Principal component analysis of refractive index spaces: from glass properties to residual colour prediction

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Abstract. Traditionally glass dispersion is described by Abbe numbers and partial dispersions, and colour correction is explained and visualized e.g. with Pg,F- and Herzberger diagrams. We have recently developed an alternative approach to colour analysis and colour correction based on principal component analysis of normalized refractive index data. The resulting diagrams can be used not only for glass selection, but also for quantitative prediction of partial refractive powers and residual colour aberrations. The method is intrinsically model-free, robust to the choice of dataset and wavelengths, and transfers easily to any spectral range.

1 Linear equations for colour correction

In contrast to classical approaches summarized in Ref. [1], we use a symmetric definition of glass dispersion with normalized index differences

$$\delta_j(\lambda_i) = \frac{n_j(\lambda_i) - \bar{n}_j}{\bar{n}_j - 1}, \tag{1}$$

comparing the refractive index $n_j(\lambda_i)$ of glass j to the mean refractive index $\bar{n}_j$ across wavelengths. This allows to write the condition for colour correction for a system of k thin lenses in contact as linear equations

$$\Delta\Phi(\lambda_i) = \sum_{j=1}^{k} \delta_j(\lambda_i)\bar{\Phi}_j = 0 \quad \forall i, \tag{2}$$

where the individual refractive powers $\bar{\Phi}_j$ of the lenses are coefficients scaling the length of the basis vectors $\delta_j(\lambda_i)$. By scaling to the total refractive power, we can directly identify the normalized index differences δ_j with the refractive power deviation of a single lens made from material j.

2 Principal component analysis

Because all glasses show the same general trend in dispersion, the quantities δ_j are highly correlated and have very different extent in different directions, making the graphical solution of eqn. (2) in a δ_j glass diagram practically unviable. We therefore use principal component analysis (PCA) to arrive at a more convenient presentation while conserving the vectorial solution properties. The PCA also transforms the basis of the δ_j-diagram. The resulting δ_{j,PC_i} are combinations of the δ_j at different wavelengths and correspond to different orders of colour aberrations. The basis functions themselves are very robust to changes in both glass catalog and wavelength sampling (see Fig. 1). For two dimensions, the resulting diagrams resemble Hoogland's transformed Pg,F-diagram [2].

3 Graphical colour correction

Equations (2) can be solved graphically using vector combinations of the δ_{j,PC_i} with relative powers $\bar{\Phi}_j/\Phi$. The location of the resulting vector in the diagram indicates the different orders of relative residual colour error $\Delta\Phi_{\mathrm{PC}_i}/\Phi$. This is shown in Fig. 2 for achromatic and apochromatic correction. By varying the relative powers of two glasses, different points along the line through the glasses can be reached, where interpolation means same sign of the powers, and extrapolation means opposite signs.

Similar graphical design methods can be used for superapochromatic lenses [3]. 3D diagrams are then used to visually identify combinations of two, three or four glasses suitable for correcting up to third order colour. As shown in Fig. 3, the glasses from different catalogs fall all in the same region for first- and second order dispersion, but there are noticeable differences in the third order which may be useful for higher order colour correction. For example, N-KZFS2 from Schott and BAF11 from Hoya are significantly off the main distribution in opposite directions on the third-order colour axis.

We believe that the visual approach using PCA can be a helpful addition to existing methods, especially as it is easily transferred to any spectral range and set of glasses.

References

[1] P.S.L. Serna, J.S.C. Revuelta, Appl. Opt. **61**, A50 (2022)

[2] J. Hoogland, *The design of apochromatic lenses*, in *Recent Developments in Optical Design*, edited by R.A. Ruloff (Perkin-Elmer, Norwalk, Conneticut, 1968), pp. 6–1–6–8

[3] H. Münz, M. Peschka, *Principal component analysis of refractive index spaces: a model-free approach to color analysis and color correction*, SPIE Optical Systems Design 2024

 https://doi.org/10.1051/epjconf/202430903019

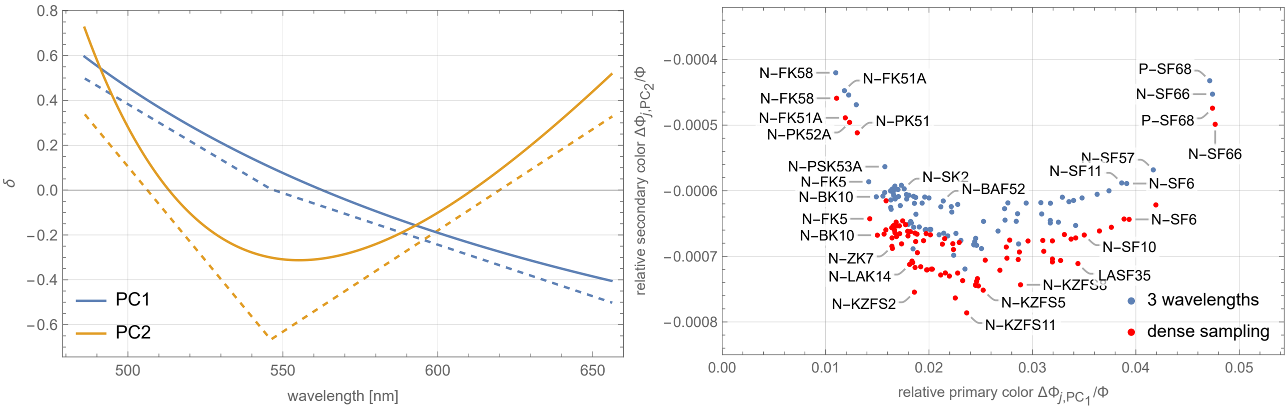

Figure 1. The left diagram shows the first two principal components for the Schott glass catalog corresponding to first and second order colour, using either the C,e,F wavelengths (dashed) or dense sampling over the same range (solid). The glass diagrams (right) are identical except for the slightly different scale on the secondary colour axis arising from the different sampling. The familiar normal line through K7 and F2 would be roughly horizontal in this plot.

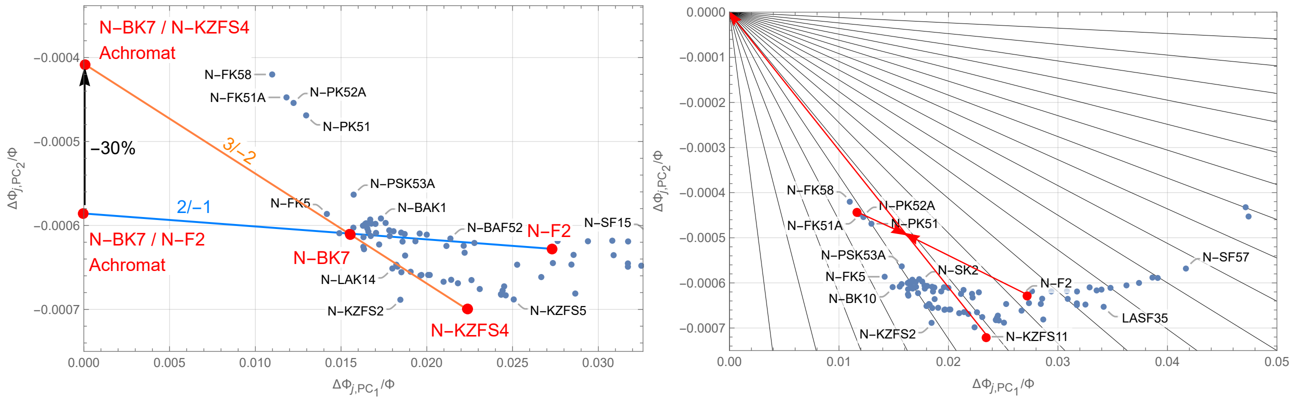

Figure 2. The refractive powers of achromatic pairs can be read from the ratio of distances along the connecting line. For N-BK7/N-F2, the powers are approximately 2:-1, and the residual secondary colour from the axis intercept is about 1/1700 of the focal length (left diagram). Changing to N-KZFS4 reduces the colour error by 30%, but requires higher individual powers of 3:-2. For apochromatic correction, three glasses can be combined to reach the origin of the diagram by inter- and extrapolation (right diagram). For N-FK51A, N-F2 and N-KZFS11 as shown here, the ratio of refractive powers is 2.5:1:-2.5. It would also be possible to use only two glasses on a line through the origin, like N-FK51A and N-KZFS2.

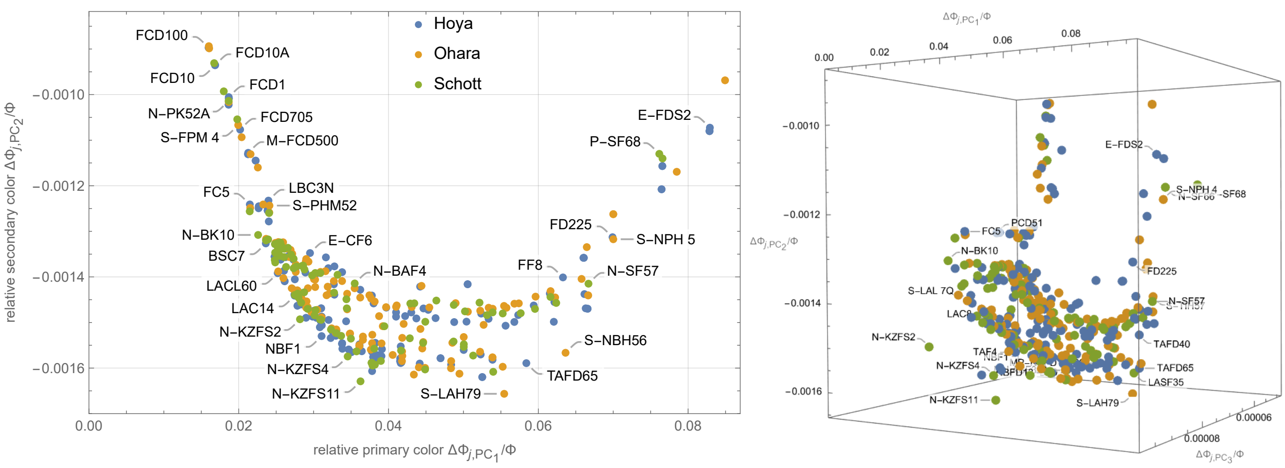

Figure 3. The first and second order dispersion diagram shows no significant differences between the Schott, Hoya and Ohara catalogs (left). Adding the third dimension reveals glasses with noticeably different third-order behaviour, like N-KZFS2, which can be useful for higher-order correction.