EPJ Web of Conferences **309**, 11011 (2024)
EOSAM 2024
https://doi.org/10.1051/epjconf/202430911011

Structured light and ultracold atoms in a driven optical cavity

Grant W. Henderson[1,★], *Gordon R. M.* Robb[1], *Gian-Luca* Oppo[1], and *Alison M.* Yao[1]

[1] *SUPA & Department of Physics, University of Strathclyde, Glasgow, Scotland G4 0NG, United Kingdom*

Abstract. We consider a far-red-detuned optical cavity, driven by a pump, which contains an ultracold atomic medium. Using coupled partial differential equations which describe the evolution of the atomic and optical fields, we demonstrate that our model leads to novel self-structuring, led by the optical field through the dipole force, within the ultracold atomic medium. Introducing OAM to the optical pump, we demonstrate that these structures may be made to rotate, forming atomic fields analogous to persistent phase currents.

1 Introduction

Atomtronics considers the possibility of creating structures of ultracold atoms that behave in an equivalent manner to that of an electric current. One such structure is a persistent current: a sustained, toroidal flow around a ring-like intensity distribution opens the possibility to form structures analogous to those used in superconducting quantum interference devices (SQUIDs) [1].

To realise useful ultracold atomic structures in Bose-Einstein condensates (BECs), precision control of their behaviour is required, typically achieved using static magnetic or optical traps acting as an ‘atomic guide’. However, an alternative approach is possible: by continuously evolving the atomic medium in the presence of a coupled, *structured* optical field [2], we observe the formation of structures, analogous to persistent phase currents. In this approach, the atomic dynamics remain under the control of the light’s characteristic features, opening the possibility of *dynamically* tailoring the atomic dynamics.

2 A Driven Optical Cavity

We consider the interactions of ultracold atomic, ψ, and structured optical, F, fields within a driven, by pump F_P, far-detuned optical cavity. A schematic of the general system is given in Fig. 1. The high reflectivity (with mirror transmittivity T) unidirectional ring cavity of length $\mathcal{L}$ contains a ‘pancake’-shaped BEC of length L, where $L \ll \mathcal{L}$. We consider pump wavelengths far detuned from the atomic transition such that absorption can be neglected, leading to the dipole interaction inducing a focusing/defocusing nonlinearity with ‘light/dark-seeking’ atoms attracted to intensity maxima/minima, dependent on the nature of the far-detuning [3].

We describe the interactions that occur between the atoms and light using coupled nonlinear partial differential equations. Building on [3, 4], we consider an atomic medium with no group velocity, and adiabatically eliminate any atomic excited states before performing a mean-field transformation upon the optical field’s evolution. This gives the scaled coupled equations

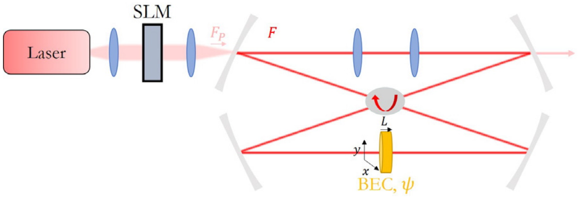

Figure 1. General schematic of the considered system. A laser beam forms a pump, F_P, that is incident upon a spatial light modulator (SLM) to add a helical phase to the beam, before being directed into an optical cavity, consisting of four highly reflective mirrors and telescope. It contains an pancake-like ultracold atomic medium (BEC), ψ, which the optical field, F, interacts with once per cavity round trip.

$$\begin{aligned}\partial_\tau F &= -(1+i\theta)F + i\alpha_F\nabla_\perp^2 F \\ &\quad - i\beta_F\left(s|\psi|^2 - \beta_{\rm dd}|\psi|^4\right)F + F_{\rm P},\end{aligned} \tag{1}$$

$$\begin{aligned}\partial_\tau\psi = \frac{\alpha_\psi}{\kappa}\Big[&i\nabla_\perp^2\psi - i\Big(s|F|^2 - 2\beta_{\rm dd}|F|^2|\psi|^2 \\ &+ \beta_{\rm col}|\psi|^2 - iL_3|\psi|^4\Big)\psi\Big].\end{aligned} \tag{2}$$

Eq. (1) is an effective Lugiato-Lefever description of the dynamics of the optical field within the cavity [5]. Its terms represent various physical processes, including the cavity detuning (term in θ), diffraction (term in $\nabla_\perp^2$), and both dipole (term in s) and higher-order dipole-dipole (term in $\beta_{\rm dd}$) coupling to the atomic field.

Eq. (2) is an effective Gross-Pitaevskii description of the atomic dynamics. It terms represent further physical processes, including the atomic kinetic energy (term in $\nabla_\perp^2$), both dipole (term in s) and dipole-dipole (term in $\beta_{\rm dd}$)

[★] e-mail: grant.henderson@strath.ac.uk coupling to the optical field, interatomic scattering (term in β_{col}, and three-body atomic loss (term in L_3).

The fields are coupled through dipole forces, whose nature depends on the parameter s, the sign of the atom-light detuning. We select $s = -1$, corresponding to far-red-detuned fields, i.e. 'light-seeking' atoms. They initially take the form of a Thomas-Fermi distribution, given by

$$\psi(r) = 1 - r^2 / \left(2w_\psi^2\right), \tag{3}$$

where r is a radial domain, w_ψ represents the beam waist of the atomic distribution relative to the transverse scaling size, and we note that we subsequently rescale the maximum amplitude of the profile to A_ψ.

3 Atom-Light Structure Formation

We initially consider an optical pump that is solely intensity structured. We take the intensity profile of a Laguerre-Gaussian mode at the beam waist, defined as [6]

$$F_{\text{P}} = A_{\text{P}} \left[\left(\text{LG}_0^m \right) / \left(\text{Max.} \left(\text{LG}_0^m \right) \right) \right], \tag{4}$$

$$\text{where} \quad \text{LG}_0^m(r, \varphi) = r^{|m|} e^{-r^2/\left(2w_F^2\right)}, \tag{5}$$

A_{P} represents the maximum amplitude of the pump, m is the OAM index of the beam, and w_F represents the beam waist of the mode relative to the transverse scaling size.

If the optical field leads the dipole interactions within the system ($A_{\text{F}} \gtrapprox 2A_\psi$), we observe the short-term formation of a ring-like atomic distribution that matches the optical, before mutual ring fragmentation occurs into coupled, coincident atom-light structures. Such structures closely match the optical-only equivalent Turing patterns in a self-focusing Kerr cavity [7]. We show them, here also in the atomic field, in Fig. 2 for the case of $m = 2$, which gives the evolution of six mutual structures in time (τ) around the coupled ring of maximum intensity.

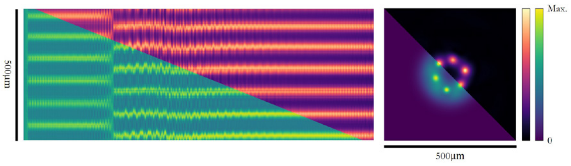

Figure 2. Coupled atom-light pattern formation. Left panel: evolution around a ring of maximum intensity in atomic (upper) and optical (lower) fields between $\tau = 0 \rightarrow 5000$. Right panel: $\tau = 5000$ amplitude distributions for atomic (upper) and optical (lower) fields. Pump as Eqs. (4)-(5) with $m = 2$, $w_F = 80\mu$m.

4 Rotating Atom-Light Structures

We now consider introducing structured phase to the optical pump. Its profile, formerly Eq. (5), is given by

$$\text{LG}_0^m(r, \varphi) = r^{|m|} e^{-r^2/\left(2w_F^2\right)} e^{im\varphi}, \tag{6}$$

where the additional term representing orbital angular momentum (OAM) gives the pump a helical phase front [2].

The initial formation procedure described without OAM remains: the atoms are attracted to the light's intensity ring, before both fields fragment into several peaks. In the presence of OAM, these peaks *rotate*, processing azimuthally around the ring: a short-term atomic *wavepacket current*. Over time, we observe that the rotational motion seeds a process of peak merging, which eventually leads to the creation of a uniform atomic ring with rotating azimuthal phase: an atomic *phase current*, exemplified in Fig. 3 for the case of an atomic winding number of five. The atomic current remains controlled by the dipole force throughout, opening the possibility of *dynamic variation* by changing the properties of the pump driving the cavity.

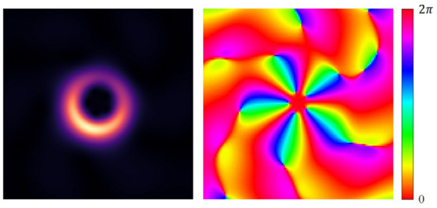

Figure 3. Atomic phase current formation, showing atomic amplitude (left) and phase (right) distributions at $\tau = 5000$. Pump as Eqs. (4) and (6) with $m = 2$, $w_F = 80\mu$m, scales as Fig. 2.

5 Conclusions

Coupling atoms to structured light in a driven optical cavity, we presented a novel model for describing the dipole-led interactions between the fields. We outlined that our model leads to co-incident structure formation for far-red-detuned fields and, in the presence of OAM, a novel approach for generating ultracold atomic phase currents. These currents may be made to *dynamically* vary, opening application to atomtronic schemes for enhanced control, and increased capabilities, of atomic currents [1].

References

[1] L. Amico et al., Rev. Mod. Phys. **94**, 041001 (2022)
[2] L. Allen, M.W. Beijersbergen, R.J.C. Spreeuw, J.P. Woerdman, Phys. Rev. A **45**, 8185 (1992)
[3] M. Saffman, D.V. Skryabin, *Coupled Propagation of Light and Matter Waves: Solitons and Transverse Instabilities* (Springer, Berlin, Heidelberg, 2001), pp. 433–447
[4] G.W. Henderson, G.R.M. Robb, G.L. Oppo, A.M. Yao, Phys. Rev. Lett. **129**, 073902 (2022)
[5] L.A. Lugiato, R. Lefever, Phys. Rev. Lett. **58**, 2209 (1987)
[6] S.M. Barnett, R. Zambrini, *Orbital Angular Momentum of Light* (Springer, New York, 2007), pp. 277–311
[7] A.M. Yao, C.J. Gibson, G.L. Oppo, Opt. Express **27**, 31273 (2019)