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Generating function approach for freeform two-reflector two-target system

Pieter Braam[1,*], *Jan* ten Thije Boonkkamp[1], *Martijn* Anthonissen[1], and *Wilbert* IJzerman[2,1]

[1]CASA, Department of Mathematics and Computer Science, Eindhoven University of Technology, The Netherlands
[2]Signify Research, High Tech Campus 7, Eindhoven, The Netherlands

Abstract. We discuss an inverse method to compute a freeform two-reflector two-target system. The optical path length constitutes an integral component and can be expressed in terms of position coordinates at the first target. The system is expressed in terms of a generating function, closed with energy balance and requires a sophisticated least-squares solver to compute the shapes of the reflectors. In a numerical example, we illustrate the algorithm's capabilities to tackle even the most intricate light distributions.

1 Introduction

To design optical systems in illumination optics, forward methods are often used to model a system through the random sampling of light rays [1]. However, convergence of these methods is slow and therefore computationally expensive. In contrast, inverse methods provide a more efficient approach by directly computing the shape of the optical surfaces [2].

In the two-reflector two-target system, light originates from a source with a certain emittance, reflects on two reflectors, arrives at the first target with a specific intensity and finally ends up at the second target with another specified intensity. We simulate this optical system using a three-stage least-squares algorithm. In the first stage, the position of each light ray at the second target will be determined as a function of the first target, which we call a 'mapping'. The second stage determines the mapping from the source to the first target. Finally, in the third stage, the algorithm finds the shapes of the two reflectors.

The contents of this paper are the following. In Section 2 we formulate the two-target system, in Section 3 we briefly discuss the least-squares algorithm, in Section 4 we give an example of the two-target system, and in Section 5 we present concluding remarks.

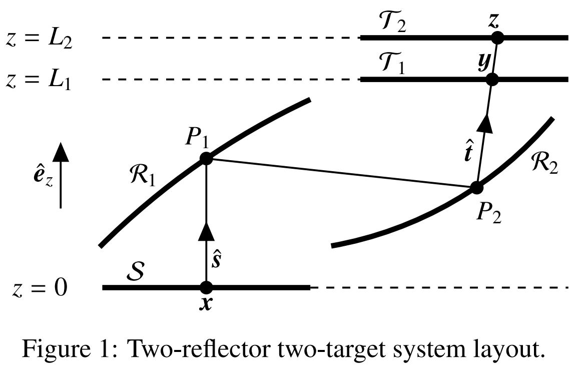


Figure 1: Two-reflector two-target system layout.

2 Formulation reflector system

In this section we formulate the two-target system. We denote vectors of unit length with a hat. As illustrated in Fig. 1, we assume that a light ray originates from a point $\boldsymbol{x}$ on the source plane $\mathcal{S}$ in $z = 0$ with direction $\hat{\boldsymbol{s}} = (0,0,1)^T$, hits a first reflector $\mathcal{R}_1$ in the point P_1, then hits a second reflector $\mathcal{R}_2$ in the point P_2, hits the first target plane $\mathcal{T}_1$ on $z = L_1$ in the point $\boldsymbol{y}$ with direction $\hat{\boldsymbol{t}} = (t_1, t_2, t_3)^T$, maintains this direction and finally hits the second target plane $\mathcal{T}_2$ on $z = L_2$ in the point $\boldsymbol{z}$. By $u_1 = u_1(\boldsymbol{x})$, $u_2 = u_2(\boldsymbol{y})$ and $V = V(\boldsymbol{y})$ we denote the distance between $\boldsymbol{x}$ and P_1, the distance between P_2 and $\boldsymbol{y}$, and the optical path length between $\boldsymbol{x}$ and $\boldsymbol{y}$ along the path of a light ray, respectively. Consequently, the position vector of the first and second reflector can be expressed as $\boldsymbol{r}_1(\boldsymbol{x}) = (\boldsymbol{x}, u_1)^T$ and $\boldsymbol{r}_2(\boldsymbol{y}) = (\boldsymbol{y}, L_1)^T - u_2\hat{\boldsymbol{t}}$. We can express u_1 as a generating function $u_1(\boldsymbol{x}) = G(\boldsymbol{x}, \boldsymbol{y}, u_2(\boldsymbol{y}))$ which has a corresponding inverse $u_2(\boldsymbol{y}) = H(\boldsymbol{x}, \boldsymbol{y}, u_1(\boldsymbol{y}))$. Both involve the optical path length, which is obtained from the equation $\nabla_{\boldsymbol{y}} V = (t_1, t_2)^T$; see [3]. We enforce a unique solution by selecting $u_1(\boldsymbol{x}) = \max_{\boldsymbol{y}\in\mathcal{T}_2} G(\boldsymbol{x}, \boldsymbol{y}, u_2(\boldsymbol{y}))$ and $u_2(\boldsymbol{y}) = \max_{\boldsymbol{x}\in\mathcal{S}} H(\boldsymbol{x}, \boldsymbol{y}, u_1(\boldsymbol{y}))$. As a result, a necessary condition is that

$$\nabla_{\boldsymbol{x}} H(\boldsymbol{x}, \boldsymbol{y}, u_1(\boldsymbol{y})) + H_z(\boldsymbol{x}, \boldsymbol{y}, u_1(\boldsymbol{y}))\nabla_{\boldsymbol{x}} u_1(\boldsymbol{x}) = \boldsymbol{0}. \quad (1)$$

We can solve for $\boldsymbol{y} = \boldsymbol{m}_1(\boldsymbol{x})$, where $\boldsymbol{m}_1$ is the mapping from $\mathcal{S}$ to $\mathcal{T}_1$, provided that the mixed Hessian matrix $\boldsymbol{C} = (\frac{\partial^2 \widetilde{H}}{\partial x_i \partial y_j})$ with $\widetilde{H}(\boldsymbol{x}, \boldsymbol{y}) := H(\boldsymbol{x}, \boldsymbol{y}, u_1(\boldsymbol{y}))$ is invertible. By substituting $\boldsymbol{y} = \boldsymbol{m}_1(\boldsymbol{x})$ in equation (1) and subsequently differentiating with respect to $\boldsymbol{x}$, we find the matrix equation $\boldsymbol{C}\mathrm{D}\boldsymbol{m}_1 = \boldsymbol{P}$, where $\boldsymbol{P} = \boldsymbol{P}(\boldsymbol{x}) = -\mathrm{D}_{\boldsymbol{xx}}\widetilde{H}$. Moreover, from global energy conservation between $\mathcal{S}$ and $\mathcal{T}_1$ we find the Monge-Ampère equation $\det(\mathrm{D}\boldsymbol{m}_1(\boldsymbol{x})) = \frac{f(\boldsymbol{x})}{g(\boldsymbol{m}_1(\boldsymbol{x}))} =: F(\boldsymbol{x})$, supplemented with the transport boundary condition $\boldsymbol{m}_1(\partial\mathcal{S}) = \partial\mathcal{T}_1$, where f denotes the emittance at $\mathcal{S}$ and g denotes the intensity at $\mathcal{T}_1$. Consequently, we have the constraint $\det(\boldsymbol{P}) = \det(\boldsymbol{C})F$ for the matrix $\boldsymbol{P}$. Similarly, we can solve for $\boldsymbol{z} = \boldsymbol{m}_2(\boldsymbol{y})$, where $\boldsymbol{m}_2$ is the mapping from $\mathcal{T}_1$ to $\mathcal{T}_2$.

*e-mail: p.a.braam@tue.nl

3 Least-squares algorithm

In this section we outline the three-stage least-squares algorithm to compute the reflector system. In the first stage, we solve for $z = \boldsymbol{m}_2(\boldsymbol{y})$. In the second stage, we solve for $\boldsymbol{y} = \boldsymbol{m}_1(\boldsymbol{x})$ and apply bilinear interpolation to find the target vector $\hat{\boldsymbol{t}}$. Moreover, we choose an average value for u_1 over the source domain $\mathcal{S}$ and a specific optical path length V for the light ray corresponding to the center of $\mathcal{S}$. Finally, in the third stage of the algorithm we solve equation (1) to find u_1, substitute this in H to find u_2 and finally shape the reflectors from the expressions of $\boldsymbol{r}_1(\boldsymbol{x})$ and $\boldsymbol{r}_2(\boldsymbol{y})$.

4 Numerical example

In this section we give an example. We assume that the system converts a uniform intensity of light, defined on $\mathcal{S} = [-8, -6] \times [-1, 1]$, onto the first target $\mathcal{T}_1 = [-3, 3]^2$ and the second target $\mathcal{T}_2 = [-3, 3]^2$, having the intensities of the gray-scale image of a pawn and the gray-scale image of a queen, respectively. We apply the least-squares algorithm for 200 iterations on a 201×201 grid, with $L_1 = 10$, $L_2 = 15$, an average value of 6 for u_1 and the optical path length for the light ray corresponding to the center of the source domain is 20. The resulting optical system can be seen in Fig. 2, where several rays have been plotted along with the desired gray-scale images of a pawn and queen. The corresponding mappings are shown in Fig. 3, where both the light distributions of a queen and a pawn can clearly be recognized.

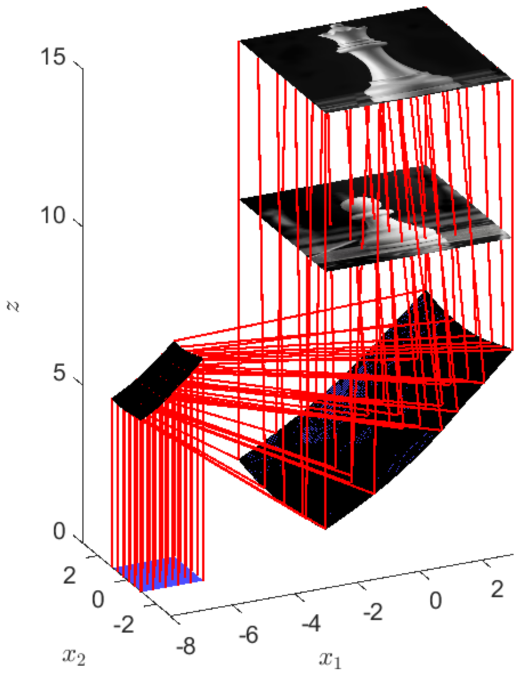
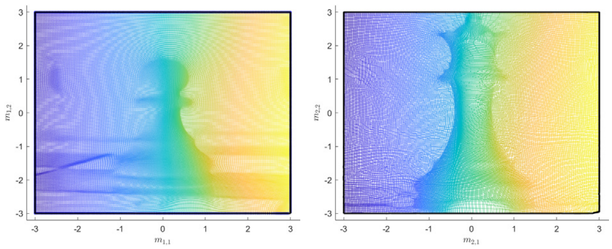


Figure 2: Two-reflector two-target system.



(a) Mapping $m_1 : \mathcal{S} \to \mathcal{T}_1$. (b) Mapping $m_2 : \mathcal{T}_1 \to \mathcal{T}_2$.

Figure 3: Mappings on the target screens.

5 Concluding remarks

We formulated an inverse model for the two-reflector two-target system using a generating function involving the optical path length. The least-squares algorithm first computed the mappings to the targets and subsequently derived the shapes of the reflectors. Specifics of the algorithm can be found in [4], [5] and [6]. To illustrate the algorithm's capabilities, we showed a numerical example with two intricate light distributions.

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