda}, |\vec{q}^{\,\mathrm{CM}}|)(\cdots), \tag{15}$$

where we consider $\vec{k} = |\vec{k}|\hat{z}$, and $\vec{q} = |\vec{q}\,|\sin\theta(\cos\phi\hat{i}+\sin\phi\hat{j})+|\vec{q}\,|\cos\theta\hat{k}$, such that $\vec{k}\cdot\vec{q} = |\vec{k}||\vec{q}\,|\cos\theta$ and, thus. the integral in $d\phi$ is trivial. In Eq. (15), F represents a product of form-factors introduced for the different vertices to take into account of the finite size of $\phi(2170)$, $f_0(980)$, etc., and Λ, $\bar{\Lambda}$ are cutoffs about 1000 MeV for the center-of-mass momentum of the particles forming these states. Typical expressions for the form factors in Eq. (15) are Lorentz [27],

$$F(\Lambda, |\vec{Q}|) = \frac{\Lambda^2}{\Lambda^2 + |\vec{Q}|^2}, \tag{16}$$

or Gaussian functions,

$$F(\Lambda, |\vec{Q}|) = e^{-\frac{|\vec{Q}|^2}{2\Lambda^2}}. \tag{17}$$

Once the coefficients appearing in the Lorentz expansion of the corresponding tensor integrals are determined, the partial decay width $\phi(2170) \to AB$ can be obtained by means of

$$\Gamma_{\phi_R\to AB} = \frac{|\vec{p}_{\mathrm{CM}}|}{24\pi m^2_{\phi_R}} \sum_{\mathrm{pol}} |t_{\phi_R\to AB}|^2, \tag{18}$$

with $|\vec{p}_{\mathrm{CM}}|$ being the modulus of the center-of-mass momentum of the particles in the final state and $\sum_{\mathrm{pol}}$ indicating the sum over the polarizations of the initial and final states.

3 Results

In Tables 1, 2 and 3 we show the results obtained within our description for the branching fractions

$$B_1 \equiv \frac{\Gamma_{\phi_R\to K^+(1460)K^-}}{\Gamma_{\phi_R\to K_1^+(1400)K^-}} = \frac{\mathcal{B}r[\phi_R \to K^+(1460)K^-]}{\mathcal{B}r[\phi_R \to K_1^+(1400)K^-]}, \tag{19}$$

$$B_2 \equiv \frac{\Gamma_{\phi_R\to K^+(1460)K^-}}{\Gamma_{\phi_R\to K_1^+(1270)K^-}} = \frac{\mathcal{B}r[\phi_R \to K^+(1460)K^-]}{\mathcal{B}r[\phi_R \to K_1^+(1270)K^-]}, \tag{20}$$

$$B_3 \equiv \frac{\Gamma_{\phi_R\to K_1^+(1270)K^-}}{\Gamma_{\phi_R\to K_1^+(1400)K^-}} = \frac{\mathcal{B}r[\phi_R \to K_1^+(1270)K^-]}{\mathcal{B}r[\phi_R \to K_1^+(1400)K^-]}. \tag{21}$$

The values listed in the above mentioned tables can be compared with those obtained from the experimental values: in Ref. [8], the values (in eV) for the products $\mathcal{B}r\Gamma_R^{e^+e^-}$ are

$$\begin{aligned}
&\mathcal{B}r[\phi_R \to K^+(1460)K^-]\Gamma_R^{e^+e^-} = 3.0 \pm 3.8,\\
&\mathcal{B}r[\phi_R \to K_1^+(1400)K^-]\Gamma_R^{e^+e^-} = \begin{cases} 4.7 \pm 3.3, & \text{Solution 1} \\ 98.8 \pm 7.8, & \text{Solution 2} \end{cases},\\
&\mathcal{B}r[\phi_R \to K_1^+(1270)K^-]\Gamma_R^{e^+e^-} = \begin{cases} 7.6 \pm 3.7, & \text{Solution 1} \\ 152.6 \pm 14.2, & \text{Solution 2} \end{cases}.
\end{aligned} \tag{22}$$

where two possible solutions for $\mathcal{B}r\Gamma_R^{e^+e^-}$ from the fits to the data were found in Ref. [8] in case of the decays $\phi(2170) \to K_1^+(1400)K^-$, $K_1^+(1270)K^-$. Using Eq. (22), we can obtain the experimental values for the B_1, B_2 and B_3 ratios of Eqs. (19)-(21), which are listed under the label “Experiment" in Tables 1-3. The theoretical values found for B_1, B_2 and B_3, as shown in Ref. [10] do not depend much on the form factor consider in the vertices involved in the mechanisms depicted in Fig. 1 and we provide here an average value of the results obtained with a Heaviside, a Lorentz and a Gaussian form factors.

As can be seen in Tables 1-3, we find compatible results with the values extracted from the experiment, however, there is a strong dependence of these ratios on the particular model used to describe $K_1(1270)$ and $K_1(1400)$. More precise data would be required to distinguish whether $K_1(1270)$ is a state generated from the pseudoscalar-vector dynamics considered in Ref. [18]. Note, however, that only if a superposition of the two poles obtained in Ref. [18] is considered in the calculation, a solution compatible with the value extracted from the experiment is obtained. Also, model B does not seem to give a good description of the ratio B_2.

Table 1. Results for the branching ratio B_1.

		B_1
Our results	Model B	0.62 ± 0.20
	Model C	0.11 ± 0.04
Experiment	Solution 1	0.64 ± 0.92
	Solution 2	0.03 ± 0.04

Table 2. Results for the ratio B_2.

		B_2	
Our results	Model A	1.3 ± 0.4	(Poles z_1, z_2)
		3.6 ± 1.2	(Pole z_1)
		8.8 ± 2.8	(Pole z_2)
	Model B	16 ± 6	
	Model C	1.2 ± 0.4	(Solution $\mathbb{S}_1$)
		0.12 ± 0.04	(Solution $\mathbb{S}_2$)
		0.05 ± 0.02	(Solution $\mathbb{S}_3$)
Experiment	Solution 1	0.40 ± 0.54	
	Solution 2	0.02 ± 0.03	

Table 3. Results for the ratio B_3.

		B_3	
Our results	Model B	0.04 ± 0.01	
	Model C	0.09 ± 0.02	(Solution $\mathbb{S}_1$)
		0.96 ± 0.16	(Solution $\mathbb{S}_2$)
		2.40 ± 0.40	(Solution $\mathbb{S}_3$)
Experiment	Solution 1	1.62 ± 1.38	
	Solution 2	1.55 ± 0.19	

The results for the ratio $R_{\eta/\eta'}$ between the widths of $\phi(2170) \to \phi\eta$ and to $\phi\eta'$ are summarized in Table 4. The results listed in this table should be compared with the ratio

$R^{\rm exp}_{\eta/\eta'} \equiv \mathcal{B}^{\phi(2170)}_{\phi\eta}\Gamma^{\phi(2170)}_{e^+e^-}/\mathcal{B}^{\phi(2170)}_{\phi\eta'}\Gamma^{\phi(2170)}_{e^+e^-}$ obtained by using the values $\mathcal{B}^{\phi(2170)}_{\phi\mathcal{P}}\Gamma^{\phi(2170)}_{e^+e^-}$ found in Refs. [6, 9]:

$$R^{\rm exp}_{\eta/\eta'} = \begin{cases} 0.034^{+0.018}_{-0.011} \text{ solution I,} \\ 1.42^{+0.58}_{-0.48} \text{ solution II,} \end{cases} \tag{23}$$

and with the results obtained for $R^{\rm exp}_{\eta/\eta'}$ by using for $\mathcal{B}^{\phi(2170)}_{\phi\eta}\Gamma^{\phi(2170)}_{e^+e^-}$ the value found in Ref. [7], which gives

$$R^{\rm exp}_{\eta/\eta'} = \begin{cases} 0.013 \pm 0.007 \text{ solution I,} \\ 0.009 \pm 0.003 \text{ solution II,} \\ 2.4 \pm 0.4 \text{ solutions III, IV.} \end{cases} \tag{24}$$

As in case of the previous decay widths, several solutions where found for this ratio by using the experimental data. Considering the values listed in Table 4, we find that mixing angles of $\simeq 22^\circ$ give rise to values for $R_{\eta/eta'}$ which are closer to the upper limit of the solution II of Eq. (23) and solutions III, IV of Eq. (24).

Table 4. Values for the ratio $R_{\eta/\eta'}$ considering different $\eta-\eta'$ mixing angles, β, and form factors. The labels L and G indicate the consideration of a Lorentz (L) or Gaussian (G) form factors, while the numbers I and II refer to the model used to calculate the $\mathcal{P}\bar{\mathcal{P}}'$ t-matrix.

β (Degree)		−15	−19.47	−22
$R_{\eta/\eta'}$	LI	5.12 ± 1.57	3.93 ± 1.21	3.39 ± 1.04
	GI	5.47 ± 1.68	4.21 ± 1.29	3.63 ± 1.11
	LII	4.21 ± 1.29	3.25 ± 1.00	2.80 ± 0.86
	GII	4.41 ± 1.35	3.40 ± 1.04	2.93 ± 0.90

It is worth stressing that, in spite of the considerable experimental uncertainty obtained for the previous obtained ratios, models considering $\phi(2170)$ as a $s\bar{s}$ states, a hybrid, etc., have real challenges in finding a good reproduction of these ratios, together with the mass and width of $\phi(2170)$.

4 Conclusions

In this work, we have summarized our findings for the branching ratios of $\phi(2170)$ to final states involving a $\bar{K}$ and a Kaonic resonance or a ϕ and an η/η' mesons. The description of $\phi(2170)$ as a $\phi f_0(980)$ molecular state produce values compatible with the experimental findings, reinforcing the interpretation of $\phi(2170)$ as a state generated by the three-body dynamics involved in the $\phi K\bar{K}$ system in isospin 0, with s-wave interactions and in which the $K\bar{K}$ subsystem resonates as $f_0(980)$. The values obtained for these ratios depend on the nature of the Kaonic resonances involved in the final state as well and more precise data are needed to disentangle whether $K_1(1270)$ is a molecular state obtained from pseudoscalar-vector dynamics and the hadron-hadron component in $K_1(1400)$.

References

[1] B. Aubert et al. (BaBar), Phys. Rev. D **74**, 091103 (2006), `hep-ex/0610018`
[2] B. Aubert et al. (BaBar), Phys. Rev. D **77**, 092002 (2008), `0710.4451`
[3] M. Ablikim et al. (BES), Phys. Rev. Lett. **100**, 102003 (2008), `0712.1143`

[4] C.P. Shen et al. (Belle), Phys. Rev. D **80**, 031101 (2009), 0808.0006
[5] M. Ablikim et al. (BESIII), Phys. Rev. D **99**, 032001 (2019), 1811.08742
[6] M. Ablikim et al. (BESIII), Phys. Rev. D **102**, 012008 (2020), 2003.13064
[7] W.J. Zhu et al. (Belle), Phys. Rev. D **107**, 012006 (2023), 2209.00810
[8] M. Ablikim et al. (BESIII), Phys. Rev. Lett. **124**, 112001 (2020), 2001.04131
[9] M. Ablikim et al. (BESIII), Phys. Rev. D **104**, 032007 (2021), 2104.05549
[10] B.B. Malabarba, X.L. Ren, K.P. Khemchandani, A. Martinez Torres, Phys. Rev. D **103**, 016018 (2021), 2011.03448
[11] B.B. Malabarba, K.P. Khemchandani, A. Martinez Torres, Phys. Rev. D **108**, 036010 (2023), 2305.03885
[12] A. Martinez Torres, K.P. Khemchandani, L.S. Geng, M. Napsuciale, E. Oset, Phys. Rev. D **78**, 074031 (2008), 0801.3635
[13] A. Martinez Torres, K.P. Khemchandani, E. Oset, Phys. Rev. C **77**, 042203 (2008), 0706.2330
[14] K.P. Khemchandani, A. Martinez Torres, E. Oset, Eur. Phys. J. A **37**, 233 (2008), 0804.4670
[15] A. Martinez Torres, K.P. Khemchandani, E. Oset, Phys. Rev. C **79**, 065207 (2009), 0812.2235
[16] M. Napsuciale, E. Oset, K. Sasaki, C.A. Vaquera-Araujo, Phys. Rev. D **76**, 074012 (2007), 0706.2972
[17] A. Martinez Torres, D. Jido, Y. Kanada-En'yo, Phys. Rev. C **83**, 065205 (2011), 1102.1505
[18] L.S. Geng, E. Oset, L. Roca, J.A. Oller, Phys. Rev. D **75**, 014017 (2007), hep-ph/0610217
[19] J.E. Palomar, L. Roca, E. Oset, M.J. Vicente Vacas, Nucl. Phys. A **729**, 743 (2003), hep-ph/0306249
[20] J.A. Oller, E. Oset, Nucl. Phys. A **620**, 438 (1997), [Erratum: Nucl.Phys.A 652, 407–409 (1999)], hep-ph/9702314
[21] J.A. Oller, E. Oset, J.R. Pelaez, Phys. Rev. D **59**, 074001 (1999), [Erratum: Phys.Rev.D 60, 099906 (1999), Erratum: Phys.Rev.D 75, 099903 (2007)], hep-ph/9804209
[22] M. Bando, T. Kugo, K. Yamawaki, Nucl. Phys. B **259**, 493 (1985)
[23] P. Herrera-Siklody, J.I. Latorre, P. Pascual, J. Taron, Nucl. Phys. B **497**, 345 (1997), hep-ph/9610549
[24] R. Kaiser, H. Leutwyler, Eur. Phys. J. C **17**, 623 (2000), hep-ph/0007101
[25] W.H. Liang, E. Oset, Phys. Lett. B **737**, 70 (2014), 1406.7228
[26] J.X. Lin, J.T. Li, S.J. Jiang, W.H. Liang, E. Oset, Eur. Phys. J. C **81**, 1017 (2021), 2109.08405
[27] D. Gamermann, J. Nieves, E. Oset, E. Ruiz Arriola, Phys. Rev. D **81**, 014029 (2010), 0911.4407