

Light Meson Decays at BESIII

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Abstract. The 10 billion J/ψ decays collected with the BESIII experiment offer a unique opportunity to investigate the decays of η and η' mesons produced in the radiative $J/\psi \rightarrow \gamma\eta^{(\prime)}$ transitions. Using this clean production mechanism, the BESIII experiment is making important contributions to precision studies of the strong and electromagnetic interactions in $\eta^{(\prime)}$ decays. Three papers are presented in which the decays $\eta \rightarrow \pi^+\pi^-\pi^0$, $\eta \rightarrow \pi^0\pi^0\pi^0$, $\eta' \rightarrow \eta\pi^0\pi^0$, and $\eta' \rightarrow \pi^+\pi^-e^+e^-$ are studied to test C and CP symmetry, as well as the predictions of different effective field theories.

1 Introduction

Since the discovery of pions [1] and kaons [2] more than seven decades ago, the study of light meson decays has played a crucial role in low-energy particle physics. The study of these decays allows for precision tests of fundamental symmetries and of effective field theories, the search for new particles and rare and forbidden decays, as well as the determination of transition form factors (TFFs).

Over the last decade, the BESIII collaboration has provided valuable experimental insights in this field. The poster presented summarized the results of three recently published BESIII papers investigating the decays of the η and η' mesons [3–5].

In Ref. [3], the matrix elements of the decay amplitudes for the isospin violating decay of η into three pions were determined. These matrix elements could then be compared with effective field theories of the strong interaction, such as chiral effective field theory [6–9] and non-relativistic effective field theory (NREFT) [9, 10]. Furthermore, additional matrix elements were added to the amplitude model to search for C -parity violations, and the $\pi^0\pi^0$ invariant mass spectra were checked for evidence of the so-called cusp effect predicted by NREFT [9]. In Ref. [4], the same cusp effect was searched for in the $\eta' \rightarrow \eta\pi^0\pi^0$ decay using the NREFT framework [10]. By considering the loop level contributions of the NREFT to model the data, the $\pi\pi$ S -wave scattering length combination was determined. Finally, a paper [5] is presented that determined the branching ratio of $\eta' \rightarrow \pi^+\pi^-e^+e^-$ and searched for CP violating asymmetries in this decay. The measurements were then compared with the predictions of effective field theories.

All of the mentioned analyses have used J/ψ data collected by the Beijing Spectrometer III (BESIII) in Beijing, China. The BESIII detector [11] records symmetric e^+e^- collisions provided by the BEPCII storage ring [12] in the center-of-mass energy range from 2.0 GeV to 4.95 GeV, with a peak luminosity of $1 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ achieved at $\sqrt{s} = 3.77 \text{ GeV}$. Details about the detector can be found in [11].

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2 Matrix elements for $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\eta \rightarrow \pi^0\pi^0\pi^0$ decays

The first analysis [3] to be presented investigated the light meson decays $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\eta \rightarrow \pi^0\pi^0\pi^0$. To this end, the η meson was produced via $J/\psi \rightarrow \gamma\eta$ using the $(10\,087 \pm 44) \times 10^6$ J/ψ events collected by BESIII [13] and the π^0 was reconstructed via $\pi^0 \rightarrow \gamma\gamma$. After applying various selection criteria on the properties of the charged tracks and photons, as well as on the χ^2 values of the kinematic fits, 631 686 candidates for $\eta \rightarrow \pi^+\pi^-\pi^0$ decays and 272 322 candidates for $\eta \rightarrow \pi^0\pi^0\pi^0$ decays were selected for subsequent analysis. The $\pi^+\pi^-\pi^0$ Dalitz plot was described by the variables [14]

$$X = \frac{\sqrt{3}}{Q_{\eta \rightarrow \pi^+\pi^-\pi^0}}(T_{\pi^+} - T_{\pi^-}), \quad Y = \frac{3T_{\pi^0}}{Q_{\eta \rightarrow \pi^+\pi^-\pi^0}} - 1 \quad (1)$$

with T_π being the kinetic energy of a given pion in the η rest frame and $Q_{\eta \rightarrow \pi^+\pi^-\pi^0} = m_\eta - m_{\pi^+} - m_{\pi^-} - m_{\pi^0}$ being the excess energy of the reaction. By exploiting the threefold symmetry of the $\pi^0\pi^0\pi^0$ final state, the Dalitz plot was parameterized using only the polar variable

$$Z = X^2 + Y^2 = \frac{2}{3} \sum_{i=1}^3 \left(\frac{3T_{\pi_i^0}}{Q_{\eta \rightarrow 3\pi^0}} - 1 \right)^2, \quad (2)$$

where $T_{\pi_i^0}$ is the kinetic energy of each π^0 and the excess energy can be calculated with $Q_{\eta \rightarrow 3\pi^0} = m_\eta - 3m_{\pi^0}$. For the excess energies, the η and pion masses m_η and m_π were taken from the Particle Data Group (PDG) [15]. Using these variables, the resulting Dalitz plots can be seen in Fig. 1.

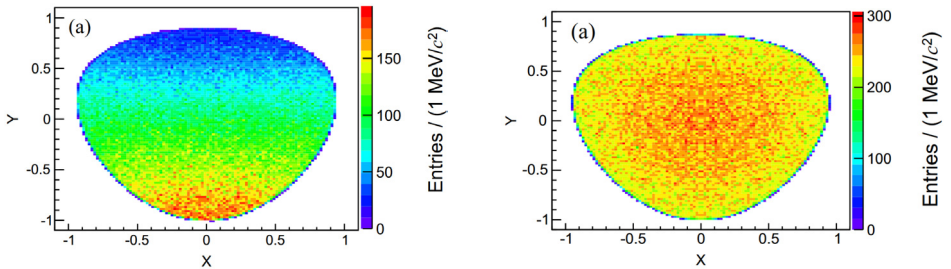


Figure 1: Dalitz plots of the selected data events for the reactions $\eta \rightarrow \pi^+\pi^-\pi^0$ (left) and $\eta \rightarrow \pi^0\pi^0\pi^0$ (right). Figures taken from Ref. [3].

Unbinned maximum likelihood fits were performed on selected data using the squared decay amplitudes [9, 16]

$$|A(X, Y)|^2 \propto 1 + aY + bY^2 + cX + dX^2 + eXY + fY^3 + gX^2Y + hXY^2 + lX^3 + \dots \quad (3)$$

for the $\eta \rightarrow \pi^+\pi^-\pi^0$ channel and

$$|A(X, Y)|^2 \propto 1 + 2\alpha Z + 2\beta(3X^2Y - Y^3) + 2\gamma Z^2 + \dots \quad (4)$$

for the $\eta \rightarrow \pi^0\pi^0\pi^0$ channel, respectively. For the fit to $\pi^+\pi^-\pi^0$ it was found that the C -parity violating parameters c , e , h and l are all consistent with zero, indicating no C -parity violation. This result is further confirmed by the subsequent tests of left-right asymmetry, quadrant asymmetry, and sextant asymmetry, all of which are consistent with zero [17–24].

For the fit to $\pi^0\pi^0\pi^0$, both the β and γ parameters were consistent with zero. To check whether the loop contributions predicted by NREFT [9] lead to a cusp at the $\pi^+\pi^-$ mass threshold in the $\pi^0\pi^0$ spectrum, an additional fit was performed using the squared decay amplitude [25]:

$$|A|^2 \propto 1 + 2\alpha Z + 2\delta \sum_{i=1}^3 \rho(s_i), \quad (5)$$

with the real part of the phase space density $\rho(s_i) = \Re\left(\sqrt{1 - s_i/4m_{\pi^\pm}^2}\right)$ and the Mandelstam variable $s_i = (p_\eta - p_i)^2$, where p_η and p_i are the momenta of the η meson and the pions, and $i = 1, 2, 3$ denotes the three pions. The projections onto the $\pi^0\pi^0$ invariant mass spectrum of the two fits using Eq. (4) and Eq. (5) can be seen in Fig. 2. No significant cusp effect was observed in this channel, with the statistical significance of the cusp parameter δ being 0.8σ .

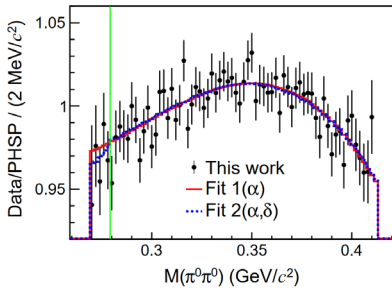


Figure 2. Number of data events (black dots with error bars) and fit projections (red line and blue dotted line) divided by the number of Monte Carlo events evenly distributed in phase space. The red curve corresponds to the fit with Eq. (4) and the blue curve with Eq. (5) as the decay amplitude. The $\pi^+\pi^-$ mass threshold is indicated by the green vertical line. Figure taken from Ref. [3].

The values obtained for the matrix elements when including only the terms with parameters a, b, d, f , and g for the $\pi^+\pi^-\pi^0$ channel and α for the $\pi^0\pi^0\pi^0$ channel in comparison with previous experimental results and various theoretical predictions can be seen in Fig. 3. Recent experimental results [14, 17, 25, 26], as well as theoretical values determined using a dispersive approach [6, 7] and next-to-next-to-leading order (NNLO) chiral perturbation theory (ChPT) [8, 9] are in agreement with the measured matrix elements.

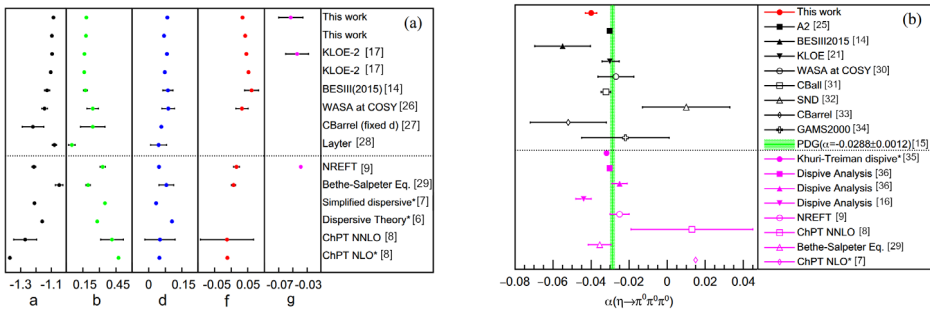


Figure 3: Comparison between different experimental results and theoretical predictions for the matrix element(s) of $\eta \rightarrow \pi^+\pi^-\pi^0$ (a) and $\eta \rightarrow \pi^0\pi^0\pi^0$ (b) [6–9, 14–17, 21, 25–36]. No uncertainty has been reported for theoretical predictions marked with *. Figures taken from Ref. [3].

3 Evidence for the Cusp Effect in η' Decays into $\eta\pi^0\pi^0$

The aforementioned cusp effect was also studied via the process $J/\psi \rightarrow \gamma(\eta' \rightarrow \eta\pi^0\pi^0)$ [4] using the same 10 billion J/ψ events as in the previous work. In this work, both the η and the π^0 candidates were reconstructed from two photons. After applying several selection criteria to the properties of the seven final state photons, as well as to the χ^2 of the kinematic fit 432 295 candidates for $\eta' \rightarrow \eta\pi^0\pi^0$ decays were left in the data for further analysis. The resulting Dalitz plot is shown on the left side of Fig. 4. Four unbinned maximum likelihood fits were performed on the Dalitz plot data using different constraints on the loop level contributions of the decay amplitude derived from NREFT. These loop contributions are parameterized by three coefficients [10], where

$$C_x = \frac{16\pi}{3}(a_2 - a_0)(1 + \xi/3) \quad (6)$$

is the cusp term corresponding to the coupling of the decay to the rescattering process $\pi^+\pi^- \rightarrow \pi^0\pi^0$, where a_0 and a_2 are the $\pi\pi$ scattering lengths of isospin 0 and 2, and $\xi = (M_{\pi^\pm}^2 - M_{\pi^0}^2)/M_{\pi^\pm}^2$ is the relative squared mass difference between the charged and neutral pions using their respective PDG values [15]. The other two loop level coefficients

$$C_{00} = \frac{16\pi}{3}(a_0 + 2a_2)(1 - \xi) \quad C_{+-} = \frac{8\pi}{3}(2a_0 + a_2)(1 + \xi), \quad (7)$$

correspond to the loop processes $\pi^0\pi^0 \rightarrow \pi^0\pi^0$ and $\pi^+\pi^- \rightarrow \pi^+\pi^-$, respectively, which do not contribute to the cusp effect. In the first fit, a_0 and a_2 were set to zero, effectively neglecting the loop-level contributions and fitting only the tree-level decay amplitude, which was parameterized similarly to the work presented in Section 2 [37]. In the second fit, both a_0 and a_2 were left as free fit parameters, while in the third fit the scattering length combination $a_0 - a_2$ was used as a free fit parameter and $(a_0 + 2a_2)$ was set to 0.1312 according to the theoretical values for a_0 and a_2 from Ref. [10]. Finally, in the fourth fit, contributions from C_{00} and C_{+-} were neglected and only $a_0 - a_2$ was used as a free fit parameter for the loop level amplitude. Projections of the four fit results onto the squared invariant mass spectrum of the $\pi^0\pi^0$ system in the mass region of the cusp effect can be seen on the right hand side of Fig. 4. Deviations between the fit using only the tree-level amplitude and the data below the $\pi^+\pi^-$ mass threshold were observed, and it was found that the fourth fit provided the best description of the data. The statistical significance of the cusp effect was thus determined to be about 3.5σ , while the scattering length combinations were measured to be $a_0 - a_2 = 0.226 \pm 0.060_{\text{stat}} \pm 0.013_{\text{sys}}$, which is in good agreement with the theoretical value calculated from NREFT in Ref. [10].

4 Search for a CP -Violating Asymmetry in $\eta' \rightarrow \pi^+\pi^-e^+e^-$

In this work [5], the decay $\eta' \rightarrow \pi^+\pi^-e^+e^-$ was studied to compare the experimental branching ratio with the theoretical predictions and to test for CP violation. As in Section 3, the η' is produced in radiative J/ψ decays using 1.31×10^9 J/ψ events [38, 39] measured by BESIII in 2009 and 2012. To extract the $\eta' \rightarrow \pi^+\pi^-e^+e^-$ candidates, several constraints on the measured properties of the charged tracks and photons, as well as the χ^2 of the kinematic fit, were applied. Additional selection criteria were applied to reduce the dominant background of $\eta' \rightarrow \pi^+\pi^-\gamma$ events, where the photons create an e^+e^- pair by interacting with the beam pipe or the inner wall of the multilayer drift chamber. After applying all the selection criteria, the signal purity was estimated with MC simulations to be 98%, and the resulting η' peak in the $\pi^+\pi^-e^+e^-$ invariant mass spectrum can be seen on the left-hand side of Fig. 5. Additionally,

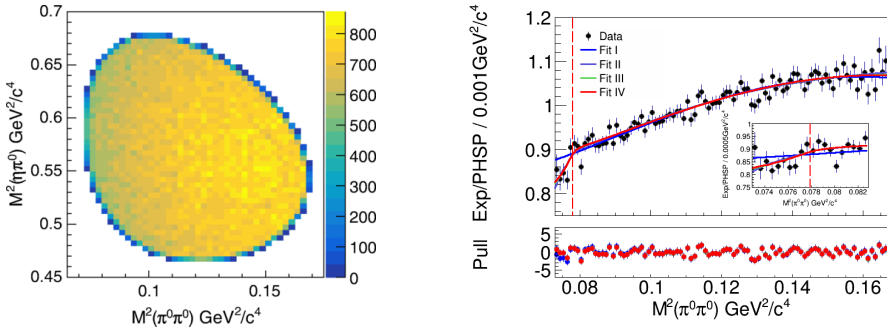


Figure 4: **Left:** Dalitz plot with the $\eta' \rightarrow \pi\pi^0\pi^0$ candidates selected from the data. **Right:** Invariant mass distributions of the data (black dots with error bars) and the projections of the four fit results (colored lines), normalized by MC events uniformly distributed in phase space. The dashed red line indicates the $\pi^+\pi^-$ mass threshold, and an inset of the region around the threshold is shown. Figures taken from Ref. [4].

one η' signal and two sideband regions were selected in the $\pi^+\pi^-e^+e^-$ invariant mass spectrum, resulting in a signal yield of 2584 ± 52 events after subtracting the sidebands. Using this signal yield, a branching ratio of $\mathcal{B}(\eta' \rightarrow \pi^+\pi^-e^+e^-) = (2.42 \pm 0.05_{\text{stat}} \pm 0.08_{\text{sys}}) \times 10^{-3}$ was obtained, which is consistent with the hidden gauge and modified vector meson dominance (VMD) model [40], as well as the unitary ChPT [41].

Assuming that the e^+e^- pair in the decay $\eta' \rightarrow \pi^+\pi^-e^+e^-$ is produced by a virtual photon, interference between the dominant CP -conserving magnetic dipole transition and a possible CP -violating electric dipole transition would be possible [42–44]. The amplitude of this interference is predicted to be proportional to $\sin 2\varphi$ [40, 43], where φ is the angle between the e^+e^- and $\pi^+\pi^-$ decay levels in the rest frame of the η' meson. Therefore, the amplitude of the CP asymmetry as a function of the number of acceptance-corrected events in a given angular range $N(x)$ can be written as:

$$\mathcal{A}_\varphi = \frac{N(\sin 2\varphi > 0) - N(\sin 2\varphi < 0)}{N(\sin 2\varphi > 0) + N(\sin 2\varphi < 0)}. \quad (8)$$

Using the sideband corrected events in the η' signal region, the CP asymmetry was calculated to be $(2.9 \pm 3.7_{\text{stat}} \pm 1.1_{\text{sys}})\%$, which is compatible with zero within 1σ , meaning that no significant CP violation was observed. The $\sin 2\varphi$ distribution is shown on the right side of Fig. 5.

5 Conclusion and Outlook

Using (part of) the 10 billion J/ψ events measured by the BESIII detector, the three papers presented have studied decays of the light η and η' mesons to test the predictions of effective field theories and to check for C and CP violations. The decay amplitude matrix elements of the isospin violating decays $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\eta \rightarrow \pi^0\pi^0\pi^0$ have been determined with high precision, and the results are in agreement with previous analyses as well as theoretical predictions from dispersive approaches [6, 7] and NNLO ChPT [8, 9]. Furthermore, no significant C violation or evidence for the cusp effect was observed. However, evidence for the cusp effect as a loop level contribution to the NREFT decay amplitude [10] was found in the decay $\eta' \rightarrow \eta\pi^0\pi^0$, with a $\pi\pi$ scattering length combination consistent with the prediction

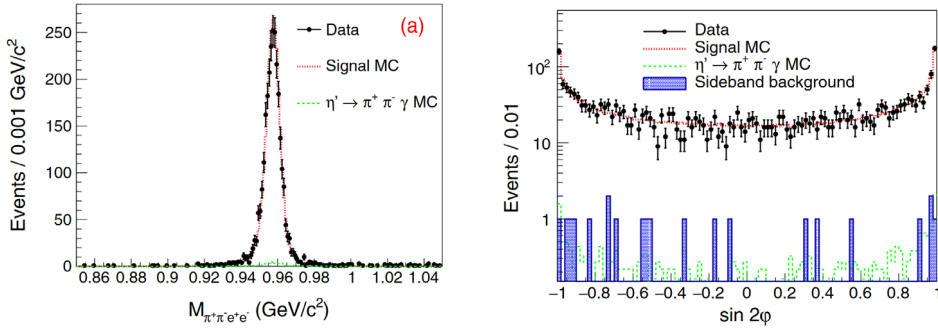


Figure 5: $\pi^+\pi^-e^+e^-$ invariant mass spectrum (left) and $\sin 2\varphi$ distribution (right) with data (black dots with error bars), $\eta' \rightarrow \pi^+\pi^-e^+e^-$ MC (red dotted line), and $\eta' \rightarrow \pi^+\pi^-\gamma$ MC (green dashed line). The $\sin 2\varphi$ distribution is filled only with events within the η' signal region, and the events in the sideband regions are shown as filled blue bars. Figures taken from Ref. [5].

from NREFT [10]. Moreover, the branching ratio for the decay $\eta' \rightarrow \pi^+\pi^-e^+e^-$ was determined, which is in good agreement with the theoretical predictions of two different VMD models [40] and unitary ChPT [41]. Using the $\sin 2\varphi$ distributions of the $\eta' \rightarrow \pi^+\pi^-e^+e^-$ signal events, CP symmetry was tested and no significant CP violation was found.

These three papers are examples of the capabilities of BESIII as a light meson factory, and the 10 billion J/ψ events available will continue to provide a solid framework for further studies such as the determination of transition form factors and the quark structure of scalar mesons. For further precision studies of $\eta^{(\prime)}$ decays, it is also possible to increase the number of events available by including hadronic J/ψ decays such as $J/\psi \rightarrow \phi\eta^{(\prime)}$ in addition to the radiative J/ψ decays used in the presented analyses.

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References

- [1] C.M.G. Lattes, H. Muirhead, G.P.S. Occhialini, C.F. Powell, *Nature* **159**, 694 (1947)
- [2] G.D. Rochester, C.C. Butler, *Nature* **160**, 855 (1947)
- [3] M. Ablikim, M.N. Achasov, P. Adlarson, R. Aliberti, A. Amoroso, M.R. An, Q. An, Y. Bai, O. Bakina, I. Balossino et al., *Precision measurement of the matrix elements for $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\eta \rightarrow \pi^0\pi^0\pi^0$ decays*, *Physical Review D* **107**, 092007 (2023), DOI: 10.1103/PhysRevD.107.092007
- [4] M. Ablikim, M.N. Achasov, P. Adlarson, M. Albrecht, R. Aliberti, A. Amoroso, M.R. An, Q. An, X.H. Bai, Y. Bai et al., *Evidence for the Cusp Effect in η' Decays into $\eta\pi^0\pi^0$* , *Physical Review Letters* **130**, 081901 (2023), DOI: 10.1103/PhysRevLett.130.081901
- [5] M. Ablikim, M.N. Achasov, P. Adlarson, S. Ahmed, M. Albrecht, A. Amoroso, Q. An, X.H. Bai, Y. Bai, O. Bakina et al., *Measurement of the branching fraction of and search for a CP -violating asymmetry in $\eta' \rightarrow \pi^+\pi^-e^+e^-$ at BESIII*, *Physical Review D* **103**, 092005 (2021), DOI: 10.1103/PhysRevD.103.092005
- [6] J. Kambor, C. Wiesendanger, D. Wyler, *Nuclear Physics B* **465**, 215 (1996)
- [7] J. Bijnens, J. Gasser, *Physica Scripta* **T99**, 34 (2002)

- [8] J. Bijnens, K. Ghorbani, *Journal of High Energy Physics* **2007**, 030 (2007)
- [9] S.P. Schneider, B. Kubis, C. Ditsche, *Journal of High Energy Physics* **2011**, 1 (2011)
- [10] B. Kubis, S.P. Schneider, *The European Physical Journal C* **62**, 511 (2009)
- [11] M. Ablikim, Z.H. An, J.Z. Bai, N. Berger, J.M. Bian, X. Cai, G.F. Cao, X.X. Cao, J.F. Chang, C. Chen et al., *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* **614**, 345 (2010)
- [12] C. Yu, Z. Duan, S. Gu, Y. Guo, X. Huang, D. Ji, H. Ji, Y. Jiao, Z. Liu, Y. Peng et al., eds., *BEPCII Performance and Beam Dynamics Studies on Luminosity: JACoW, Geneva, Switzerland* (2016)
- [13] M. Ablikim, M.N. Achasov, P. Adlarson, S. Ahmed, M. Albrecht, R. Aliberti, A. Amoroso, M.R. An, Q. An, X.H. Bai et al., *Chinese Physics C* **46**, 074001 (2022)
- [14] M. Ablikim, M.N. Achasov, X.C. Ai, O. Albayrak, M. Albrecht, D.J. Ambrose, A. Amoroso, F.F. An, Q. An, J.Z. Bai et al., *Physical Review D* **92**, 012014 (2015)
- [15] R.L. Workman, V.D. Burkert, V. Crede, E. Klempt, U. Thoma, L. Tiator, K. Agashe, G. Aielli, B.C. Allanach, C. Amsler et al., *Progress of Theoretical and Experimental Physics* **2022** (2022)
- [16] K. Kampf, M. Knecht, J. Novotný, M. Zdráhal, *Physical Review D* **84**, 114015 (2011)
- [17] A. Anastasi, D. Babusci, G. Bencivenni, M. Berłowski, C. Bloise, F. Bossi, P. Branchini, A. Budano, L. Caldeira Balkeståhl, B. Cao et al., *Journal of High Energy Physics* **2016**, 1 (2016)
- [18] J.G. Layter, J.A. Appel, A. Kotlewski, W. Lee, S. Stein, J.J. Thaler, *Physical Review Letters* **29**, 316 (1972)
- [19] M.R. Jane, B.D. Jones, N.H. Lipman, D.P. Owen, B.K. Penney, T.G. Walker, M. Gettner, P. Grannis, H. Uto, J. Anderson et al., *Physics Letters B* **48**, 260 (1974)
- [20] M. Gormley, E. Hyman, W. Lee, T. Nash, J. Peoples, C. Schultz, S. Stein, *Physical Review Letters* **21**, 402 (1968)
- [21] F. Ambrosino, A. Antonelli, M. Antonelli, F. Archilli, P. Beltrame, G. Bencivenni, C. Bini, C. Bloise, S. Bocchetta, F. Bossi et al., *Physics Letters B* **694**, 16 (2010)
- [22] S. Gardner, J. Shi, *Physical Review D* **101**, 115038 (2020)
- [23] H. Akdag, T. Isken, B. Kubis, *Journal of High Energy Physics* **2022**, 1 (2022)
- [24] H. Akdag, B. Kubis, A. Wirzba, *Journal of High Energy Physics* **2023** (2023)
- [25] S. Prakhov, S. Abt, P. Achenbach, P. Adlarson, F. Afzal, P. Aguar-Bartolomé, Z. Ahmed, J. Ahrens, J.R.M. Annand, H.J. Arends et al., *Physical Review C* **97**, 065203 (2018)
- [26] P. Adlarson, W. Augustyniak, W. Bardan, M. Bashkanov, F.S. Bergmann, M. Berłowski, H. Bhatt, A. Bondar, M. Büscher, H. Calén et al., *Physical Review C* **90**, 045207 (2014)
- [27] A. Abele, J. Adomeit, C. Amsler, C. Baker, B. Barnett, C. Batty, M. Benayoun, A. Berdoz, K. Beuchert, S. Bischoff et al., *Physics Letters B* **417**, 197 (1998)
- [28] J.G. Layter, J.A. Appel, A. Kotlewski, W. Lee, S. Stein, J.J. Thaler, *Physical Review D* **7**, 2565 (1973)
- [29] B. Borasoy, R. Nibler, *The European Physical Journal A* **26**, 383 (2005)
- [30] C. Adolph, M. Angelstein, M. Bashkanov, U. Bechstedt, S. Belostotski, M. Berłowski, H. Bhatt, J. Bisplinghoff, A. Bondar, B. Borasoy et al., *Physics Letters B* **677**, 24 (2009)
- [31] S. Prakhov, B.M.K. Nefkens, P. Aguar-Bartolomé, L.K. Akasoy, J.R.M. Annand, H.J. Arends, K. Bantawa, R. Beck, V. Bekrenev, H. Berghäuser et al., *Physical Review C* **79**, 035204 (2009)
- [32] M.N. Achasov, K.I. Beloborodov, A.V. Berdyugin, A.G. Bogdanchikov, A.V. Bozhenok, D.A. Bukin, S.V. Burdin, A.V. Vasil'ev, T.V. Dimova, A.A. Drozdetskiĭ et al., *Journal of Experimental and Theoretical Physics Letters* **73**, 451 (2001)

- [33] A. Abele, J. Adomeit, C. Amsler, C. Baker, B. Barnett, C. Batty, M. Benayoun, A. Berdoz, K. Beuchert, S. Bischoff et al., *Physics Letters B* **417**, 193 (1998)
- [34] D. Alde, F. Binon, C. Bricman, S.V. Donskov, P. Duteil, M. Gouanre, A.V. Inyakin, D.B. Kakauridze, V.A. Kachanov, G.V. Khaustov et al., *Zeitschrift für Physik C Particles and Fields* **25**, 225 (1984)
- [35] M. Albaladejo, B. Moussallam, *The European Physical Journal C* **77**, 1 (2017)
- [36] G. Colangelo, S. Lanz, H. Leutwyler, E. Passemar, *Physical Review Letters* **118**, 022001 (2017)
- [37] M. Ablikim, M.N. Achasov, S. Ahmed, M. Albrecht, A. Amoroso, F.F. An, Q. An, J.Z. Bai, Y. Bai, O. Bakina et al., *Physical Review D* **97**, 012003 (2018)
- [38] M. Ablikim, M.N. Achasov, X.C. Ai, O. Albayrak, M. Albrecht, D.J. Ambrose, A. Amoroso, F.F. An, Q. An, J.Z. Bai et al., *Chinese Physics C* **41**, 013001 (2017)
- [39] M. Ablikim, M.N. Achasov, P. Adlarson, S. Ahmed, M. Albrecht, M. Alekseev, A. Amoroso, F.F. An, Q. An, Y. Bai et al., *Chinese Physics C* **44**, 040001 (2020)
- [40] T. Petri, Ph.D. thesis (12.10.2010)
- [41] B. Borasoy, R. Nißler, *The European Physical Journal A* **33**, 95 (2007)
- [42] C.Q. GENG, J.N. NG, T.H. WU, *Modern Physics Letters A* **17**, 1489 (2002)
- [43] D.N. GAO, *Modern Physics Letters A* **17**, 1583 (2002)
- [44] L. Gan, B. Kubis, E. Passemar, S. Tulin, *Physics Reports* **945**, 1 (2022)