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Impact of Malfunctioning Sensors on Data-Driven Reduced Order Modelling: Application to Molten Salt Reactors

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Abstract. Over the years, the development of Data-Driven Reduced Order Modelling (DDROM) techniques has paved the way for novel approaches to combine the physical knowledge built in *high-fidelity* simulations with the physical observations from experimental measurements. On the one hand, these approaches allow updating and correcting the background information obtained from the physical model; on the other hand, they allow overcoming the sparsity of observations for a global state estimation. For these reasons, these approaches are of interest for applications where one of the two sources of information is incomplete: for example, for applications related to Circulating Fuel Reactors, such as the Molten Salt Fast Reactor. These reactors are characterised by a hostile and harsh environment and by the absence of solid structures inside the core, making the monitoring of the quantities of interest inside the core a challenging task. Many works of literature on DDROM assume that experimental data represent the truth, and although extensive research has been done on noisy sensors, few works of literature analyse what happens to the state estimation when one or more sensors malfunction. Then, the robust and reliable application of DDROM techniques, requires first investigating how their performance is affected by malfunctioning sensors. This work aims to investigate this aspect in the context of modelling and simulating the system response during an accidental transient occurring in a Molten Salt Fast Reactor, considering the impact of failed sensors on the performance of Data-Driven Reduced Order Modelling techniques. Quite importantly, this work also proposes a strategy based on Supervised Learning to compensate for the failed sensors.

1 Introduction

In the field of nuclear reactor engineering, and complex engineering systems in general, the state-of-the-art for the analysis techniques of the system behaviour rely on physics-based *high-fidelity* simulations. However, the computational burden demanded for these simulations makes certain tasks such as real-time control and dynamic monitoring unfeasible. Researchers have developed various approaches to reduce the computational burden of numerical simulations, in particular for real-time applications. One example is given by Reduced Order Modelling (ROM) techniques [1]. Among the available ROM techniques proposed in the literature, this work focuses on linear dimensionality reduction by a Reduced Basis approach: given a *high-fidelity* model, called the Full Order Model (FOM), and some solutions

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of the FOM, the fundamental modes [2], representing the dominant physics of the problem can be extracted. These modes define a surrogate low-dimensional space, much smaller than the number of degrees of freedom of the FOM, in which a Reduced Order Model is built. Indeed, this reduction makes it possible to obtain a fast and efficient model to estimate physical parameter values not included into the dataset of solutions of the FOM, without having to solve the FOM itself again. In addition, reduction techniques can help speed up Data Assimilation (DA) [2] algorithms with experimental data by reducing the complexity of the optimisation problem, which represents the bottleneck for most state-of-the-art DA methods.

The methods applied in this work belong to the Data-Driven Reduced Order Modelling (DDROM) framework defined by the authors in [3], which can be seen as a branch of Scientific Machine Learning [4]. In particular, DDROM focuses on updating and correcting the background model [5] using physical observations and on the optimal positioning of experimental sensors to collect measurement data [6]. Within the framework of DDROM, the Generalised Empirical Interpolation Method (GEIM) [6] and the Parameterised-Background Data-Weak (PBDW) formulation [5] are important examples used in the field of nuclear reactor engineering [3, 7–9]. Yet, most methods consider the (noisy) measurements obtained from sensors as the ground truth, and there is no consideration of malfunctioning sensors and their impact on the performance of the methods. For engineering applications, this holds significant importance as the methods need to be reliable in a variety of scenarios, both nominal and accidental. Therefore, it is crucial to understand how these methods behave in the presence of drifted or non-physical measurements from malfunctioning sensors and how to reconstruct the missing information from the model using auxiliary data [10]; this research is motivated by the ongoing interest in autonomous systems (including nuclear reactors) that need to recognise whether a sensor has failed or not, and retrieve the optimal state estimate even in such a situation. This work builds from [9], in which DDROM techniques were used for the monitoring and control of a Molten Salt Fast Reactor, introducing the possibility to deal with malfunctioning sensors, assuming that the state of each sensor is known *a-priori*. Specifically, this work proposes an improvement of the DDROM algorithm used in [9] by adding a Machine Learning (ML) step to include the missing information caused by the malfunctioning sensor through Gaussian Process Regression (GPR) [11]. As a numerical test case, this work considers an accidental transient in the Molten Salt Fast Reactor (MSFR) [12], and focuses on how the performance of GEIM and PBDW is affected by one or more malfunctioning sensors and on how to eventually retrieve the missing information. Following a brief presentation of the two DDROM methods used in this work (Section 2), their implementation under the conditions described above is analysed in detail in Section 3. Section 4, then, discusses the main findings from this work together with general considerations and future applications.

2 Data-Driven Reduced Order Modelling

Reduced Order Modelling (ROM) techniques allow the reduction of the computational complexity of *high-fidelity* numerical simulations by compressing the essential information while maintaining the desired level of accuracy. DDROM shares the typical offline-online decomposition [1] of ROM techniques, adding sensor placement and enabling the inclusion of the measured data. Figure 1 shows the overall scheme of these methods.

In the **offline phase**, the FOM is solved several times for different realisations of the model parameters $\boldsymbol{\mu}$ to generate a set of training solutions called *snapshots*, $u_{FOM}(\mathbf{x};\boldsymbol{\mu}_n) \in \mathbb{R}^{\mathcal{N}_h \times p}$, depending on the space, discretised on a mesh with $\mathcal{N}_h$ degrees of freedom, and the vector of model parameters $\boldsymbol{\mu} \in \mathcal{D} \subset \mathbb{R}^p$. This training dataset allows retrieving a set of basis functions $\psi_n(\mathbf{x})$ (whose physical interpretation varies according to the algorithm used),

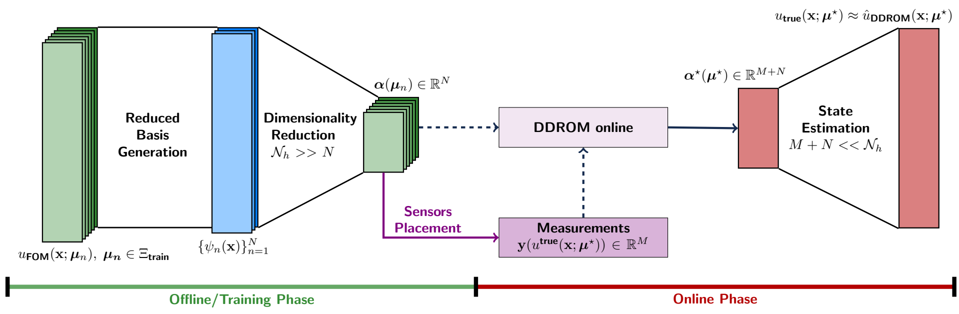

Figure 1: Scheme of DDROM [3]

which span a linear subspace of the solution space and represent the spatial dependence of the solutions. Projection of the FOM onto this surrogate space enables a Reduced Order Model, from which the parametric coefficients $\alpha(\mu_n)$ can be obtained. In addition, the reduced coordinates are used to find optimal configurations for experimental sensors to maximise the amount of information extracted from the system. In the **online phase**, measurements $\mathbf{y}\left(u^{true}(\mathbf{x}, \mu^\star)\right)$ are collected by sensors, and the ROM is solved quickly and accurately due to its much lower complexity. Therefore, its output can be used to obtain the full state of the system $\hat{u}_{DDROM}(\mathbf{x}; \mu^\star)$ approximating the true solution.

The DDROM framework aims to combine the knowledge embedded in the mathematical model with the physical information contained in the data measured by sensors. Compared to standard ROM methods, state prediction is not necessarily bounded by model accuracy; on the contrary, it can be improved, even during operation, by integrating measurements of the physical fields of interest to include unforeseen uncertainties and non-modelled physics [5].

Within the DDROM framework, this work uses the Generalised Empirical Interpolation Method and the Parameterised-Background Data-Weak formulation; both have been implemented in the Python language within the *pyforce* package [3, 9], which is part of the ROSE framework (Reduced Order multi-phySics data-drivEn) developed by the authors at the ERMETE Lab, openly available under the MIT License.

2.1 Generalised Empirical Interpolation Method

The Generalised Empirical Interpolation Method (GEIM) is a *non-intrusive* DDROM technique first proposed in [6] and based on a greedy algorithm for the selection of basis functions and sensors. In particular, GEIM builds hierarchical spaces to approximate a given function u with a suitable interpolant

$$u(\mathbf{x}; \mu) \simeq \mathcal{I}_M[u](\mathbf{x}; \mu) = \sum_{m=1}^{M} \beta_m(\mu) \cdot q_m(\mathbf{x}) \qquad \text{s.t.} \qquad \{v_m(u) = v_m\left(\mathcal{I}_M\right)\}_{m=1}^M \tag{1}$$

where the spatial behaviour is given by the **magic functions** $\{q_m(\mathbf{x})\}_{m=1}^M$ and the parametric dependence by the coefficients $\{\beta_m(\mu)\}_{m=1}^M$. Each magic function q_m is associated with a **magic sensor** $v_m(\cdot; \mathbf{x}_m,\ s)$, mathematically represented by a linear functional centred in $\mathbf{x}_m \in \Omega$ and with a point-spread $s \in \mathbb{R}^+$, representing the area onto which the sensor collects data (sensors have a physical dimension and therefore do not collect point-wise information) [7].

In the offline phase, the greedy procedure takes the training dataset as input and returns a set of magic functions and sensors by minimising the interpolation error at each step. The magic sensors are selected from a library Υ, representing the set of possible locations in the domain to maximise the amount of information extracted from the system [3, 6]. In the online phase, the coefficients are, then, determined using the data collected by the system $\mathbf{y} \in \mathbb{R}^M$:

$$y_m = v_m(u^{\text{true}}(\mathbf{x});\ \mathbf{x}_m,\ s) + \epsilon_m \qquad \text{with } m = 1, \dots M \tag{2}$$

given $\epsilon_m \sim \mathcal{N}(0, \sigma^2)$ as the uncorrelated Gaussian random noise, with σ^2 as the noise level. The reduced coefficients $\boldsymbol{\beta} \in \mathbb{R}^M$ are the solution of a linear system of dimension $M << \mathcal{N}_h$ resulting from the interpolation condition (1): to avoid an unbounded error due to noisy measurements, this work considers the regularised version of GEIM, called TR-GEIM, developed by the authors in [7], which weakens the interpolation condition by adding a penalty term; the equivalent linear system of dimension M^2 is, then

$$\left(\mathbb{B}^T\mathbb{B} + \lambda\mathbb{T}^T\mathbb{T}\right)\boldsymbol{\beta} = \mathbb{B}^T\mathbf{y} + \lambda\mathbb{T}^T\mathbb{T}\overline{\boldsymbol{\beta}} \tag{3}$$

given $\mathbb{B}_{ij} = v_i(q_j)$, $\overline{\boldsymbol{\beta}}$ the sample mean of the training coefficients $\boldsymbol{\beta}$, $\mathbb{T}$ as the regularisation matrix depending on the standard deviation of the training coefficients $\boldsymbol{\beta}$ as in [7] and λ as the regularisation parameter whose optimal value is σ^2, for unconstrained sensors [9].

2.2 Parameterised-Background Data-Weak formulation

The Parameterised-Background Data-Weak (PBDW) formulation [5], derived from the general DA statement [13], provides a general framework for Reduced Basis techniques coupled with data. In short, the PBDW approximates the state $u(\mathbf{x};\boldsymbol{\mu})$ by a linear combination of two contributions, the background knowledge z_N and the update η_M

$$u(\mathbf{x};\boldsymbol{\mu}) \simeq z_N(\mathbf{x};\boldsymbol{\mu}) + \eta_M(\mathbf{x};\boldsymbol{\mu}) = \sum_{n=1}^{N} \alpha_n(\boldsymbol{\mu}) \cdot \zeta_n(\mathbf{x}) + \sum_{m=1}^{M} \theta_m(\boldsymbol{\mu}) \cdot g_m(\mathbf{x}), \tag{4}$$

where $\{\zeta_n\}_{n=1}^N$ is the basis of the reduced space of dimension N that approximates the information of the mathematical model, $\{\alpha_n\}_{n=1}^N$ is the weighting coefficient for the background space, and $\{g_m\}_{m=1}^M$ is the basis of the update space of dimension M, which represents the non-modelled physics, with $\{\theta_m\}_{m=1}^M$ the weighting coefficient for the update space.

This work uses the Proper Orthogonal Decomposition (POD) [1] to build the reduced space for the FOM. On the other hand, the bases of the update space refer to the sensors; the same procedure used by TR-GEIM builds the update space with the aim of minimising the reconstruction error of the method to select the optimal configuration of the sensors $\{v_m\}_{m=1}^M$. The update basis functions g_m are the Riesz representation of the linear functional v_m [5, 9].

During the online phase, the coefficients $\boldsymbol{\alpha} \in \mathbb{R}^N$ and $\boldsymbol{\theta} \in \mathbb{R}^M$ are computed as the solutions of the following linear system of dimension $(N + M)^2$

$$\begin{bmatrix} \xi M\mathbb{I} + \mathbb{A} & \mathbb{K} \\ \mathbb{K}^T & 0 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\theta} \end{bmatrix} = \begin{bmatrix} \mathbf{y} \\ \mathbf{0} \end{bmatrix} \qquad \text{given } \begin{cases} \mathbb{A}_{mm'} = (g_m,\ g_{m'})_{L^2(\Omega)} \\ \mathbb{K}_{mn} = (g_m,\ \zeta_n)_{L^2(\Omega)} \end{cases} \tag{5}$$

with $m, m' = 1, \dots M$ and $n = 1, \dots, N$, ξ a hyperparameter to be tuned using cross-validation to improve the algorithm performance [9] and $\mathbb{I}$ as the identity matrix. Contrary to GEIM, PBDW is stable in the presence of random noise.

2.3 ML-Improved DDROM For Malfunctioning Sensors

TR-GEIM [6] and PBDW [5] are reliable and efficient in providing an accurate state estimation of the quantities of interest, even using noisy data [3, 7, 9]. However, their performance in the presence of malfunctioning sensors has yet to be proven. This work considers two classes of sensor malfunctions [10]: a drift from the average value and spikes not related to random noise, which can be expressed in the measurement vector of field T as follows

$$y_m^T = v_m(T^{\text{true}}(\mathbf{x});\ \mathbf{x}_m,\ s) + \epsilon_m + \mathcal{N}(\kappa, \rho) \qquad \text{with } m = 1, \dots M \tag{6}$$

in which high values of κ correspond to drifting, whereas high values of ρ indicate spikes. As a preliminary step, the algorithm assumes that a single sensor fails at the beginning of the transient; thus, the algorithm *a-priori* knows the state of each sensor that represents an external input. Hence, this work is the first step of a methodological pathway for engineering applications where the algorithm itself can recognise the state of each sensor in time (e.g., using, for example, classification or unsupervised learning methods) and retrieve the information from each failed sensor when needed to develop autonomous systems from the point of view of monitoring, control and diagnosis.

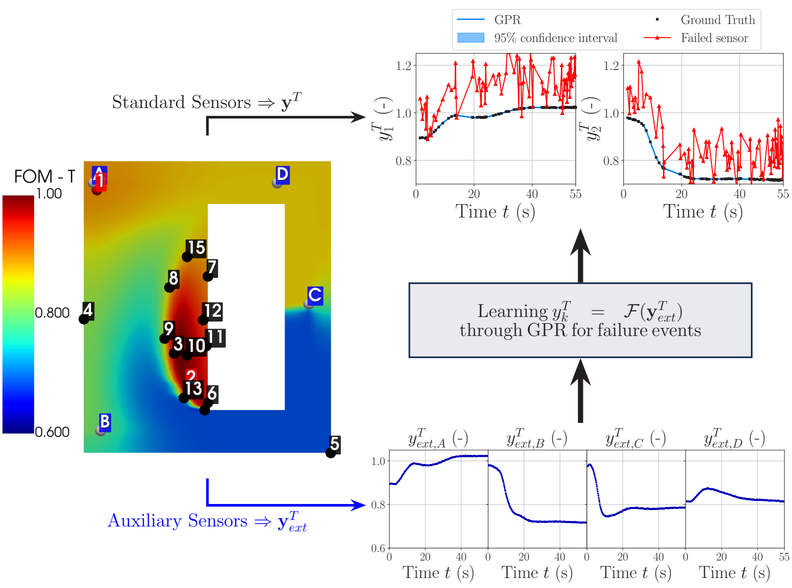

Figure 2: Recovery of missing information of failed sensor y_k^T (in black in the Figure on the left) from external sensors $\mathbf{y}^T_{ext}$ (in blue), for the temperature field T.

Supervised Learning (SL) methods can help retrieve the missing information about the measurement of the malfunctioning sensor with the help of auxiliary sensors. In nuclear reactor applications, these sensors should be located outside the core, where the environment is less hostile and, thus, the probability of failure is lower. This work proposes a possible strategy for this step. Let $\mathbf{y}^T_{ext}(t)$ be the vector of measurements of an observable field T from external sensors as a function of time t and let $y_k^T(t)$ be the measurement for the sensor k that is known to have entered a failure state. SL methods such as Gaussian Process Regression [11] can learn the input-output relationship $y_k^T(t) = \mathcal{F}(\mathbf{y}^T_{ext})$ and, thus, represent a possible recovery strategy for the missing information of the failed sensors. Instead of relying on the output of the sensor, the output of an aptly-trained surrogate model based on GPR replaces the missing information when the system detects a malfunctioning sensor. Figure 2 shows this idea.

More precisely, let $\mathcal{X} = \{(\mathbf{x}_i, y_i) : i = 1, 2, \dots, N_s\}$ be the training set of N_s observations, $\mathbf{x}_i \in \mathbb{R}^d$ being the input with d entries and y_i the corresponding output. SL algorithms aim to find a model f that can infer the relationship between $\mathbf{x}$ and y. The Gaussian Process Regression (GPR) method [11] infers a probabilistic distribution over functions from the observed data, which can be used for predictions with unknown input values $\mathbf{x}$. Given a mean value $\mu(\mathbf{x})$ and a covariance function (kernel) $\mathcal{K}(\mathbf{x}, \mathbf{x}')$, the prior distribution of f is a GP that can be expressed as $f(\mathbf{x}) \sim \text{GP}\,(\mu(\mathbf{x}), \mathcal{K}(\mathbf{x}, \mathbf{x}'))$, where $\mu(\mathbf{x})$ is usually assumed to be a constant function found based on the training data. Several kernels can be used, each with some hyper-parameters that need to be tuned to improve the predictive performance of the GPR, such as the Radial Basis Function (RBF) or the exponential sinusoidal kernel; this work uses the former.

GPR models embed knowledge contained in the training data $\mathcal{X}$ into the prior distribution to obtain a posterior distribution. Hence, given a set of training input vectors $\mathbb{X} = [\mathbf{x}_1|\mathbf{x}_2|\dots\mathbf{x}_{N_s}] \in \mathbb{R}^{d\times N_s}$ with correspondent output $\mathbf{y} = [y_1, y_2, \dots y_{N_s}]^T \in \mathbb{R}^{N_s}$ and a set of new test input data $\mathbb{X}^* \in \mathbb{R}^{d\times M}$, predictions $\mathbf{y}^* \in \mathbb{R}^M$ can be made by exploiting the theorem of conditional Gaussian, whereby the conditional distribution of the predicted GP is given by $\mathbf{y}^* = f(\mathbb{X}^*|\mathbf{y}, \mathbb{X}, \mathbb{X}^*) \sim \text{GP}\left(\hat{\mu}(\mathbb{X}^*), \hat{\kappa}(\mathbb{X}, \mathbb{X}^{*'})\right)$ in which the knowledge of the mean and covariance functions update as follows [11]:

$$\hat{\mu}(\mathbb{X}^*) = \mu(\mathbb{X}^*) + \mathbb{K}_*\mathbb{K}^{-1}\left(\mathbf{y} - \mu(\mathbb{X})\right), \qquad \hat{\mathcal{K}}(\mathbb{X}, \mathbb{X}^{*'}) \;= \mathbb{K}_{**} - \mathbb{K}_*\mathbb{K}^{-1}\mathbb{K}_*^T \tag{7}$$

given $\mathbb{K} = \mathcal{K}(\mathbb{X}, \mathbb{X}'), \mathbb{K}_* = \mathcal{K}(\mathbb{X}^*, \mathbb{X}'), \mathbb{K}_*^T = \mathcal{K}(\mathbb{X}, \mathbb{X}^{*'}), \mathbb{K}_{**} = \mathcal{K}(\mathbb{X}^*, \mathbb{X}^{*'})$. The kernel functions depend on the hyper-parameters $\boldsymbol{\vartheta}$, whose values must be tuned by maximising the log-marginal likelihood loss function as in [11]:

$$\boldsymbol{\vartheta}_{\text{opt}} = \arg\max_{\boldsymbol{\vartheta}} \log p(\mathbf{y}|\mathbb{X}) = \arg\max_{\boldsymbol{\vartheta}} \left[-\frac{1}{2}\mathbf{y}^T\mathbb{K}^{-1}(\boldsymbol{\vartheta})\mathbf{y} - \frac{1}{2}\log|\mathbb{K}(\boldsymbol{\vartheta})| - \frac{N_s}{2}\log(2\pi)\right] \tag{8}$$

where $p(\mathbf{y}|\mathbb{X})$ is the conditional density function of $\mathbf{y}$ given $\mathbb{X}$ known also as marginal likelihood. This work adopts the GPR implementation of the Python package GPy (http://github.com/SheffieldML/GPy), in which the optimisation is incorporated.

3 Numerical Results

This work analyses the performance of TR-GEIM and PBDW in the presence of malfunctioning sensors considering an accidental transient scenario for the MSFR, namely an Unprotected Loss of Fuel Flow (ULOFF) following the failure of the primary pump. In particular, this work considers the 2D axisymmetric wedge of the EVOL geometry of the MSFR [12] with an external Hastelloy layer acting as a reflector. For sensor placement, auxiliary sensors can be located only in this region, whereas regular sensors can be located anywhere in the domain.

The FOM also includes a momentum source and a heat sink, representing the primary pump and the heat exchanger, respectively. More details on the FOM can be found in [14]: the model is solved using the multi-physics (neutronic and thermal-hydraulics) numerical solver developed at Politecnico di Milano in the OpenFOAM environment [15], adapted for a multi-region problem to include also the reflector: at the interface between regions, the solver imposes the continuity of variables, whereas on the external walls vacuum and adiabatic boundary conditions are imposed, respectively, on neutron fluxes and temperature fields [14].

The FOM dataset for the selected ULOFF transient includes 275 snapshots, with time domain [0 : 0.2 : 55] seconds. This dataset Ξ splits into three subsets: a **train set** Ξ^{train} for generating the basis functions and sensor positions for both algorithms (75% of the dataset Ξ up to the 250th snapshot), a **test set** Ξ^{test} for cross-validation (25% of the dataset Ξ up to the 250th snapshot) and a **predict set** Ξ^{predict} to analyse the extrapolation capabilities of the reconstruction algorithm (last 25 snapshots of Ξ). The split into training and test sets follows the random 75-25 split of scikit-learn. For brevity, this work reports only the results for temperature T, albeit the algorithm is general. This is because temperature is a significant quantity for monitoring and control during accidental scenarios, and its dynamic evolution is usually more complex than that other parameters like of the neutron flux.

The sensors are represented by linear functionals with a Gaussian kernel centred in the sensor position and with a point-spread of $s = 0.025$, as in [9]. Moreover, the FOM snapshots normalise to the maximum value of the initial state, i.e. $T(\mathbf{x};t) \leftarrow T(\mathbf{x};t)\cdot[\max_{\mathbf{x}\in\Omega} T(\mathbf{x};0)]^{-1}$, levelling all data to the same order of magnitude to improve the performance and the training stability. Finally, measurements are synthetically generated during the online phase of TR-GEIM and PBDW from the FOM snapshots using the sensors selected by Eq. 2 with $\sigma^2 = 10^{-3}$ as the variance of the noise level: this value is defined referring to the maximum value of the initial condition since each temperature snapshot is normalised.

3.1 Performance of TR-GEIM and PBDW With Malfunctioning Sensors

As mentioned above, a single sensor is supposed to fail at time $t = 0$. The failure corresponds to its output y_k being corrupted as in Eq. (6) using $\kappa = 0.1$ and $\rho = 0.5$, respectively simulating malfunctioning by shifting the actual measure from the true value and by significantly increasing the noise for producing spikes. These values have been chosen to simulate a small fixed drift from the true value with κ and oscillations with ρ, as reported in Figure 2. The performance of TR-GEIM and PBDW are measured using the reconstruction error, employing as number of sensors $M = 15$, chosen during the offline phase by measuring the reducibility of the problem [1, 9]. Let $\mathcal{P}_M[T(\cdot;t)]$ be the reconstruction operator for TR-GEIM, with M measurements for the temperature $T(\cdot;t)$ and let $r[T(\cdot;t)](\mathbf{x}) = |T(\cdot;t) - \mathcal{P}_M[T(\cdot;t)]|$ be the average absolute E^k error measured in L^2-norm, with sensor k failed, such that the error is:

$$E^k[T] = \frac{1}{\dim(\Xi^*)}\sum_{t\in\Xi^*}\|r(\mathbf{x};t)\|_{L^2(\Omega)} \tag{9}$$

given $\Xi^* \subset \mathcal{D}$ a subset of the parameter space with unseen data (test and predict sets).

Figure 3 shows the reconstruction error for TR-GEIM and PBDW when a single sensor k fails, compared with the case where all sensors work perfectly (the reference case). As expected, the errors increase compared to the reference case, meaning that the reconstruction becomes less and less accurate and physical, causing non-physical and non-negligible oscillations in the reconstructed field $\mathcal{P}_M[T(\cdot;t)]$. The performance of TR-GEIM and PBDW in presence of failed sensors is similar; however, compared to their respective reference case the latter shows greater deterioration. Generally, the error for the specific sensor k depends on the variability of the measure itself: sensors that during the specific transient show less variability influence less the performance of the algorithm, even though a drop in performance is still observed. Thus, even a single output from a failed sensor can induce a completely wrong state estimation, making the DDROM method less reliable: this highlights the need for some recovery strategy for the signals of the failed sensor.

In principle, the algorithm could remove the corrupted $k-$th measure from the associated linear systems of TR-GEIM and PBDW; this can lead to different results: for the former, the removal of the computed $k-$th measure implies also the deletion of the associated $k-$th basis function, and since GEIM uses a greedy approach, the loss of information is inversely proportional to the index of the sensor; the PBDW is conceived to circumvent this issue by adopting a non-greedy procedure to select the sensors starting from a reduced basis $\{\zeta_n\}_{n=1}^N$: however, if more than one sensors fail, it may occur that $M < N$, resulting in a loss of stability and convergence of the PBDW algorithm [5].

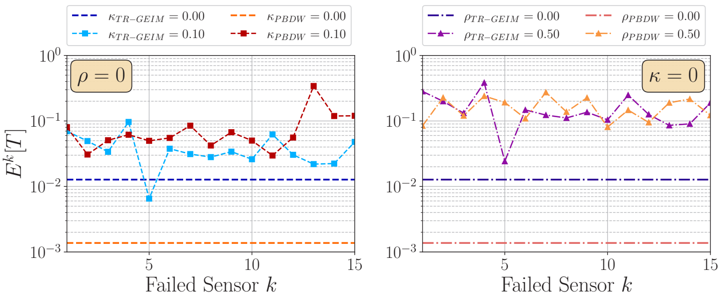

Figure 3: Reconstruction errors with drifted measure, see Eq. (6), at sensor k.

3.2 Recovery of the Failed Measure Using GPR

Given the $k-$th sensor failing at time $t = 0$, GPR is used to learn the input-output relations between the external auxiliary measures $\mathbf{y}_{ext}^T$ and the output of the malfunctioning sensors y_k^T, in the framework represented in Figure 2; hence, from the mathematical standpoint the prediction of the GPR can fill the missing quantity in the measurements vector $\mathbf{y}^T$ appearing in the right-hand side of TR-GEIM (3) and PBDW (5) linear systems.

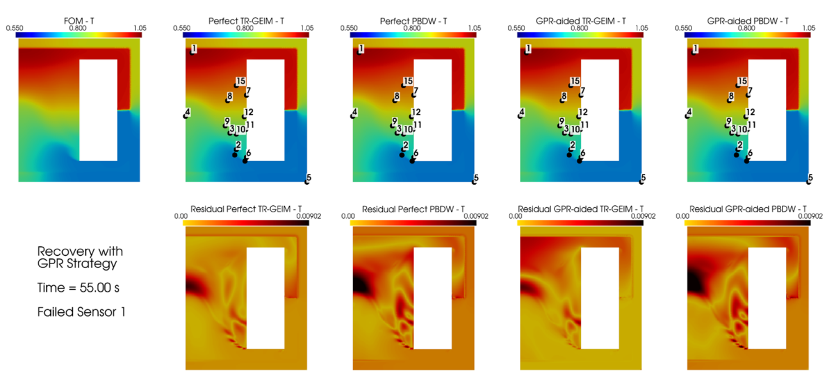

Figure 4: Contour plots of the temperature field T obtained by the FOM, TR-GEIM and PBDW (both in perfect and GPR-aided conditions, for sensor $k = 1$) at time $t^\star = 55\ s \in \Xi_{\text{predict}}$.

Figure 4 shows the contour plots of the temperature T at the final time instant $t^\star = 55\ s \in \Xi_{\text{predict}}$ for the FOM and the TR-GEIM and PBDW reconstruction, with their associated residual fields $r[T(\cdot;t)](\mathbf{x}) = |T(\cdot;t) - \mathcal{P}_M[T(\cdot;t)]|$, computed either considering the reference condition in which all sensors are working or the case in which a sensor has failed: in this case, $k = 1$, since for the greedy selection procedure the first sensor is the most important one in terms of amount of information it contains. Figure 4 shows how, even for a time instant outside the training dataset, the recovery using GPR is sufficient to provide a correct state estimation of the temperature field, thus showing a possible strategy to cure the inaccurate response of TR-GEIM and PBDW when a sensor fails even in a prediction scenario. Similar results, not reported here for sake of brevity, have been obtained for the cases in which other sensors have entered the failure state.

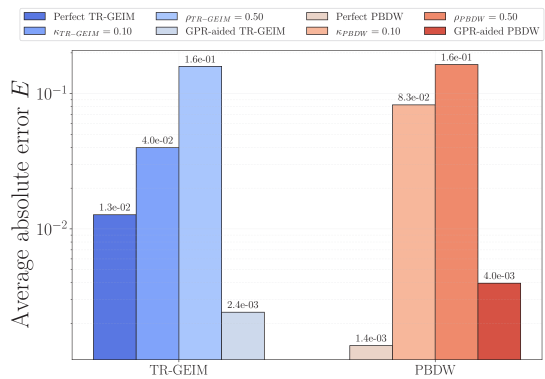

Figure 5: Comparison of the average absolute errors using different algorithms (the case GPR-aided refers to the sum of the two error contributions).

Figure 5 shows the average absolute error $E = \frac{1}{M}\sum_{k=1}^{M} E^k[T]$, given $E^k[T]$ defined in Eq. (9), for TR-GEIM and PBDW in different scenarios: perfect case in which all sensors are in working status, drifted measurements with $\kappa = 0.1$, measurements polluted by random spikes with $\rho = 0.5$ and the GPR-aided case in which GPR has learned the input-output map through external auxiliary sensors, thus recovering the output of the failed sensors (this latter case considers the sum of the two measurement error contributions). The strategy proposed in this work can cure the issues related to drift or spikes, either combined or independent: the reconstruction error is comparable with the perfect case. Moreover, TR-GEIM is more aided than PBDW because the recovery made by GPR creates smoother measurements, which is beneficial due to the less noise introduced.

4 Conclusions

This paper proposes to analyse the behaviour of Data-Driven Reduced Order Modelling methods, designed to combine *high-fidelity* numerical simulation and measurements collected by sensors installed, when some sensors fail and their outputs become non-physical. The test case selected for this purpose is an accidental scenario of the Molten Salt Fast Reactor, the Unprotected Loss of Fuel Flow, onto which the performance of the Generalised Empirical Interpolation Method and the Parameterised-Background Data-Weak formulation are investigated.

From the proposed analysis, even a single drifted measure coming from a failed sensor significantly worsens the estimation capabilities of the DDROM techniques, resulting in unbounded errors and non-physical results. In order to overcome this situation, Gaussian

Process Regression has been used to recover the information of the drifted measurements using some auxiliary sensors placed outside the reactor, learning the input-output relations; the output of the GPR, then, fills the missing information associated with the failed sensors in the DDROM algorithms formulation. The results obtained with this additional step are very similar to the case in which all the sensors are perfectly working, thus ensuring the possibility of retrieving the missing information with this recovery strategy.

The promising results of this work offer a first step towards the development of autonomous reactors, in which artificial intelligence will recognise the presence of failed measurements and thus activate recovery strategies to retrieve the missing information. The final goal will be to have a completely autonomous system that will ensure complete control of the nuclear reactor, even in the scenarios of untrustworthy measurements.

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