# A Novel Fission-Matrix Based Radial Boundary Correction Methodology and its Implementation into the RAPID Code System

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**Abstract.** The RAPID code system utilizes a novel combined fission matrix methodology to rapidly solve the whole-core eigenvalue problem. To represent axial and radial reflector regions, the fission density distribution in the boundary assemblies, a fission density correction factor methodology was devised and demonstrated. Although, accurate results were achieved, because of the need for a full core Monte Carlo calculation, this technique requires significant computing resources. To address this issue, this paper introduces a novel FM based methodology, which achieves accurate results, however at very small computing expense. The methodology has been implemented into the RAPID code system, and successfully demonstrated for the Watts Bar whole core OECD/NEA

## 1 Introduction

Nuclear reactors necessitate regular modeling for reactor core design and analysis. These models are typically simulated using a Monte Carlo code [1], or a deterministic code based on the Linear Boltzmann Equation (LBE) or transport equation [2, 3]. Achieving high-fidelity solutions is highly challenging for Monte Carlo and deterministic codes and requires parallel computing resources.

The RAPID code has been developed based on the Multistage Response function Transport (MRT) methodology [4]. This methodology involves partitioning a problem into stages or sub-problems that can be simulated using response functions or coefficients. By pre-calculating these functions/coefficients based on different parameters using a Monte Carlo code, solutions can be obtained using a linear system of equations in seconds or minutes on a single computer core.

The current version of the RAPID code is capable of solving various neutronics problems, including critical and sub-critical systems [5–7], kinetics, burnup [8, 9], control rod movements [7], and detector response [5, 6]. RAPID has been evaluated for use in reactor core calculations, and achieved accurate axially-dependent pin-wise fission neutron distributions within seconds or minutes on a single computer core. To treat the boundary assemblies correctly, a 'correction factor' technique was devised and successfully demonstrated [10]. This

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technique, however, relies on whole-core reactor Monte Carlo calculations (actual model and two extended models) that are computationally expensive, i.e., they require 100 hours of CPUs. To address this issue, this paper introduces a novel fission-based methodology. This methodology has been implemented into the RAPID code system and examined using the Watts Bar Unit 1 benchmark.

## 2 RAPID Code Methodology

The RAPID code utilizes the MRT methodology to divide a problem into stages, as illustrated in [4]. These stages can be categorized into pre-calculation stages for obtaining response functions/coefficients, and real-time calculations for obtaining problem solutions. For pre-calculation and post-processing, RAPID includes a routine known as pRAPID.

For criticality calculations, RAPID code [5, 11] utilizes a fission matrix (FM) formulation to carry out multiplication calculations. This formulation is derived from the LBE, which can be written in matrix form [1] as

$$\boldsymbol{F} = \frac{1}{k}\boldsymbol{A}\boldsymbol{F} \tag{1}$$

Its discretized form for N fission regions is given by,

$$F_i = \frac{1}{k}\sum_{j=1}^{N} a_{ij}F_j \tag{2}$$

where

$F_i$ = fission neutron source in spatial region i
$a_{i,j}$ = fission matrix coefficient
$N$ = total number of spatial cells in the model

The FM coefficients, $a_{i,j}$, represent the number of the fission neutrons produced in spatial cell *i* per source neutron in cell *j*.

To be able to obtain accurate solutions for models containing various types of assemblies, a novel combined FM (CFM) methodology [5] is developed and effectively used in RAPID.

## 3 Radial Boundary Correction Factors Methodology

Assemblies located at the core boundary differ from other assemblies because they are surrounded by a reflector region. A boundary correction factor methodology is implemented to account for this difference [5].

To calculate the correction factors, we consider two fixed fission source models and simulate them using the Serpent Monte Carlo code. The first model represents the reactor core with a reflector, while the second model represents a reactor model with an extended core. We obtain the fission neutrons distribution, $f(x, y)$, from the first model and the fission neutrons distribution, $\tilde{f}(x, y)$, from the second model. The correction factor in mesh cell $i$ is then obtained as:

$$bnd(x_i, y_i) = f(x_i, y_i)/\tilde{f}(x_i, y_i) \tag{3}$$

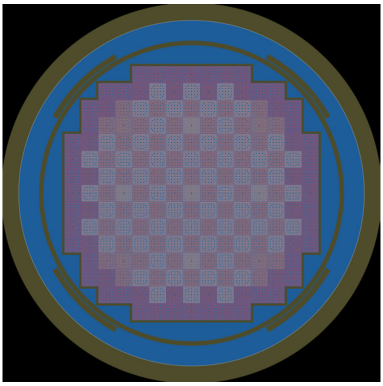


Figure 1: Watts Bar Unit 1 Core Radial Cross Section

This factor is used to correct the FM coefficients as follows,

$$a_{ij} = bnd(x_i, y_i)\tilde{a}_{ij} \tag{4}$$

where $\tilde{a}_{ij}$ is refer to FM coefficients without boundary correction.

## 4 Fission Matrix Based-Boundary Correction Methodology

Based on our previous discussion, the assumption we used to calculate the fission matrix coefficients breaks down at the boundary due to the finite nature of the core. To utilize the FM method to represent the reflector region, we consider an assembly containing water with a minimal concentration of U235. Then, we determine the FM coefficients for this "reflector" assembly. This means that the "reflector assembly" is another type of assembly, and it surrounds the core.

## 5 Watts Bar Benchmark Model

The Watts Bar benchmark was developed by the OECD/NEA and is based on the Watts Bar Unit 1 reactor. The benchmark is designed for neutronics and thermal hydraulics codes [12]. For this paper, we are interested in exercise 2: Hot Full Power (HFP), Beginning of Cycle (BOC). One modification was made to the model, instead of using the designated start-up rod positions, the model was run in an All Rods Out (ARO) configuration to simplify the calculations.

The core is loaded with 193 Westinghouse assemblies as shown in Figure 1. Each assembly contains 264 fuel rods, utilizing $UO_2$ fuel, 24 Zircaloy-4 guide tubes for control rod insertion, and a central instrument thimble tube. The rod pitch for the assembly is 1.26 cm, and the assemblies have a half gap of 0.04 cm.

The active region of the fuel rods is 365.74 cm long with a Zircaloy-4 cladding. The control rods are comprised of a Silver-Indium-Cadmium (AIC) region and a boron carbide (B4C) region [12] with a Stainless Steel-304 (SS304) cladding. The Pyrex rods are made of borosilicate annular glass containing 6.24 mg/cm of $^{10}B$, with SS304 cladding. There are 10 assembly configurations, three with no Pyrex rods and enrichments of 2.11%, 2.619%, and 3.10%, three at 2.619% enrichment with 16, 20, or 24 Pyrex rods, and four at 3.10% with 8, 12, 20, or 24 Pyrex rods. All the unique configuration is shown in Figure 2, representing 0, 8, 12, 16, 20, and 24 Pyrex rods respectively. Regions with the Pyrex rods are shown with orange and white concentric rings.

The reactor model includes the core's structural components, including the baffle, the barrel, 4 neutron pads and the pressure vessel. The model also uses natural boron as a chemical shim, with a concentration of $1291ppm$ for HZP, ARO, BOC criticality. The moderator initial temperature is specified as 565 $^{\circ}K$ with a density of 0.743 $g/cm^3$.

## 6 Fission Matrix Database

To determine FM coefficients in the CFM methodology, we are considering octal symmetry and an axial translation approach [5]. Since there are 39 fuel pins (in an assembly with octal symmetry) and 10 unique assembly types plus one "reflector" assembly, 429 fixed-source Serpent calculations are performed. For each calculation, $10^9$ particles are simulated. Table 1 provides the timing for the pre-calculation of FM coefficients.

| Calculation | Time | Processors |
|---|---|---|
| Single Source | $\approx 12min$ | 4 |
| 429 Sources | $\approx 1.65days$ | 56 |

Table 1: FM pre-calculation times

Since fixed source calculations are independent, table 1 indicates that in this study the pre-calculations could be accomplished in about 12 min using 1716 computer cores.

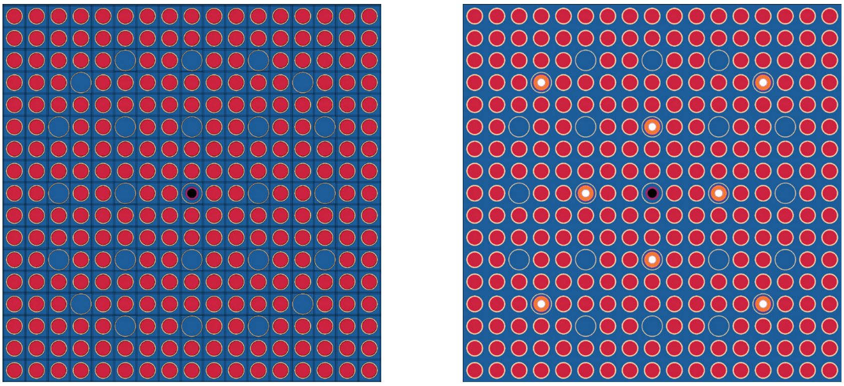
## 7 RAPID Results

The RAPID code system utilizes the CFM methodology, with FM coefficients generated through Serpent fixed-source calculations. Figures 3 and 4 display the axially-dependent (144 levels) pin-wise fission neutrons distribution for the Watts Bar Unit 1 core as determined by the RAPID code.

To verify the RAPID core results, we created a Serpent model. The criticality parameters specified for the Serpent model were $5 \times 10^6$ neutrons per cycle (NPS), 1000 active cycles (NAC), and 500 skip cycles (NSC). To ensure a reasonable level of uncertainty, we tallied the axially dependent, assembly-wise fission neutrons. The fission neutrons distribution for Watts Bar Unit 1 core from Serpent calculations is shown in Figure 5.

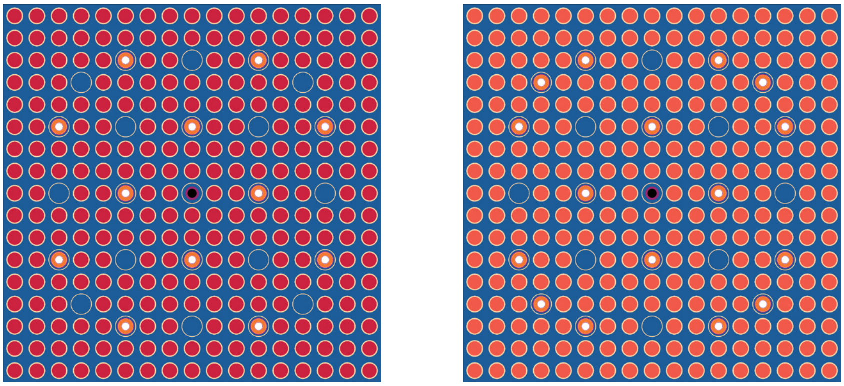
Table 2 compares the core eigenvalue calculated by Serpent to those calculated with RAPID without boundary correction and with boundary correction at different concentrations of U235 (0.31%, 0.031%, 0.0031%). It is demonstrated that the relative differences of the three cases with boundary conditions are similar and are within 10 pcm as compared to the Serpent calculations.

Figure 2: Westinghouse 17x17 assembly configurations



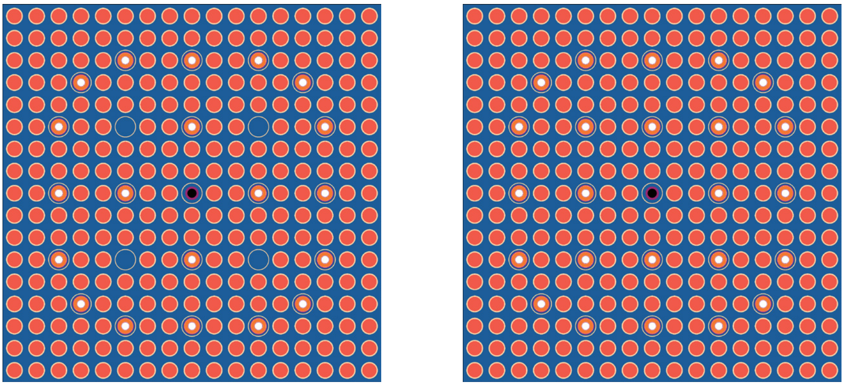
(a) Unfilled guide tubes

(b) 8 Pyrex Rods



(c) 12 Pyrex Rods

(d) 16 Pyrex Rods



(e) 20 Pyrex Rods

(f) 24 Pyrex Rods

Table 3 shows the timing and computer resources used for RAPID and Serpent simulations for the Watts Bar Unit 1 reactor core.

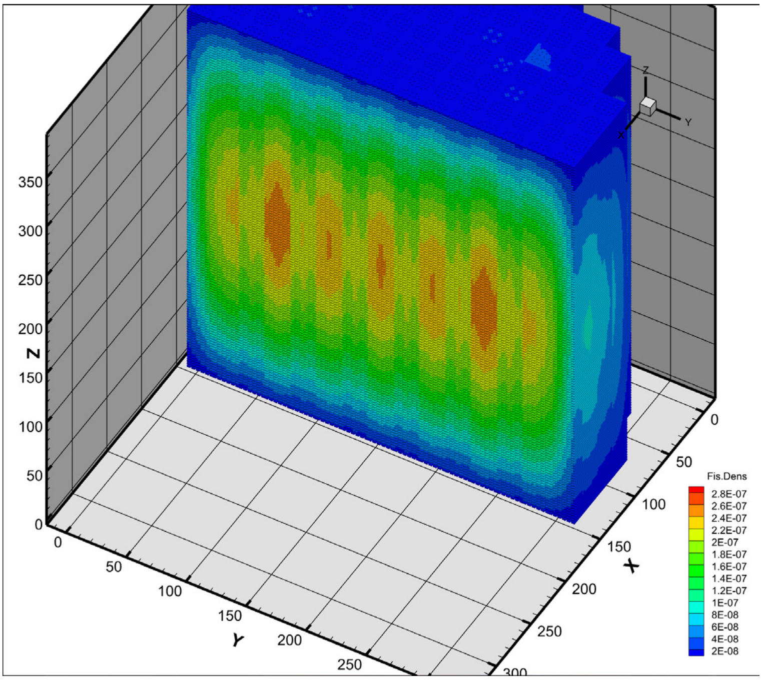


Figure 3: RAPID pin-wise Axial fission neutrons density for Watts Bar

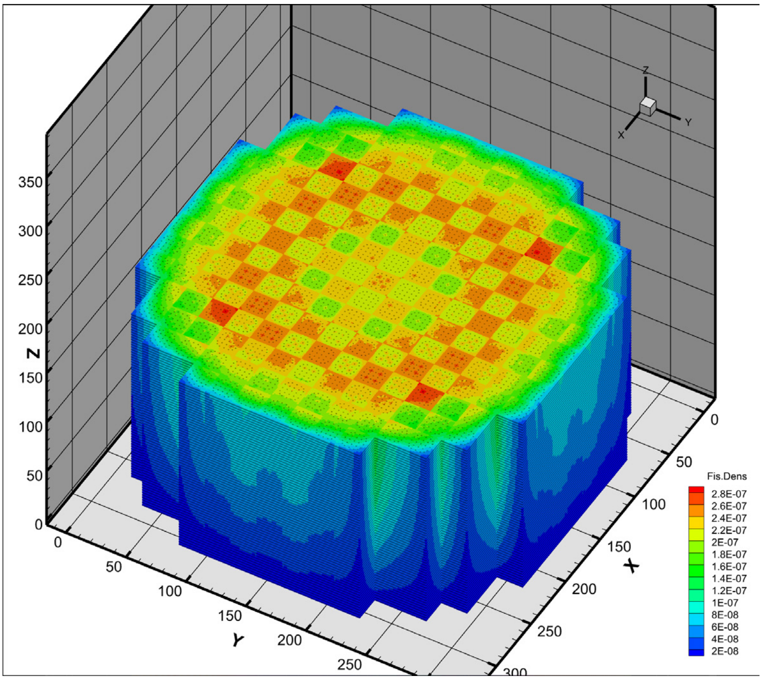


Figure 4: RAPID pin-wise Radial fission neutrons density for Watts Bar

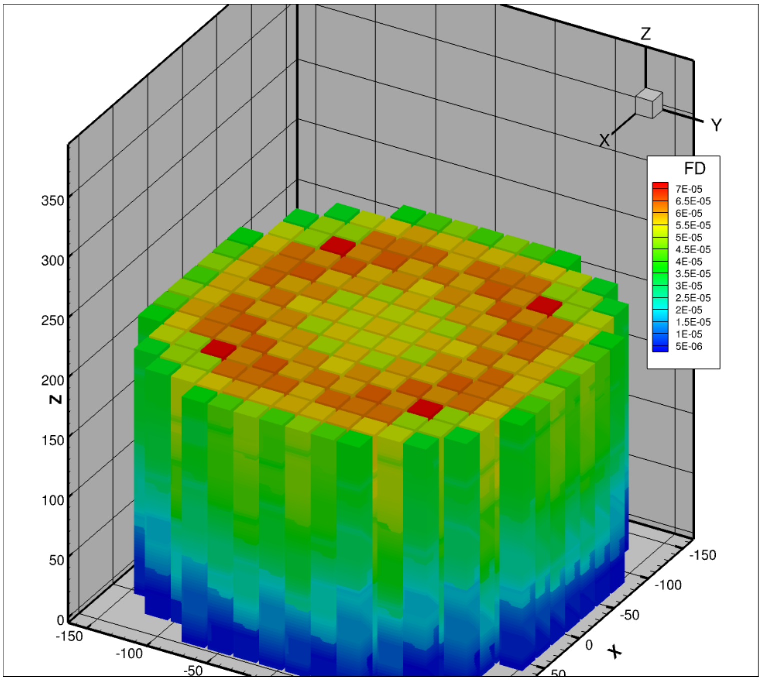


Figure 5: Serpent assembly-wise fission neutrons density for Watts Bar

| Code | $k_{eff}$ | \|Rel. Diff\| |
|---|---|---|
| Serpent | 0.998287 ± (.56 pcm) | – |
| RAPID | 0.997555 | 73.3 pcm |
| RAPID+BND(0.31%) | 0.998381 | 9.4pcm |
| RAPID+BND(0.031%) | 0.998381 | 9.4pcm |
| RAPID+BND(0.0031%) | 0.998148 | 13.9pcm |

Table 2: System eigenvalue results

| Code | # of Cores | Run Time | Speed-up |
|---|---|---|---|
| Serpent | 40 | 10,944 min | 1 |
| RAPID | 1 | 7.11min* | 1539 |
| RAPID(0.31%) | 1 | 7.11min* | 1539 |
| RAPID(0.031%) | 1 | 7.11min* | 1539 |
| RAPID(0.0031%) | 1 | 7.11min* | 1539 |

*This does not include the pre-calculation time

Table 3: Calculation run times

To examine the effectiveness of the new methodology, we evaluated the ratio of calculated fission neutrons with and without boundary correction. Figure 6 shows the distribution of the fission neutron ratio throughout the core. This distribution indicates that the impact of

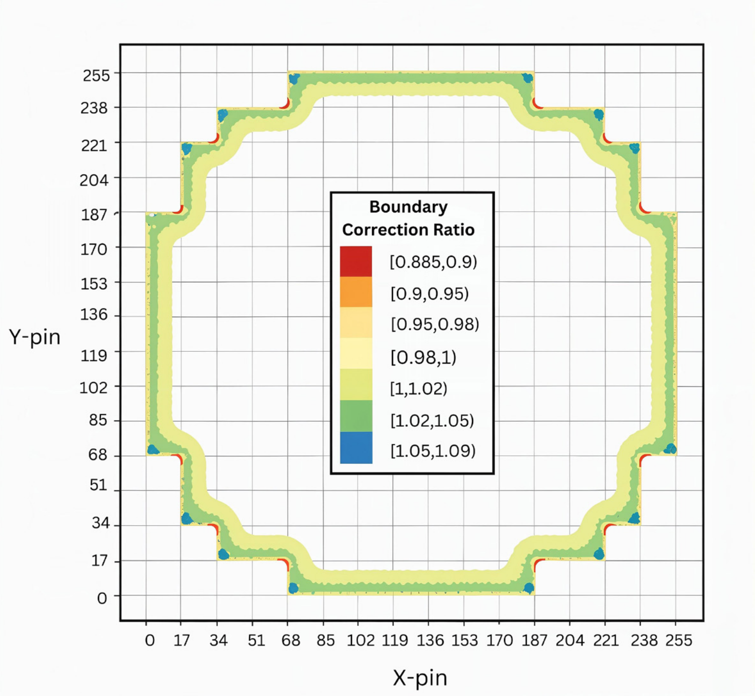


Figure 6: Radial Correction Ratios

boundary correction is only observed in the boundary assemblies. Also, the observed ratio changes within ± 10%, as reported in a previous similar study [10].

## 8 Conclusion and Ongoing Work

This paper presents a new methodology for addressing the boundary condition in the RAPID code system. We compared this methodology with the ratio correction methodology and found that both methods yield consistent results. However, integrated with the RAPID code system, the proposed methodology demonstrates significantly faster processing times than the ratio correction method. Our future work will focus on testing this methodology across various core models.

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