Closing the debate on the transverse orbital angular momentum of spatiotemporal optical vortices

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Abstract. We present a theory of the transverse orbital angular momentum (OAM) of spatiotemporal wave packets that explains the different values of the transverse OAM of spatiotemporal optical vortices (STOVs) provided by several authors as belonging to different canonical STOVs. The theory also rules out inaccurate values contributed by other authors, closing the debate on this issue.

1 Introduction

Spatiotemporal optical vortices (STOVs) are rapidly gaining attention among the diverse forms of structured light. Despite the enormous progresses in the generation and applications of STOVs, fundamental aspects of their nature remain obscure. We show here that the controversy on the transverse orbital angular momentum (OAM) of STOVs originated from different choices of ideal or "canonical" STOV in different works. References [1, 2] consider STOVs with elliptical symmetry in three-dimensional space (3D for short) at an instant of time, while Ref. [3] considers STOVs with elliptical symmetry at a cross section and time t, or in space-time (ST for short). As seen in Figs. 1 and 2, 3D elliptical STOVs at a time t [1(a)] are no longer elliptical when viewed in ST at a cross section z [1(b)], and the elliptical STOVs in ST at a cross section z [2(b)] are not elliptical when viewed in 3D at a time t [2(a)]. These are therefore two distinct families of STOVs whose transverse OAM is indeed different.

We have developed a dual theory of the transverse OAM of arbitrary spatiotemporal wave packets in which they can be described in space flying in time, as usual in mechanics, or at a transversal plane and time propagating along z, as more usual in optics. When applied to both families of STOVs, the values provided in [1, 2] for elliptical STOVs in 3D, and in [3] for elliptical STOVs in ST, are all reproduced from both perspectives and are all correct. In contrast, the authors of Ref. [4, 5] believe that elliptical STOVs in ST —those they can generate in their experiments—, are also elliptical in 3D, erroneously attributing to the elliptical STOVs in ST the OAM properties of elliptical STOVs in 3D.

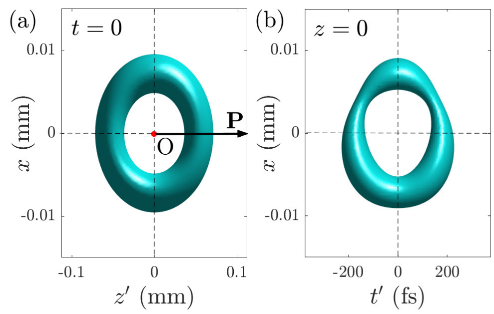

Figure 1. For the STOV in Eq. (1) with $\omega_0 = 2.8$ rad/fs, $l = 1$, $w_0 = 0.01$ mm, and $z_0 = 0.075$ mm (ellipticity $\gamma = z_0/w_0 = 7.5$), iso-energy-density surfaces (80% of the maximum value) (a) in (x, y, z') at $t = 0$ and (b) in (x, y, t') at the plane $z = 0$ where the STOV is placed at $t = 0$,). The y-axis is outward from the screen.

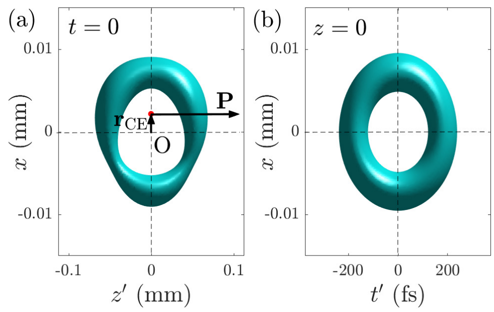

Figure 2. For the STOV in Eq. (4) with $\omega_0 = 2.8$ rad/fs, $l = 1$, $w_0 = 0.01$ mm, and $t_0 = 250$ fs (ellipticity $\gamma = ct_0/w_0 = 7.5$), iso-energy-density surfaces (80% of the maximum value) (b) in (x, y, t') at $z = 0$ and (a) in (x, y, z') at the time of arrival $t = 0$ at $z = 0$.

2 Transverse OAM of elliptical STOVs in space

Specifically, for an elliptical STOV in 3D propagating along the z direction at the velocity of light c, with line

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phase singularity of topological charge l along the y direction, such as

$$\psi(x,y,z') = e^{-\frac{y^2}{w_0^2}} e^{-\frac{x^2}{w_0^2}} e^{-\frac{z'^2}{z_0^2}} \left(\frac{z'}{z_0} + i\,\mathrm{sign}(l)\frac{x}{w_0}\right)^{|l|}, \quad (1)$$

($z' = z - ct$) the transverse OAM about the center of symmetry of the energy distribution in 3D, and the extrinsic and intrinsic parts, when expressed per unit energy W, are found to be

$$\frac{J_y}{W} = \frac{l}{2\omega_0}\gamma, \quad \frac{J_y^{(e)}}{W} = 0, \quad \frac{J_y^{(i)}}{W} = \frac{l}{2\omega_0}\gamma, \quad (2)$$

where ω_0 is the carrier frequency and $\gamma = z_0/x_0$ is the STOV ellipticity. These are the values erroneously attributed in [4] to the elliptical STOVs in ST. Our formulation trivially allows us to shift the definition of the center of the STOV from the center of the energy to the enter of photon wave function at $x = cl/2\gamma\omega_0$, to recover the results

$$\frac{J_y}{W} = \frac{l}{2\omega_0}\gamma, \frac{J_y^{(e)\prime}}{W} = -\frac{l}{2\omega_0}\frac{1}{\gamma}, \frac{J_y^{(i)\prime}}{W} = \frac{l}{2\omega_0}\left(\gamma + \frac{1}{\gamma}\right) \quad (3)$$

in Refs. [1, 2]. Figure 1(a) shows an example of elliptical STOV in space at an instant of time. Adequate temporal evolution formulas allows us to visualize the same STOV at a transversal plane and in time in Fig. 1(b), where it is evident that elliptical STOVs in 3D are no longer elliptical in ST.

3 Transverse OAM of elliptical STOVs in space-time

In all experiments, however, STOVs are not shaped in a volume, but created to purportedly have elliptical symmetry at a plane and in time, e. g., the focal plane of a lens [4, 6]. An example is

$$\psi(x,y,t') = e^{-\frac{y^2}{w_0^2}} e^{-\frac{x^2}{w_0^2}} e^{-\frac{t'^2}{t_0^2}} \left(\frac{t'}{t_0} - i\,\mathrm{sign}(l)\frac{x}{w_0}\right)^{|l|}, \quad (4)$$

($t' = t - z/c$). For these STOVs, our theory yields

$$\frac{J_y}{W} = 0, \quad \frac{J_y^{(e)}}{W} = -\frac{l}{2\omega_0}\gamma, \quad \frac{J_y^{(i)}}{W} = \frac{l}{2\omega_0}\gamma, \quad (5)$$

where $\gamma = ct_0/x_0$ is the STOV ellipticity, for the transverse OAM about the center of symmetry of the energy distribution in ST, and the extrinsic and intrinsic parts. These are the values reported in [3]. The fact that these STOVs are different from those in the previous section is evidenced in Fig. 2. The elliptical symmetry of the energy density in time at a transversal plane is illustrated in Fig. 2(b). Adequate propagation formulas in z, allow us to visualize in Fig. 2(a) the non-elliptical structure of same STOV in 3D space at the time of arrival at the plane z. The existence of the extrinsic OAM, which is hidden in (b), is evident in (b) as the product $\mathbf{r}_{\mathrm{CE}} \times \mathbf{P}$, where $\mathbf{r}_{\mathrm{CE}}$ is the center of the energy.

4 Conclusion

All results reported in [1, 2] and [3] for different STOVs are concluded to be correct. We argue, however, that STOVs with elliptical symmetry in ST, and the description of general wave packets in ST propagating along z, are closer to the STOVs generated in experiments and to the description and experiments with spatiotemporal wave packets in optics. Pulse and beam shaping techniques are usually aimed at producing a wave packet with desired characteristics at a transversal section and in time, as elliptical, or purportedly elliptical STOVs. Characterisation techniques are designed to retrieve optical fields with spatial resolution at a transversal section and in time. It is no coincidence that these optical fields are referred to as spatiotemporal fields. This is why we believe that Eqs. (5) should be taken as the reference for canonical STOVs, and the theory in [3] should be used when evaluating the transverse OAM of general spatiotemporal fields.

Of course, the transverse OAM and its intrinsic part depend on the choice of the transverse axis, but in the end all this discussion on the transverse OAM of STOVs about its center of symmetry has led to firmly establish the values of the intrinsic transverse OAM of the different types of canonical STOVs, also with different choices of the STOV center, and in the way, to reveal the innermost structure of these wave objects.

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