Analytical model for dispersion measurement in integrated waveguides using michelson interferometry effects

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Abstract. We present an analytical model for measuring the dispersion of integrated waveguides, leveraging the Michelson interferometry effects observed in devices with chirped Bragg gratings. Building on our previous experimental work, we derived a theoretical framework that simulates the group delay and subsequent dispersion values from the reflected spectrum of a device under test (DUT) which is a linearly chirped Bragg grating fabricated on a silicon-on-insulator (SOI) platform. This model incorporates the principles of interference fringes generated by reflections within the waveguide, enabling a precise calculation of group delay ($\boldsymbol{\tau}$) in the DUT as a function of frequency. Our model predicts the dispersion by determining the spacing between the peaks ($\Delta \boldsymbol{f}$) from the local period of the interferometric fringes, with $\boldsymbol{\tau}$ being inversely proportional to $\Delta \boldsymbol{f}$. Simulations were conducted on a DUT that is designed to produce a dispersion of **-45.9 ps²**. The model yielded a dispersion of **-45.6 ± 0.67 ps²**, demonstrating close alignment with both the theoretical design and our experimental results, which recorded a dispersion of **-45.5 ± 11.2 ps²** from **7** different DUTs that were measured.

Introduction

In this paper, we analytically demonstrate an approach that we believe can serve as an alternative to established methods [1] for measuring the dispersion of light reflected in integrated optical devices. This method leverages the principle that light reflected from the end facet of an integrated waveguide interferes with light reflected from points within the device under test (DUT), effectively forming a Michelson interferometer. The spacing between the fringes of this interferometric signal is directly related to the group delay experienced in the DUT, enabling a fast and straightforward measurement of waveguide dispersion.

In the context of Bragg gratings, the dominant interactions, which induce the reflected spectrum occur around a wavelength for which there is reflection of a mode of amplitude A into an identical counter-propagating mode of amplitude B. The evolution of A and B along the z-axis of the waveguide can therefore be simplified by retaining only the terms involved in the interactions with the amplitudes of the particular mode [2].

Method

For an optical cavity with a free spectral range of $\Delta \boldsymbol{f}$, the group delay ($\boldsymbol{\tau}$) is inversely proportional to $\Delta \boldsymbol{f}$ [3]. By finding the local period in the reflected spectrum, $\boldsymbol{\tau}$ can be found as a function of frequency and from this, the dispersion as the slope of $\boldsymbol{\tau}$**.** The combined intensity ($\boldsymbol{I}$) contained in the two beams reflected from the mirrors of Michelson interferometer is given by:

$$I = |E_1 exp[i(kz + \omega t + \emptyset_1)] + E_2 exp[i(kz + \omega t + \emptyset_2)]|^2 \quad (1)$$

$\omega =$ angular frequency, $k = 2\pi/\lambda$ represents the wavenumber, $\lambda =$ wavelength, $\emptyset_n =$ grating chirp, and is a function of position (z) and time (t). E_1 and E_2 are the amplitudes of the two beams [4]. Using coupling mode theory, we can obtain a quantitative measure of the spectral dependence on the Bragg gratings, and the analytical reflected spectrum can be generated. Following the derivations from [2], the amplitudes E_1 and E_2 are defined as:

$$E_1 \equiv A\left[exp\left(i\delta z - \frac{\emptyset}{2}\right)\right] \quad (2)$$

$$E_2 \equiv B\left[exp\left(-i\delta z + \frac{\emptyset}{2}\right)\right] \quad (3)$$

$$\delta = 2\pi n_{eff}\left(\frac{1}{\lambda} - \frac{1}{\lambda_D}\right) \quad (4)$$

Where λ_D is the design wavelength and n_{eff} is the effective index seen by the propagating mode inside the DUT. Parameters used to obtain the analytical spectrum are shown in the table below:

Table showing parameters for simulation

Parameter	Value	Remarks
L	$5248\mu m$	Length used to fabricate DUT
$\emptyset$	7.5nm	Grating chirp used to fabricate DUT
n_{eff}	2.8	
λ	1530nm to 1550nm	Wavelength sweeping range during experiment
λ_D	1550nm	Design wavelength for DUT
z	L-200nm	Total length for Gratings

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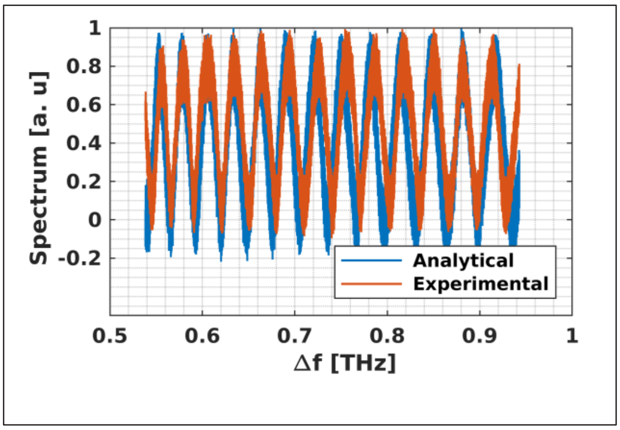
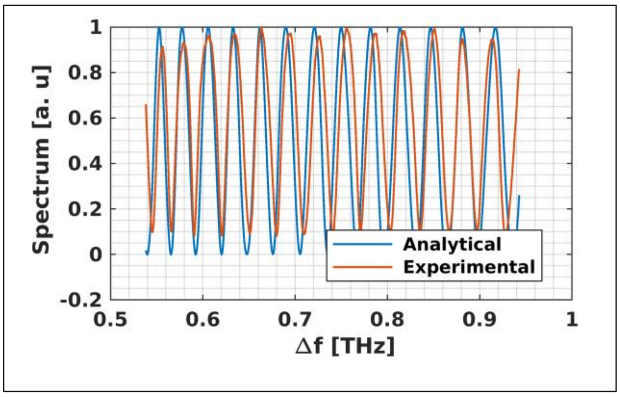
Results

The plots of the experimental and analytical reflected spectra for both the unfiltered and filtered cases are shown in the figures below:

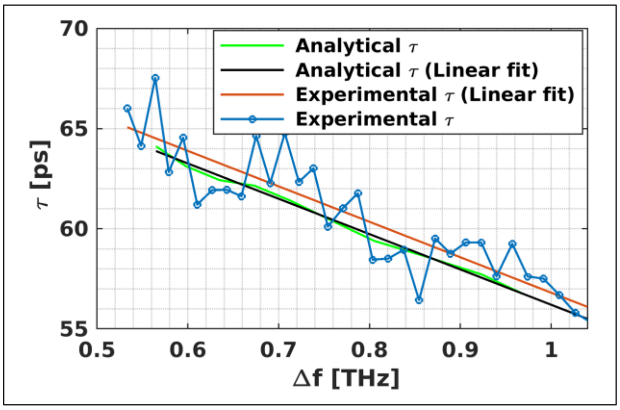


Reflected spectra for the unfiltered case



Reflected spectra for the filtered case

From the plots shown above, a closer look at the peaks of the spectra reveals that the spacing between the peaks (Δf) varies as the frequency increases, and τ was calculated as the inverse of Δf. The group delay (τ), as a function of frequency or wavelength is a linear function, with some additional oscillations. Again, the group delay ripple is usually observed experimentally [5,6], and in a particular case of chirped Bragg gratings, it is partly as a result of the perturbation induced in the effective index, which can be reduced by decreasing the corrugation width [7]. Using the curve fit strategy, the slope (which is the dispersion of the device) on the group delay curve for the DUT was calculated. The combined plots of the group delay for the experimental measurement and the analytical model is shown in the figure below:



The experimental and analytical τ

The DUT is a linearly chirped Bragg grating designed to generate a dispersion of **-45.9 ps**^{2}. Simulations conducted on the DUT yielded a dispersion of **-45.6 ± 0.67 ps²**, demonstrating close alignment with both the theoretical design value and our experimental results, which recorded a dispersion of **-45.5 ± 11.2 ps²** from **7** different DUTs that were measured.

Conclusion

This significant reduction in uncertainty highlights the robustness of our model, confirming its potential as a reliable and efficient alternative to traditional dispersion measurement methods [3]. By offering a comprehensive analytical approach, this work paves the way for rapid and accurate assessment of dispersive integrated devices for various applications such as true time delays and optical time lenses. Future work will focus on further validation and refinement of the model.

References

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