Framework for canonical quantum plasmonics for finite structure in three dimensions

Georgii Semin[1,⋆], *Hans-Rudolf* Jauslin[1], and *Stéphane* Guérin[1]

[1] *Laboratoire Interdisciplinaire Carnot de Bourgogne, UMR CNRS 6303, Université de Bourgogne, BP 47870, F-21078 Dijon, France*

Abstract. We provide the framework and the tools for canonical quantization of plasmon polaritons from metallic or dielectric finite nanostructures. They allow one to diagonalize the Hamiltonian and to exactly determine the quantized electromagnetic field and an imaginary Green's tensor identity satisfying the Sommerfeld radiation boundary conditions.

1 Introduction

We consider the microscopic model of the classical electromagnetic field interacting with a linear medium. One can derive such a system's Hamiltonian from the standard form's Lagrangian [1]. We consider the model in three dimensions, where we represent the medium as a finite non-magnetic dielectric that occupies the volume V, with the dielectric coefficient $\varepsilon(\mathbf{x}, \nu)$. The full Hamiltonian reads

$$H = H_{\rm em} + H_{\rm med} + H_C + H_S, \quad (1)$$

where

$$H_{\rm em} = \frac{1}{2}\int_{\mathbb{R}^3} {\rm d}^3 r \left[\frac{1}{\varepsilon_0}\mathbf{\Pi}_A^2(\mathbf{r}) - c^2\varepsilon_0 \mathbf{A}(\mathbf{r})\cdot\Delta\mathbf{A}(\mathbf{r})\right] \quad (2)$$

is the free Hamiltonian of the electromagnetic field expressed in terms of the vector potential $\mathbf{A}$ of the field and its conjugate momentum $\mathbf{\Pi}_A$,

$$H_{\rm med} = \frac{1}{2}\int_0^\infty {\rm d}\nu \int_V {\rm d}^3 r \left[\mathbf{\Pi}_X^2 + \nu^2\mathbf{X}^2\right] \quad (3)$$

is the free Hamiltonian of the medium expressed in terms of the position field $\mathbf{X}$ of the oscillators and momentum field $\mathbf{\Pi}_X$ with frequency ν and position r,

$$H_C = \frac{1}{\varepsilon_0}\int {\rm d}^3 r\, \mathbf{\Pi}_A \cdot \mathcal{P}^\perp \int_0^\infty d\nu\, \alpha\mathbf{X}. \quad (4)$$

is the cross-interaction term between the electromagnetic field and the medium which includes the coupling coefficient $\alpha(\mathbf{x}, \nu)$, and

$$H_S = \frac{1}{2\varepsilon_0}\int {\rm d}^3 r \left[\int_0^\infty d\nu\, \alpha\mathbf{X}\right]^2. \quad (5)$$

is the self-interaction term.The dielectric coefficient $\varepsilon(\mathbf{x}, \nu)$ obeys the Kramers-Kronig relation. We consider the structure with the constant dielectric coefficient in each volume $V_i \subset V$.

⋆e-mail: Georgii_Semin@etu.u-bourgogne.fr

2 Diagonalization and quantization

In this work, we find the creation-annihilation operators for the electromagnetic field in the presence of the finite dielectric using the Fano-diagonalization method [2]. To find the diagonal form of the given Hamiltonian, we consider continuous spectra of the electromagnetic field and the medium. These sets generate two types of creation and annihilation operators that obey the canonical commutation relations. The quantized Hamiltonian reads [3]

$$\hat{H} = \frac{1}{2}\int d\kappa\, \hbar\omega_\kappa \hat{C}_\kappa^{e\dagger}\hat{C}_\kappa^e + \frac{1}{2}\int d\mu\, \hbar\nu_\mu \hat{C}_\mu^{m\dagger}\hat{C}_\mu^m. \quad (6)$$

where the operator $\hat{D}_\kappa$ with $\kappa = (\omega_\kappa, d_\kappa)$ is the annihilation operator of the field with the frequency ω_κ and degeneracy indices d_κ and the operator $\hat{C}_\mu$ with $\mu = (\nu_\mu, x_\mu, j_\mu)$ is the annihilation operator of the medium oscillator with the frequency ν_μ at the position x_μ and component index j_μ. They satisfy the canonical commutation relations.

3 Results

3.1 Electromagnetic field

The operator of the electromagnetic field is defined as

$$\hat{\mathbf{E}}(\mathbf{r}) = \hat{\mathbf{E}}^e(\mathbf{r}) + \hat{\mathbf{E}}^m(\mathbf{r}), \quad (7)$$

where

$$\hat{\mathbf{E}}^e(\mathbf{r}) = -\int d\kappa \sqrt{\frac{\hbar}{2\varepsilon_0\omega_\kappa}}\left[\mathbf{e}_\kappa(\mathbf{r})\hat{C}_\kappa^e + \text{H.c.}\right], \quad (8)$$

$$\hat{\mathbf{E}}^m(\mathbf{r}) = -\int d\mu \sqrt{\frac{\hbar}{2\varepsilon_0\nu_\mu}}\left[\mathbf{m}_\mu(\mathbf{r})\hat{C}_\mu^m + \text{H.c.}\right]. \quad (9)$$

We call $\mathbf{e}_\kappa(\mathbf{r})$ and $\mathbf{m}_\mu(\mathbf{r})$ as field coefficients. They can be found through the following integral equations

$$\mathbf{e}_\kappa(\mathbf{r}) = \omega_\kappa \mathbf{\Phi}_\kappa(\mathbf{r}) + \frac{\omega_\kappa^2}{c^2}\int_V \mathrm{d}^3x(\varepsilon(\mathbf{x},\omega_\kappa)-1)\bar{\bar{G}}_0(\mathbf{r},\mathbf{x},\omega_\kappa)\mathbf{e}_\kappa(\mathbf{x}), \quad (10)$$

$$\mathbf{m}_\mu(\mathbf{r}) = -\tilde{\alpha}(\nu_\mu,\mathbf{x}_\mu)\frac{\nu^2}{c^2}\bar{\bar{G}}_0(\mathbf{r},\mathbf{x}_\mu,\nu_\mu)\mathbf{n}_{j_\mu} + \frac{\nu_\mu^2}{c^2}\int_V \mathrm{d}^3z(\varepsilon(\nu_\mu,\mathbf{z})-1)\bar{\bar{G}}_0(\mathbf{r},\mathbf{z},\nu_\mu)\mathbf{m}_\mu(\mathbf{z}) \quad (11)$$

that follow from the diagonalization of the Hamiltonian. Here $\mathbf{\Phi}_\kappa(\mathbf{r})$ is a eigenfunction of the operator $\nabla\times\nabla\times -\omega_\kappa^2/c^2$, $\bar{\bar{G}}_0(\mathbf{r},\mathbf{x},\omega_\kappa)$ is the Green tensor of the free space, $\mathbf{n}_{j_\mu}$ is a unit vector that represents an axis of the chosen coordinate system. The connection between $\mathbf{m}_\mu(\mathbf{r})$ and the Green tensor of the inhomogeneous space is as follows

$$-\tilde{\alpha}(\nu_\mu,\mathbf{x}_\mu)\frac{\nu_\mu^2}{c^2}\bar{\bar{G}}(\mathbf{r},\mathbf{x}_\mu,\nu_\mu) = \sum_{j_\mu}\mathbf{m}_\mu(\mathbf{r})\otimes\mathbf{n}_{j_\mu}. \quad (12)$$

3.2 Imaginary Green tensor identity

The integral equations of the field coefficient allows to obtain an important identity, which we call as imaginary Green tensor identity [4]:

$$\mathrm{Im}[\bar{\bar{G}}(\mathbf{x},\mathbf{y},\omega)] = \frac{\pi c}{2\omega^3}\sum_{d_\kappa}\mathbf{e}_{\omega_\kappa,d_\kappa}(\mathbf{x})\otimes\mathbf{e}_{\omega_\kappa,d_\kappa}(\mathbf{y}) + \int \mathrm{d}^3z\,\bar{\bar{G}}(\mathbf{x},\mathbf{z},\omega)\bar{\bar{G}}(\mathbf{z},\mathbf{y},\omega). \quad (13)$$

The imaginary Green tensor identity allows to express the spontaneous emission rate and the local density of freedom in terms of the field coefficients.

References

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