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Dilute nuclear matter with light clusters and in-medium effects: towards a unified dynamical framework

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Abstract. Understanding the dynamics of dilute nuclear matter is of crucial importance in several contexts, ranging from nuclear fragmentation to supernova collapse and gravitational-wave signal emission. However, within a unified dynamical framework, describing the concurrent appearance of light clusters, emerging from few-nucleon correlations, and heavier fragments formed due to large-scale correlations related to liquid-gas phase instabilities, remains a significant challenge. Within a linearized Vlasov dynamics, we show that light clusters, and in-medium effects in their propagation, have a strong influence on the growth and characteristics of the unstable modes that prelude the fragmentation of the system. These findings might pave the way for novel avenues in the study of dilute composite matter, envisioning intriguing consequences for heavy-ion collisions and in the broader astrophysical context.

1 Introduction

Heavy-ion reactions are a powerful tool for probing transient states of nuclear matter (NM) under conditions far from saturation [1]. These reactions provide crucial insights into the nuclear equation of state (EOS), which is closely linked to astrophysical phenomena [2, 3]. Specifically, during the expansion phase following the initial compression in central heavy-ion collisions (HICs) at Fermi/intermediate energies (beam energies $E/A \approx$ 30–300 MeV/nucleon), regions of low density and moderate temperatures in the NM phase diagram can be explored [4]. In these conditions, large-scale correlations may arise from volume (spinodal) instabilities associated with the liquid-gas phase transition, contributing to multifragmentation processes [5]. Moreover, at low densities, few-body correlations driven by short-range nucleon-nucleon interactions become significant, leading to the formation of light clusters such as deuterons or α particles [6].

Phenomenological models using energy density functionals (EDFs) provide a convenient framework for describing these processes from a thermodynamic perspective, incorporating clusters as explicit degrees of freedom (DOF) and characterizing dilute NM as a mixture of nucleons and nuclei [6, 7]. On the other hand, transport models are the preferred choice for investigating out-of-equilibrium processes in dynamical studies. Nevertheless, the formulation of a transport model accounting on equal footing for both light clusters emerging from few-body correlations and volume instabilities leading to heavier fragments remains a complex challenge for many-body theories [8–11].

To complicate matters, light clusters are expected to dissolve at high densities (beyond the so-called Mott density) due to in-medium effects, which are primarily driven by Pauli blocking [12]. Beyond this density, bound clusters can exist only if their momentum exceeds a critical value, the Mott momentum [12]. Moreover, few-body correlations might persist beyond the Mott density as continuum correlations, which are often not included in EDF-based models due to the complexity of modeling NM at high densities [13].

In this context, we present a novel approach to solve the nuclear dynamics in the heterogeneous sub-saturation regime, within a linear response analysis. Our goal is to develop a unified theoretical framework that accounts for various mechanisms of fragment formation in HICs, considering the interplay between nucleonic and light cluster DOF and including in-medium effects. Specifically, we investigate how light clusters, which mainly arise in the compression phase, influence the development of spinodal instabilities, occurring in the expansion stage and leading to the fragmentation dynamics, of a composite system initialized at low baryon density ρ_b and at a given temperature T. The paper is organized as follows: Section 2 summarizes the theoretical formalism initially proposed in Ref. [14]. Section 3 presents the main results obtained. Finally, Section 4 outlines the conclusions and future directions.

2 Theoretical formalism

The theoretical formalism was developed in Ref. [14] and we refer readers to that work for more details. Here, we provide only a summary of the main formulas and ideas.

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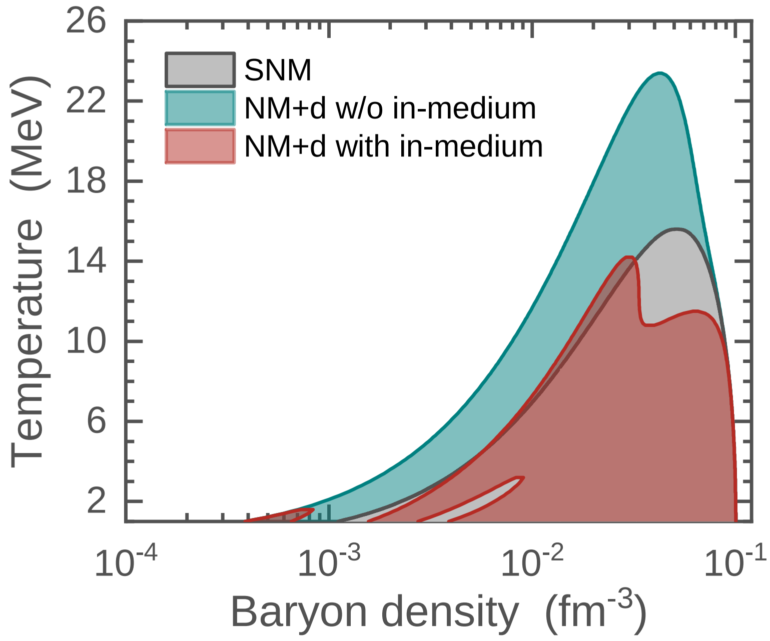

Figure 1. Spinodal region in the (ρ_b, T) plane for three scenarios: 1) pure nucleonic matter (SNM, black); 2) nuclear matter with deuterons, including in-medium effects along the dynamics (red); 3) nuclear matter with deuterons, neglecting in-medium effects along the dynamics ($\Phi_\lambda^{dj} = 0$, cyan).

Let us consider then a system of nucleons—neutrons (n) and protons (p)—and one light cluster species (d), in thermodynamic equilibrium at temperature T. The density ρ_j for each species is given by

$$\rho_j = g_j \int_{\Lambda_j} \frac{d\mathbf{p}}{(2\pi\hbar)^3} f_j, \qquad j = n, p, d \tag{1}$$

where g_j is the spin-degeneracy, f_j is the phase-space distribution function and Λ_j is the Mott momentum, which might generally depend on densities and temperature, introduced for clusters ($\Lambda_q = 0$ for $q = n, p$) to account for in-medium effects [10, 12, 14]. The total baryon density would be then given by $\rho_b = \sum_j \rho_j A_j$, as expressed in terms of the densities ρ_j and mass numbers A_j of the three constituents ($j = n, p, d$) considered.

By applying a small perturbation δf_j to the initial distribution functions f_j, within a linear response framework, the collisionless (Vlasov) limit of the Boltzmann equation becomes:

$$\partial_t(\delta f_j) + \nabla_{\mathbf{r}}(\delta f_j) \cdot \nabla_{\mathbf{p}}\varepsilon_j - \nabla_{\mathbf{p}} f_j \cdot \nabla_{\mathbf{r}}(\delta\varepsilon_j) = 0, \tag{2}$$

The single-particle energy $\varepsilon_j = \frac{(2\pi\hbar)^3}{g_j}\frac{\delta\mathcal{E}}{\delta f_j(\mathbf{p})}$ (and correspondingly its variation $\delta\varepsilon_j$) is derived from the EDF $\mathcal{E}$, whose potential part $\mathcal{U}$ comes from a (momentum-independent) Skyrme-like effective interaction.

Notably, using Eq. (1), the fluctuation in density can be expressed as:

$$\delta\rho_j(\mathbf{r}, t) = g_j \int_{\Lambda_j} \frac{d\mathbf{p}}{(2\pi\hbar)^3}\delta f_j - \delta_{jd}\sum_l \Phi_\lambda^{dl}\delta\rho_l, \tag{3}$$

where δ_{jd} is the Kronecker function, while the second term on the right-hand side accounts for the variation in the local density of light nuclei due to changes of in-medium effects driven by the density-dependent cut-off. However, to better assess the role of in-medium effects, the opposite scenario will be also considered in the calculations, where the cut-off remains constant during the propagation of density fluctuations ($\Phi_\lambda^{dj} = 0$).

Equation (2) allows for plane-wave solutions where δf_j oscillates with a frequency ω and wave vector $\mathbf{k}$, represented as $\delta f_j \sim \sum_{\mathbf{k}} \delta f_j^{\mathbf{k}}\, e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$. Using the Landau procedure [15], this leads to the system:

$$\delta\rho_j = -\chi_j \sum_l \tilde{F}_{0\lambda}^{jl}\delta\rho_l - \delta_{jd}\sum_l \Phi_\lambda^{dl}\delta\rho_l, \tag{4}$$

where $\chi_j = \chi_j(\omega, \mathbf{k})$ is the Lindhard function with the cut-off and where $\tilde{F}_{0\lambda}^{jl}$ are properly modified Landau parameters [14].

3 Results

We present results for the simplest case of symmetric nuclear matter (SNM) with deuterons. In this context, we assume chemical equilibrium in the initial conditions as a baseline to incorporate in-medium effects and to determine the parameters for the deuteron Mott momentum density-temperature parameterization [14]. The chemical equilibrium condition reflects a balance between the chemical potentials of different species. Specifically, in the framework of the mean-field interaction and the density dependence of the cut-off considered here, the equilibrium condition simplifies to $\mu_d^* = \mu_n^* + \mu_p^* + B_d$, where μ_j^*, $j = n, p, d$ are the non-relativistic effective chemical potentials (not including the rest mass) and B_d is the deuteron binding energy in vacuum. This approach yields a density-dependent behavior for the deuteron mass fraction, $X_d = A_d\rho_d/\rho_b$, which

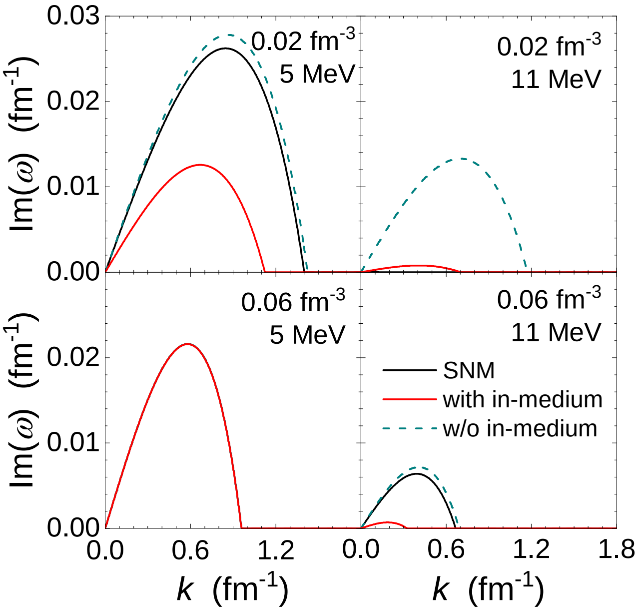

Figure 2. Growth rate of the instability, Im(ω), as a function of the wave number k, for the same cases as in Fig. 1, at various density and temperature values. Readapted from Ref. [14].

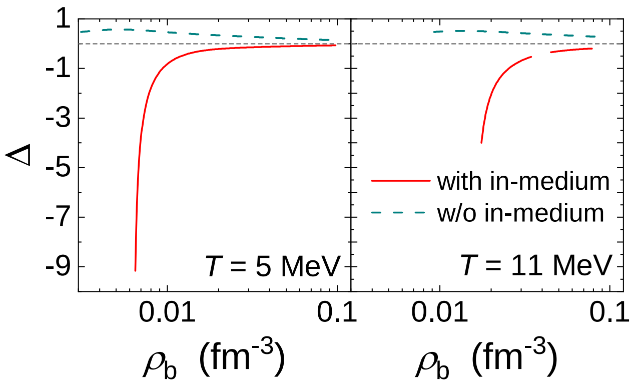

Figure 3. The relative ratio Δ (see text) as a function of the total baryon density ρ_b for nuclear matter with deuterons, neglecting (cyan) or including (red) in-medium effects in the dynamics, for three temperature values. Lines are drawn only for the density values lying inside the spinodal region. Readapted from Ref. [14].

aligns with the benchmark calculations from Ref. [6] in the relevant temperature range. However, chemical equilibrium is only an initial condition for our calculations, as it may not always be achieved during the expansion phase of a nuclear reaction. We can thus release this assumption in our calculations without affecting the overall conclusions, while keeping only the deuteron Mott momentum density-temperature parameterization.

The results are derived by solving the homogeneous system defined by Eq. (4), which yields the dispersion relation connecting frequency ω to wave number k. The onset of the spinodal region, where ω becomes imaginary and density fluctuations are amplified with time, leading to the disassembly of the system into fragments, is identified by solving the system for $\omega = 0$ [5, 16]. The main panel of Fig. 1 shows the spinodal region in the (ρ_b, T) plane, with the red area representing the case where the local density dependence of the cut-off is included in the dynamics. This is compared to the cyan area, where the density dependence of the cut-off is entirely neglected. Including light clusters as explicit DOF significantly influences the extent of the spinodal region. When in-medium effects are fully considered, the spinodal border of the composite system closely aligns with that of pure nucleonic matter (black line). On the other hand, neglecting in-medium effects ($\Phi_\lambda^{dj} = 0$) would expand the instability region due to the stronger attraction from the deuteron mean-field potential. Notably, an isolated instability region below 0.002 fm^{-3} and a meta-stable region at higher densities appear at low temperatures in the full calculations, echoing recent findings from Ref. [17].

Inside the spinodal region, a pure imaginary ω will be obtained, which quantifies the growth rate of the unstable modes. The latter is plotted as a function of the wave number k in Fig. 2, for the same cases as in Fig. 1, at various density and temperature values. One observes that, the growth rate exhibits a maximum, which means that the system favors the growth of the density fluctuations with a given k. However, the maximum growth rate is generally quenched and shifted to lower k-values by in-medium effects, which slow down the instability growth and eventually lead to the dominance of different fragmentation modes. Once again, the opposite scenario would occur if in-medium effects were neglected in the dynamics. Moreover, the impact of this effect would be strongly mitigated for densities beyond $\rho_0/3$, up to moderate temperatures.

A deeper insight into the direction of the unstable modes in the space of density fluctuations is provided by the $(\delta\rho_S/\delta\rho_d)$ ratio, where $\rho_S = \rho_n + \rho_p$ is the total isoscalar nucleonic density. In Fig. 3, the relative ratio $\Delta = (\delta\rho_S/\delta\rho_d)/(\rho_S/\rho_d)$ is plotted as a function of ρ_b inside the spinodal region for the two options regarding the in-medium effects considered so far. Positive or negative values indicate that nucleons and deuterons fluctuations move in or out of phase, respectively. As illustrated in the sketch shown in Fig. 4, when neglecting in-medium effects along the dynamics, light clusters move in phase with the nucleons, favoring the growth of instabilities and possibly cooperating to the formation of massive fragments. On the contrary, once in-medium effects are taken into account, deuterons move out-of phase with respect to nucleons, thus migrating towards lower density regions while the nucleon density fluctuations grow and fragments emerge. Local in-medium effects produce then a sort of “distillation” mechanism, which induce deuterons to be eventually separately emitted, possibly increasing their yield in MF-based simulations of HIC at intermediate energies, according to recent experimental evidences [18].

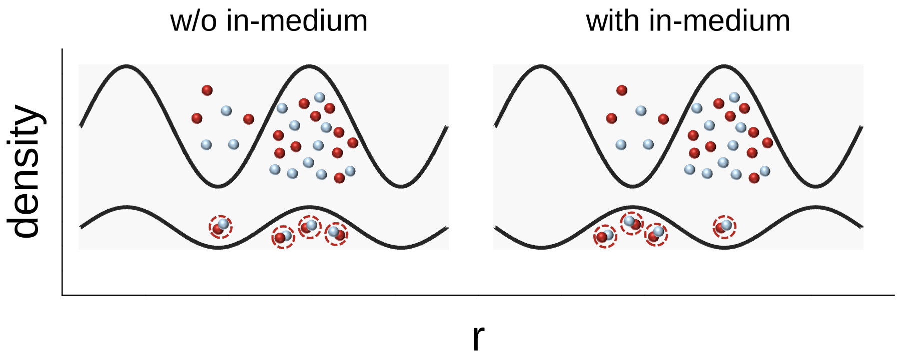

Figure 4. Spatial fluctuations of nucleon density (top) and deuteron (bottom) density, neglecting (left) or including (right) in-medium effects during the dynamics. The sketch illustrates the "distillation" mechanism of light clusters when in-medium effects are considered throughout the propagation.

4 Summary, outlooks and conclusions

In summary, using a linearized Vlasov approach, we have developed a novel framework to investigate the dynamics of dilute systems composed of nucleons and light clusters, while simultaneously accounting for in-medium Mott effects that lead to cluster dissolution.

Our results demonstrate that light clusters, particularly with in-medium effects, significantly influence the unstable modes driving the system disassembly in multifragmentation. Without in-medium effects, clusters move with nucleons, facilitating fragment formation. However, in-medium effects trigger a "distillation" mechanism, causing clusters to migrate to lower-density regions, which slows the growth of instabilities and alters the dominant fragmentation modes.

A complete understanding and unified description of the mechanisms behind fragment formation will require moving beyond the current quasi-analytical approach to fully numerical calculations. Nonetheless, our work opens a new avenue for studying out-of-equilibrium processes in HICs and in broader astrophysical contexts.

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