Investigation of random laser effect in structural disordered aperiodic multilayer

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Abstract. The gap states of a random Fibonacci-on-average multilayer exhibit strong localization and high Q-factors, indicating potential suitability for random lasing applications. This study investigates the gain threshold behavior of random lasing within the visible spectrum. Using the scattering matrix method with complex refractive indices, our simulations examine the relationship between the gain threshold, structural disorder, and the number of layers. The results contribute to the understanding of random lasing behavior of this type of quasi-crystal.

1 Introduction

In recent years, following Yablonovitch's discovery of photonic band-gap (BG) materials and the Shechtman founding of physical structures with similar properties but non-crystallographical symmetry, interest in photonic crystals (PhC) and *quasi-crystals* (QCs) has grown significantly. PhCs exhibit structural periodicity with translational symmetry, leading to sharp peaks (δ-like) in the spectrum. But they aren't the only one kind of structures that exhibit this such a property. In the early 1980s, it was discovered that aperiodic point sets also produce sharp diffraction peaks. QCs don't have translational symmetry, instead, they exhibit other kind of symmetries. Their spectrum can be classified througth the Fourier transform of the electronic density, according to *Lebesgue decomposition theorem*, in terms of three primitive kind of spectrum: pure-point, singular continuous (both are commonly conisdered QCs), and absolutely continuous. Focusing on one-dimensional (1d) structures, aperiodic configurations can be generated from binary sequences (e.g. Fibonacci, Rudin-Shapiro) by applying substitution rules, although not all of them have a spectra classifiable as QCs. In the late 1980s research on 1d quasi-periodic photonic structures started, Fibonacci QC was the first to be fabricated [1].

The hallmark of Fibonacci structures is the self-similarity of their singular continuous energy spectrum, where eigenvalues form a Cantor set with zero Lebesgue measure. The spatial extension of these eigenmodes is classified as multifractal, bridging the gap between extended and localized states [2]. It is well known that localized states can be formed in the BG of periodic multilayer (1d PhCs) by introducing structural disorder, a phenomenon known as Anderson localization in electronic systems. One might wonder if a similar phenomenon occurs in structures different from PhCs. Fibonacci multilayers offer a good framework to observe localization-delocalization transition in QCs.

Our analysis about localization properties involving passive ($n'' = 0$) and active ($n'' > 0$) multilayer are presented in section 3 and 4.

2 Model and computational method

Electromagnetic propagation through multilayered structures has been calculated using the scattering matrix method [3]. All calculations were carried out in terms of the dimensionless frequency ω_n. The reflection and transmission coefficient $r_j(\omega_n)$, and $t_j(\omega_n)$ of the j-th layer are obtained recursively as follows:

$$t_j = \frac{2n_j\phi_j}{(n_j + n_{j+1}) + (n_j - n_{j+1})r_{j+1}\phi_{j+1}(\omega_n)} \quad (1)$$

$$r_j = \frac{(n_j - n_{j+1}) + (n_j + n_{j+1})r_{j+1}\phi_{j+1}(\omega_n)}{2n_j} t_j \quad (2)$$

where $\phi_j = \exp(ik_0\omega_n n_j d_j)$ is the propagation phase inside the j-th layer, with $k_0 = 2\pi/\lambda_0$, $\lambda_0 = 1.55\mu$m, and refractive indexes n_j. To account for homogeneous active layers, we consider also a complex part n'' for the layer A, so that $n_A = n' - in''$. This negative complex part led to an amplification of the transmitted output.

In order to study average properties of an ensemble of random configurations at different level of disorder, we introduced a disorder strength parameter W, which varies the thickness of one type of layer A, according to $d = d_0(1 + gW)$, where g is a uniform random variable with $< g >= 0$ and $var(g) = 1/12$.

3 SPST violation in Fibonacci-on-average multilayers

One dimensional periodic-on-average multilayer behave similarly to the one-dimensional Anderson model, demon-

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strating *single parameter scaling* (SPS) and universality. Analogously, the same model can be applied to Fibonacci multilayers. According to the SPS theory (SPST), certain physical systems are characterized by single parameter. In electronic systems, SPST governs conductivity. In photonic systems, the fundamental parameter is the Lyapunov Exponent (LE) γ, which determines the typical decay length of $t(\lambda)$ along the structure [4]. In the limit of a long structure (longer than the localization length), the LE is defined as: $\gamma = -\lim_{L\to\infty} \frac{\partial|t}{\partial L}$. In a recent study [5], we analyzed SPST in Fibonacci-on-average multilayers and observed a threshold behavior: when the disorder is equal to or below a disorder threshold, SPST is violated; for values above this threshold, SPST is restored. The modes formed within the BG, similar to band-edge modes, are localized, and the number of such modes increases with disorder. The physical mechanism behind the violation of SPST is linked to the density of these modes. In a random ensemble of configurations with few or no modes inside the BG (low W), the average LE is influenced only by a small subset of the ensemble. When W increases, more modes appear within the BG, allowing the averaging process to incorporate a larger portion of the ensemble, thereby restoring SPST. Being localized, the photons are repeatedly reflected and transmitted in a finite spatial region, forming a virtual optical cavity [6]. These considerations direct research on such structures towards laser applications.

4 Random lasing threshold

It is well known that in disordered periodic structures fabricated with active materials, the localized modes formed within the band-gap are well-suited for random lasing applications. Specifically, when a minimum level of gain is achieved n_t'', these modes begin to lase [7]. With sufficient gain, the wave can overcome losses due to backscattering and propagate through the system with increased intensity [8]. In this study, we extended the analysis to Fibonacci multilayers. As shown in Figure 1, the average n_t'' for lasing modes increases with W and decreases as the total number of layers N_l. In Figure 2, where we can see that for low level disorder, some random samples even don't exhibit modes in the BG. For some random d set, W is not sufficient to allow the formation of modes.Considering a fixed amplification, the total energy is distributed among various resonant modes affecting n_t'', more modes need more gain. Additionally, as N_l increases, the traveling photons interact with more active layers, thus lowering the minimum gain needed to achieve lasing. However, there exists a level of disorder where only a few modes (or even just one) form, resulting in a minimum n_t''.

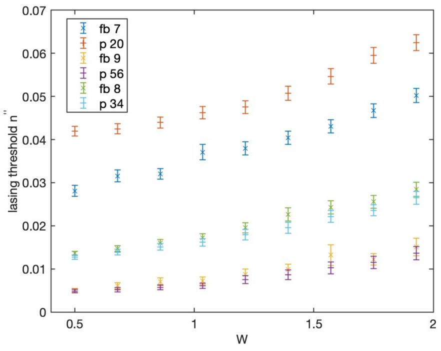

Figure 1. Gain threshold for different structures. Fibonacci's multilayers *"fb"* of different order are confronted with the periodic *"p"* multilayer with about the same number of layers.

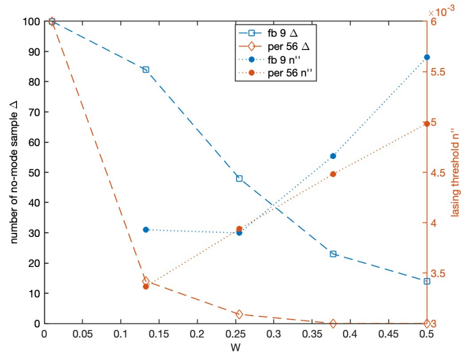

Figure 2. Low disorder details of the lasing threshold of periodic and fibonacci on average structures. The left y-axis represents the number of the samples for fixed disorder that don't exhibits modes in the BG.

Conclusion

Fibonacci-on-average passive multilayers exhibit peculiar spectral and localization properties that are worth studying to assess the suitability of the band-gap modes of active multilayers for random lasing applications. Our analysis, focused on a transmission coefficient-related quantities (n_t'' and the number of modes), suggests that they can achieve threshold behavior comparable with periodic-on-average modes, encouraging further investigations.

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