Quarkyonic matter and applications in neutron stars

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Abstract. The structure and basic properties of dense nuclear matter still remain one of the open problems of physics. In particular, the composition of the matter that composes neutron stars is under theoretical and experimental investigation. Among the theories that have been proposed, apart from the classical one where the composition is dominated by hadrons, the existence or coexistence of free quark matter is a dominant guess. An approach towards this solution is the phenomenological view according to which the existence of quarkyonic matter plays a dominant role in the construction of the equation of state (EOS). In this paper we propose a phenomenological model for quarkyonic matter, borrowed from corresponding applications in hadronic models, where the interaction in the quarkyonic matter depends not only on the position but also on the momentum of the quarkynions. This consideration, as we demonstrate, can have a dramatic consequence on the shape of the EOS and thus on the properties of neutron stars.

1 Introduction

One of the fundamental problems of physics remains the composition of dense nuclear matter as well as its basic properties both at zero and at finite temperature [1–4]. In particular, the equation of state of neutron star matter is the key quantity to study these objects. In this effort, a key problem that often arises is the inability of the EOSs to predict maximum masses for neutron stars that are compatible with recent observations (well above two solar masses) without simultaneously violating the sound-speed causality.

An interesting attempt in this direction is the consideration of a hybrid state of dense nuclear matter called quarkyonic matter. Following the analysis of Ref. [5, 6] the basic assumption of quarkyonic matter is that at large Fermi energy, the degrees of freedom inside the Fermi sea may be treated as quarks, and confining forces remain important only near the Fermi surface where nucleons emerge through correlations between quarks. In this case one can consider that quarks, confinement at the Fermi surface, occupying a momentum shell of width $\Delta \simeq \Lambda_{QCD}$, produce triplets with spin $1/2$, that are the baryons [5, 7, 8].

To better understand this idea, we illustrate schematically the momentum space in Fig. (1), where we indicate, with different colors, the low and high momentum states which occupied by quarks and baryons respectively.

The structure of the paper is as follows: Section 2 introduces the model used to study the quarkionic matter in neutron stars. Section 3 is focused on presenting the results and providing relevant discussion. Lastly, Section 4 concludes our investigation with final remarks.

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2 The model

2.1 Pure neutron matter

We start our calculations with a pure neutron model to compare it with the quarkyonic one. First of all we have to compute the number density of neutrons which will be in the form,

$$n_n = g_s \int_0^{k_{F_n}} \frac{d^3k}{(2\pi)^3} = \frac{g_s}{6\pi^2} k_{F_n}^3 \tag{1}$$

We obtain the energy density in the following form as,

$$\epsilon_n = \frac{g_s}{2\pi^2} \int_0^{k_{F_n}} k^2 \sqrt{(\hbar c k)^2 + m_e^2 c^4}\, dk + V(n_n) \tag{2}$$

where the first term is the kinetic part and is obtained in the relativistic form and $V(n_n)$ is the potential energy. After that, the chemical potential of neutrons will be given from the familiar thermodynamic relation

$$\mu_n = \frac{\partial \epsilon_n}{\partial n_n} \tag{3}$$

and the total pressure will be,

$$P = \mu_n n_n - \epsilon_n \tag{4}$$

so to construct the equation of state.

As an initial effort, we assume that neutrons interact via a momentum dependent potential in the following form [1],

$$\begin{aligned} V_{\rm int}(n_n) &= \frac{1}{3} A n_s (1 + x_0) u^2 + \frac{\frac{2}{3} B n_s (1 - x_3) u^{\sigma+1}}{1 + \frac{2}{3} B' n_s (1 - x_3) u^{\sigma-1}} \\ &+ u \sum_{i=1,2} \frac{1}{5} \Big[6C_i - 8Z_i \Big] \mathcal{J}_n^i \end{aligned} \tag{5}$$

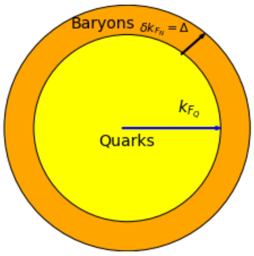

Figure 1. The momentum space of quarkyonic matter. Quarks occupy all states from zero momentum up to a quark Fermi momentum (k_{F_Q}) forming a Fermi sea. Baryons occupy higher momentum states, forming a Fermi shell in the momentum space with a width Δ.

where,

$$\mathcal{J}_n = \frac{2}{(2\pi)^3}\int d^3k g(n,\Lambda_i) f_\tau = \frac{2}{(2\pi)^3}\int_0^{k_{F_n}} 4\pi\left[1+\left(\frac{k}{\Lambda_i}\right)^2\right]^{-1} k^2 dk \quad (6)$$

The parameterization we used for the momentum dependent potential (Eq. (5)) is the following: $A = -46.65$, $B = 39.45$, $B' = 0.3$, $\sigma = 1.663$, $C_1 = -83.84$, $C_2 = 23$, $\chi_0 = 1.654$, $\chi_3 = -1.112$, $Z_1 = 3.81$, $Z_2 = 13.16$, $\Lambda_1 = 1.5k_{Fn_0}$, $\Lambda_2 = 3k_{Fn_0}$, $u = n_n/n_0$, the saturation density $n_0 = 0.16$ fm^{-3} and k_{Fn_0} is the neutron Fermi momentum at the saturation density.

The first two terms in Eq. (5) are both momentum independent and correspond to an attractive and a repulsive interaction respectively. The last term of the potential energy is momentum dependent, corresponds to an attractive interaction and expresses the finite range interaction of the potential.

We construct the equation of state for each model and we compute the sound velocity ,

$$\frac{c_s}{c} = \sqrt{\frac{\partial P}{\partial \epsilon}} \quad (7)$$

so to investigate if causality is violated or not.

2.2 The NDU quarkyonic model with momentum dependent interaction forces

To study quarkyonic matter and its effects in the neutron star properties, we start with a simple model in which we consider a neutron star that consists of only neutrons, up and down quarks ((NDU) model) [5, 9]. Also we assume that neutrons interact via a momentum dependent potential and quarks are asymptotically free. In this case, the equation for charge neutrality will take the simple form,

$$n_d = 2n_u \quad (8)$$

or with respect to fermi momentum,

$$k_{F_d} = 2^{1/3} k_{F_u} \quad (9)$$

The equation for chemical equilibrium between quarks and neutrons will become,

$$\mu_n = \mu_u + 2\mu_d \quad (10)$$

where

$$\mu_n = \sqrt{(\hbar c k_{Fn})^2 + m_N^2 c^4} + \frac{\partial V(n_n)}{\partial n_n} \quad (11)$$

is the neutron chemical potential. The quarks chemical potentials will be

$$\mu_u = \sqrt{(\hbar c k_{Fu})^2 + m_Q^2 c^4} \quad (12)$$

and

$$\mu_d = \sqrt{(\hbar c k_{Fd})^2 + m_Q^2 c^4} \quad (13)$$

for up and down quarks respectively. Quark masses are obtained by $m_Q = m_N/N_c$ where m_N is the nucleon mass and N_c is the number of colors.

In this model we assume that the system is not in chemical equilibrium so we will not impose relation (10). Instead of this, we consider a more simple expression for the Fermi momentum of down quarks and neutrons as in Ref. [5, 10], which derived by the basic assumption of the quarkyonic scenario,

$$k_{F_d} = \frac{k_{F_n} - \Delta}{3} \quad (14)$$

where Δ is the width of the momentum shell where baryons reside

$$\Delta = \frac{\Lambda_{Qyc}^3}{\hbar^3 c^3 k_{F_N}^2} + \kappa_{Qyc}\frac{\Lambda_{Qyc}}{\hbar c N_c^2} \quad (15)$$

We set the parameters $\Lambda_{Qyc} \approx \Lambda_{QCD}$ and $\kappa_{Qyc} = 0.3$ and we compute the energy density, the number density and chemical potentials as before,

$$n_Q = \frac{g_s N_c}{2\pi^2}\sum_{i=u,d}\int_0^{k_{F_i}} k^2 dk \quad (16)$$

$$\epsilon_Q = \frac{g_s N_c}{2\pi^2}\sum_{i=u,d}\int_0^{k_{F_i}} k^2\sqrt{(\hbar c k)^2 + m_Q^2 c^4}dk \quad (17)$$

$$n_n = \frac{g_s}{2\pi^2}\int_{k_{F_n}-\Delta}^{k_{F_n}} k^2 dk \quad (18)$$

$$\epsilon_n = \frac{g_s}{2\pi^2}\int_{k_{F_n}-\Delta}^{k_{F_n}} k^2\sqrt{(\hbar c k)^2 + m_n^2 c^4}dk + V_{int}(n_n, k_{F_n}) \quad (19)$$

for quarks and neutrons respectively. The interaction term for neutrons energy density will be in the form of Eq. (5), where now,

$$\begin{aligned}\mathcal{J}_n &= \frac{2}{(2\pi)^3}\int d^3k g(n,\Lambda_i) f_\tau \\ &= \frac{2}{(2\pi)^3}\int_{k_{F_n}-\Delta}^{k_{F_n}} 4\pi\left[1+\left(\frac{k}{\Lambda_i}\right)^2\right]^{-1} k^2 dk \qquad (20)\end{aligned}$$

We set the number of colors and the degeneracy of the spin equal to $N_c = 3$ and $g_s = 2$ respectively. The total baryon density and total energy density will be,

$$n_B = n_n + \frac{(n_u + n_d)}{3} = \frac{1}{3\pi^2}(k_{F_n}^3 - (k_{F_n} - \Delta)^3 + k_{F_u}^3 + k_{F_d}^3) \quad (21)$$

and the total energy density will be,

$$\epsilon_{tot} = \epsilon_n + \epsilon_Q \quad (22)$$

The chemical potential for each speace of matter will be in the form,

$$\mu_i = \frac{\partial \epsilon_{tot}}{\partial n_i} \quad (23)$$

and the total pressure will be

$$P = -\epsilon_{tot} + \sum_{i=n,u,d} \mu_i n_i \quad (24)$$

where index i express neutrons, up and down quarks respectively. After that, we compute the sound velocity from relation (7),

3 Results and Discussion

The first result we extracted is the speed of sound for each model and for various values of the microscopic parameters. The reason is to see if the equations of state we provided are causal. We investigate several values of the transition density (n_{tr}) as well as for the parameter Λ_{Qyc}. We present these results in Fig. (2).

It is important to note that for the different values of transition density, the pick in the speed of sound as a function of the baryon density is not affected at all. On the other hand, we can see that as the parameter Λ_{Qyc} increases, the maximum speed of sound also increases, leading to a violation of causality for values $\Lambda_{Qyc} > 210$ MeV.

After the construction of equations of state for the pure neutron case as well as for quarkyonic model, we solve Tolman-Oppenheimer-Volkoff (TOV) equations so to calculate mass, radius, tidal deformability and other bulk quantities of a neutron star. This system of equations for a static, spherically symmetric neutron star has the following form,

$$\begin{aligned}\frac{dm(r)}{dr} &= 4\pi r^2 \rho(r), \\ \frac{dP(r)}{dr} &= -\rho(r)c^2\left(1+\frac{P(r)}{\rho(r)c^2}\right)\frac{d\phi(r)}{dr}, \\ \frac{d\phi(r)}{dr} &= \frac{Gm(r)}{c^2r^2}\left(1+\frac{4\pi P(r)r^3}{m(r)c^2}\right)\left(1-\frac{2Gm(r)}{rc^2}\right)^{-1} \qquad (25)\end{aligned}$$

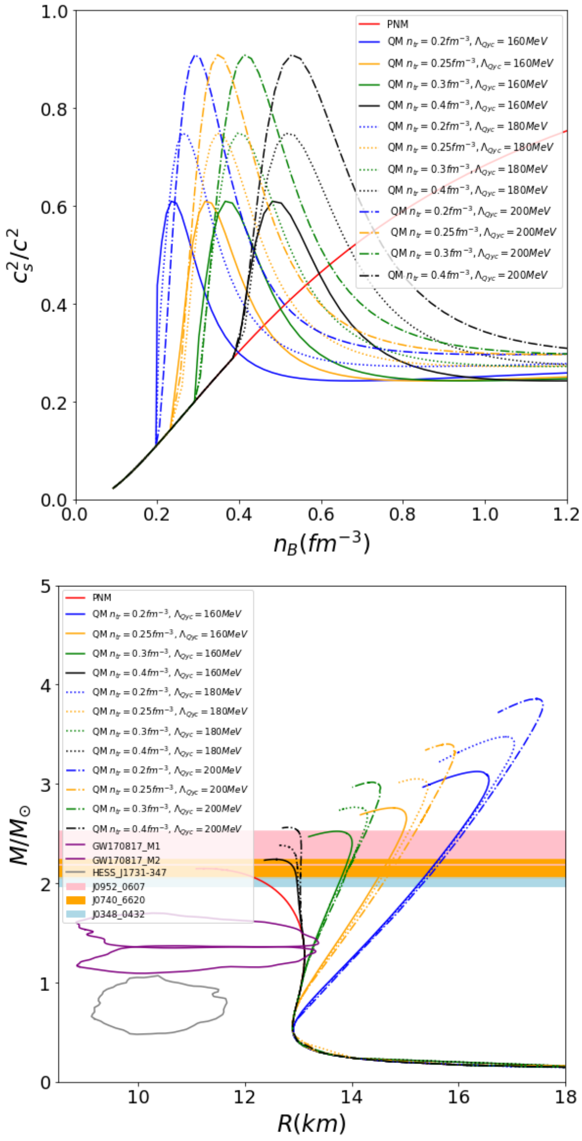

Figure 2. The sound velocity for the quarkyonic matter (QM) interacting via the momentum depended interaction (upper figure) as a function of baryon density and the mass - radius diagrams for the same model (lower figure), for $n_{tr} = 0.2, 0.25, 0.3, 0.4$ fm^{-3} (blue , yellow, green and black lines respectively) and for $\Lambda_{Qyc} = 160, 180, 200$ MeV (solid, dotted and dashed - dotted lines respectively). The solid red line corresponds to the pure neutron matter model (PNM). The shaded regions in the lower panel correspond to possible constraints on the maximum mass from the observation of PSR J0348+0432, PSR J0740+6620 and PSR J0952+0607 [11–15].

where $P(r)$ is the total pressure, $m(r)$ is the enclosed mass of the star and $\rho(r)$ is the total mass density.

Another interesting result to notice is that for values of transition density near the saturation density and below, the masses and the radius provided are very large, and as we increase the transition density, the quarkyonic model tends to be equivalent to the pure neutron case. In figures 2 and 3, we present our results for the mass, the radius and tidal deformability for a cold neutron star. We assume for transition density the values $n_{tr} = 0.2, 0.25, 0.3, 0.4$

fm^{-3} and for parameter Λ_{Qyc} we set Λ_{Qyc} = 160, 180, 200 MeV. Also we include some recent observational data from LIGO and HESS experiments, so to make a comparison with our results. Our goal is to make some constrains for the transition density and Λ_{Qyc}.

One can see in the lower diagram of Fig. (2) that quarkyonic model for n_{tr} = 0.3 and 0.4 fm^{-3} as well as the pure neutron matter model, predict neutron stars with masses around 1.4 solar masses to have radius about 13-13.5 km, which is compatible with the observational data resulting from LIGO. If we set the transition density to be 0.25 fm^{-3} and below, our predictions are far away from the observational data.

4 Conclusions

After this initial effort we can note some interesting features of quarkyonic matter. First of all, quarkyonic matter provides the sound speed as a non-monotonic function of the baryon density, without exceeding the speed of light and it is reaching asymptotically the value $1/\sqrt{3}$ which is the conformal limit. This fact, along with the maximum neutron star masses predicted by quarkyonic equations of state, constitutes an important tool for explaining the properties of dense nuclear matter as well as the bulk properties of compact objects.

Also, quarkyonic matter may bridge the gap between hadronic and quark matter and explain a possible phase transition between these two phases. In future work we have to extend our model to include protons and electrons to impose β-equilibrium and to apply quarkyonic matter in finite temperature neutron stars.

Also we have to investigate if there is any fundamental theory which can provide this state of matter [2, 3]. We expect that future gravitational wave observations from binary neutron star systems will give us information to constrain further some of the microscopic parameters of our model, so that to test and to improve our equations of state.

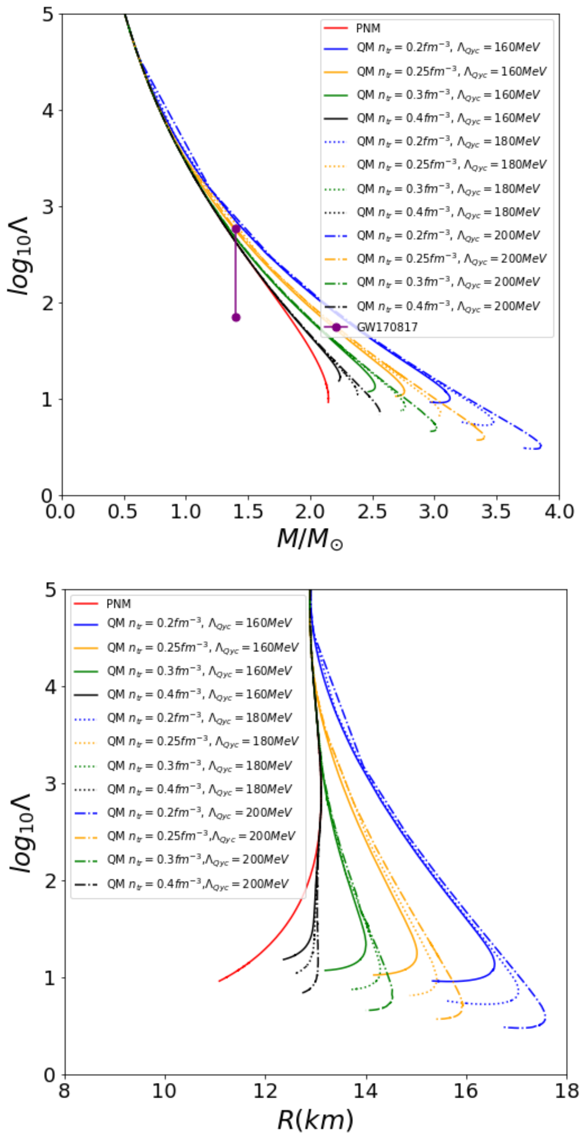

Figure 3. Tidal deformability as a function of neutron star mass for the quarkyonic matter (QM) interacting via the momentum depended interaction (upper figure) and verus radius (lower figure), for n_{tr} = 0.2, 0.25, 0.3, 0.4 fm^{-3} (blue, yellow, green and black lines respectively) and for Λ_{Qyc} = 160, 180, 200 MeV (solid, dotted and dashed - dotted lines respectively). The solid red line corresponds to the pure neutron matter model (PNM). The purple line in upper figure corresponds to the event GW170817 (see Ref. [16]).

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