Very large neutron-nucleus scattering lengths

F. Miguel Marqués[1,*] and *Emeline* Oliveira[1]

[1]LPC Caen, ENSICAEN, CNRS/IN2P3, Université de Caen, Normandie Université, 14050 Caen, France

Abstract. The interaction of neutrons and nuclei at low energies may potentially lead to strongly resonant states, characterized by scattering lengths several orders of magnitude larger than the effective range of the interaction. In order to explore the scattering of neutrons off unstable nuclei, we describe a simple analytical formalism to characterize the scattering parameters of a neutron-nucleus system created following the fast removal of a few nucleons from a slightly heavier beam. The case of the n-^{17}B system is considered, and the implications of a potentially very large scattering length on the structure of ^{18,19}B discussed.

1 Low-energy scattering

Scattering experiments are a unique tool to probe subatomic structures since the discovery of the nucleus by Rutherford. In particular, two-particle scattering provides information about the interaction potential between both particles. If the scattering takes place through a potential of range R at a given momentum $k = \sqrt{2\mu E}$ (μ is the reduced mass of the system and E the relative energy), the partial waves that will contribute to the scattering are those with angular momentum $\ell \lesssim kR$.

At sufficiently low energies (some MeV for nucleons and light nuclei), the only significant contribution comes from $\ell = 0$, the s wave. The asymptotic wave function and the scattering cross-section can be then expressed as a function of the s-wave phase shift δ_0 as [1]:

$$\varphi_k(r) = \frac{\sin(kr + \delta_0)}{kr} \qquad (1)$$

$$\sigma(k) = 4\pi \frac{\sin^2(\delta_0)}{k^2} \qquad (2)$$

As an example, in the scattering of a particle off a hard sphere of radius R, the scattering wave function must vanish at $r = R$ and thus $\delta_0 = -kR$ and $\sigma(0) = 4\pi R^2$.

The phase shift behavior at low energies is usually expanded in even powers of k according to the effective-range approximation [1]:

$$k \cot\delta_0 = -\frac{1}{a_s} + \frac{r_e}{2} k^2 + O(k^4) \qquad (3)$$

The first term governs the strength of the interaction towards zero energy, with a parameter a_s known as the scattering length of the system so that $\sigma(0) = 4\pi a_s^2$. The second term describes the energy dependence of the phase shift, through a parameter r_e known as the effective range of the interaction.

In principle, the low-energy scattering of any pair of particles can be characterized by this (a_s, r_e) couple. Negative values of a_s correspond to unbound states of the system for attractive potentials, while positive values correspond to bound/unbound states for attractive/repulsive potentials [1]. The values of $|a_s|$ and r_e are associated respectively to the strength and range of the interaction, and for nucleons scattering off light nuclei both are typically of the order of several fm, and thus $|a_s|/r_e \sim 1$.

2 Neutron scattering off exotic nuclei

Low-energy scattering of nucleons off nuclei should be a natural probe of the nuclear properties of few-nucleon systems. However, in the scattering of protons the structure of the system is completely shadowed by the dominant proton-nucleus Coulomb repulsion, that requires the use of higher energies and more complex formalisms. The neutron becomes thus the natural nuclear probe. Even if neutral, neutrons produced in some nuclear reactions can be collimated and sent on stable targets, and their scattering measured at different energies and angles.

The scattering parameters (a_s, r_e) so obtained for neutron scattering off stable nuclei are all in the range of several fm. But in order to extend this technique to exotic, unstable nuclei, the technique must adopt a different perspective. Short-lived nuclei, as well as the neutron itself, cannot be used as targets, and therefore beam on target scattering experiments are no longer possible. The closest possible scenario is the creation of the neutron and nucleus of choice in the final state of a reaction with a range of (low) relative energies.

If we assume an initial state in which the neutron and the nucleus (of mass number A) are bound within a projectile of similar size (of mass number $B \gtrsim A$), with only a few extra nucleons, and we remove the latter using a high-energy fast removal reaction, we can use the sudden approximation and calculate the scattering amplitude of the

*e-mail: marques@lpccaen.in2p3.fr

final neutron-nucleus system as the overlap integral:

$$O(k) = \int_0^\infty \psi_i(r)\,\varphi_k^*(r)\,r^2 dr \tag{4}$$

$$\sigma(k) \propto k\,|O(k)|^2 \tag{5}$$

For a given initial state, the Schrödinger equation can be solved to determine the initial wave function of the neutron $\psi_i(r)$ and then the scattering amplitude (4) and cross-section (5) evaluated.

3 Large n-nucleus scattering lengths?

Starting with $A = 1$, the neutron-neutron scattering parameters $(a_s, r_e) = (-18.5, 2.8)$ fm are an exception within the known neutron-nucleus ones, all with scattering lengths of the order of the corresponding effective ranges of a few fm. When $|a_s|/r_e \gg 1$, for systems in which the strength of the interaction overwhelms its range, the system is considered to exhibit what is known as universal properties, i.e. properties that no longer depend on the details of the interaction potential [2]. Paradoxically, at the extreme case known as the unitary limit, $|a_s| \to \infty$, the properties of the system do not depend on the potential at all.

In this context, the question whether any neutron-nucleus system may possess a large scattering length, of tens or even hundreds of fm, is relevant. Not only the system would be universal, but a three-body system of that nucleus and two neutrons, built up from two large scattering lengths, would be a favorable candidate to exhibit a very exotic form of matter, Efimov trimers [2]. These systems have already been observed in atomic physics experiments, in which the scattering length of the interaction between atoms, generated by a magnetic field, can be tuned to values much larger than the interaction range.

In nuclear physics, however, we cannot tune the interactions, Nature did. But a simple exercise tells us that we should not exclude the possibility of finding very large scattering lengths. The scattering length of a particle scattering off a square-well potential of radius R and depth V_0, as a function of $x = R\sqrt{2\mu V_0}$, is given by:

$$a_s = R\left(1 - \frac{\tan x}{x}\right) \tag{6}$$

In Fig. 1 we have plotted the dimensionless quantity $|a_s|/R$ in logarithmic scale in order to show more explicitly how, for some very specific values of the potential leading to odd multiples of $\pi/2$, the scattering length can in fact diverge towards $\pm\infty$. Does such a neutron-nucleus system exist in Nature?

4 The n-^{17}B case

The elementary neutron-nucleon scattering lengths are of the order of -20 fm, and one could expect similar or higher values when adding up other nucleons to the system. However, the Pauli principle blocks the states already occupied by other neutrons and counters the pure s-wave attraction that the incoming neutron should feel. This delicate balance between n-N attraction and n-n Pauli repulsion leads

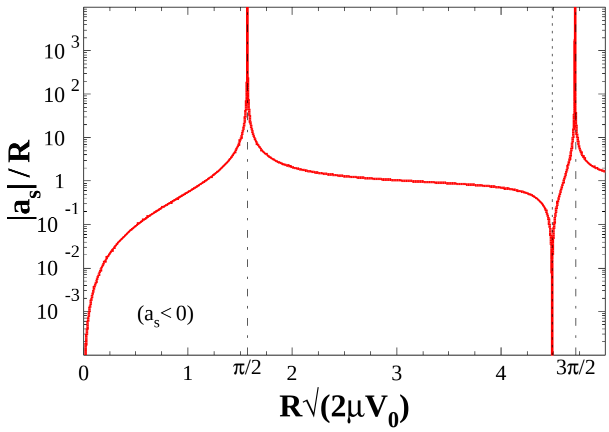

Figure 1. The absolute value of a_s for a particle scattering off a square-well potential of radius R and depth V_0 (in units of R) as a function of $x = R\sqrt{2\mu V_0}$. The scattering length a_s is negative for $x < \pi/2$ and changes sign after each crossing of the dashed/dotted lines, diverging towards $\pm\infty$ when x reaches odd multiples of $\pi/2$.

for $A > 1$ to neutron-nucleus scattering lengths of only a few fm (in fact the dominant $|a_s|/r_e \sim 1$ regime in Fig. 1), even oscillating between net attraction and repulsion [3].

In a search for resonances in the ^{18}B system, ^{17}B+n coincidences were measured following proton removal off a ^{19}C beam at 62 MeV/nucleon [4]. The observation of a sharp increase towards zero energy was attributed to a strongly resonant s-wave, although experimental resolution and acceptance limitations did not allow to determine a precise value of the associated scattering parameters. The effective range was not considered and, even if the data were compatible with a scattering length of -100 fm, only an upper limit $a_s < -50$ fm was proposed [4].

This upper limit opens the possibility of a nuclear system actually being in the very narrow divergence windows of Fig. 1. Moreover, ^{19}B is a well-known two-neutron halo nucleus, with a characteristic ^{17}B-n-n three-body structure [5]. If the n-^{17}B scattering length were large enough, of hundreds (or even thousands) of fm, ^{19}B states might exist in the form of Efimov trimers [6]. But would such a large scattering length be measurable?

5 Asymptotic wave functions

As we have seen in Eq. (4), in the case of exotic nuclei the neutron-nucleus low-energy spectrum will be the result of the scattering parameters of the system folded with the initial state of the reaction. A low-energy neutron ($\ell = 0$) is not subject to Coulomb or centrifugal repulsions and does not experience any potential barrier. It is thus reasonable to assume for the whole initial wave function its asymptotic part (in the region where the potential vanishes):

$$\psi_i(r) \approx e^{-\alpha r}/r \tag{7}$$

with $\alpha = \sqrt{2\mu_i S_n}$, μ_i the reduced mass of the neutron and the rest of the initial system (of mass number $B - 1$), and S_n the neutron separation energy in that system.

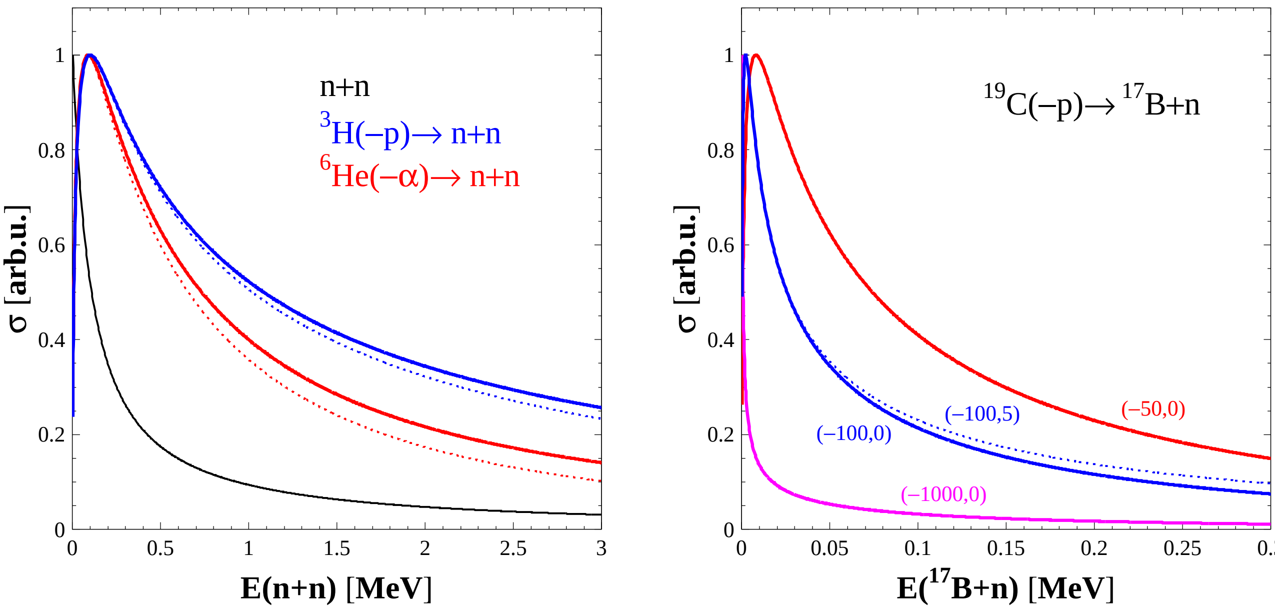

Figure 2. On the left, the *n*-*n* energy spectrum with $(a_s, r_e) = (-18.5, 2.8)$ fm for an hypothetical *n*+*n* scattering (black) using Eq. (2), and for fast removal of a proton from ^{3}H (blue) and of an α particle from ^{6}He (red) using Eq. (8). In the two latter cases, the dotted lines correspond to Eq. (12). On the right, the *n*-^{17}B energy spectrum for fast proton removal from ^{19}C using Eq. (8) for different couples (a_s, r_e) in fm. Note the factor 10 between the energy scales of both panels.

Using the also asymptotic scattering wave function (1) and the asymptotic initial one (7), the overlap integral of the scattering amplitude in Eq. (4) becomes:

$$\begin{aligned} O(k) &= \frac{1}{k}\int_0^\infty \sin(kr+\delta_0)\, e^{-\alpha r}\, dr \\ &= \left[\cos\delta_0 + \frac{\alpha}{k}\sin\delta_0\right]\frac{1}{\alpha^2+k^2} \end{aligned} \tag{8}$$

The energy spectrum depends on the initial state through α (S_n) and on the final state through δ_0 (a_s and r_e).

6 Square-well potential

We can consider the internal part of the initial neutron wave function within nucleus B analytically with a square-well potential of radius $R = 1.2\,(B-1)^{1/3}$ and a depth V_0 that binds the neutron at $-S_n$, a good approximation for the potential felt by a neutron with $\ell = 0$:

$$\psi_i(r) = \begin{cases} \dfrac{\sin(K_i r)}{r\,\sin(K_i R)} & , r < R \\ e^{-\alpha(r-R)}/r & , r > R \end{cases} \tag{9}$$

with $K_i = \sqrt{2\mu_i(V_0 - S_n)}$. Using this wave function in the overlap integral of Eq. (4), we have now two contributions:

$$\begin{aligned} O(k) &= \frac{R}{2k\,\sin x_i}\times \\ &\left[\frac{\sin(x-x_i-\delta_0)+\sin\delta_0}{x-x_i} - \frac{\sin(x+x_i+\delta_0)-\sin\delta_0}{x+x_i}\right] + \\ &\frac{1}{\alpha^2+k^2}\times \\ &\left[\cos\delta_0\left(\cos x + \frac{\alpha}{k}\sin x\right) + \sin\delta_0\left(\frac{\alpha}{k}\cos x - \sin x\right)\right] \end{aligned} \tag{10}$$

with $x = kR$ and $x_i = K_i R$. The first term corresponds to the integral from 0 to R, and the second one to the integral of the asymptotic wave functions from R to ∞. The energy spectrum depends now on the initial state through S_n (x_i in the first term and α in the second one) but also V_0 (x_i). It is straightforward to check that when $R \to 0$ the first term vanishes and the second one reduces to Eq. (8).

Still, Eq. (1) is the asymptotic form of the scattering wave function, which due to the phase shift δ_0 does not vanish at the origin. Within the sudden approximation of fast removal reaction, we can assume the same potential well characterized by R and V_0 acting also at the formation of the final state, and calculate the full scattering wave function out from that well:

$$\varphi_k(r) = \begin{cases} \dfrac{\sin(K_f r)}{r}\times\dfrac{\sin(kR+\delta_0)}{k\,\sin(K_f R)} & , r < R \\ \dfrac{\sin(kr+\delta_0)}{kr} & , r > R \end{cases} \tag{11}$$

with $K_f = \sqrt{2\mu(V_0+E)}$. Finally, using both wave functions from the square well (9,11) in Eq. (4), the scattering amplitude becomes:

$$\begin{aligned} O(k) &= \frac{R\sin(x+\delta_0)}{2k}\times \\ &\left[\frac{\cot x_f - \cot x_i}{x_i - x_f} - \frac{\cot x_f + \cot x_i}{x_i + x_f}\right] + \frac{1}{\alpha^2+k^2}\times \\ &\left[\cos\delta_0\left(\cos x + \frac{\alpha}{k}\sin x\right) + \sin\delta_0\left(\frac{\alpha}{k}\cos x - \sin x\right)\right] \end{aligned} \tag{12}$$

with $x_f = K_f R$. In the following, we will see examples using the asymptotic expression of the scattering amplitude (8) and the one derived from the full wave functions from a square well in the initial and final states (12).

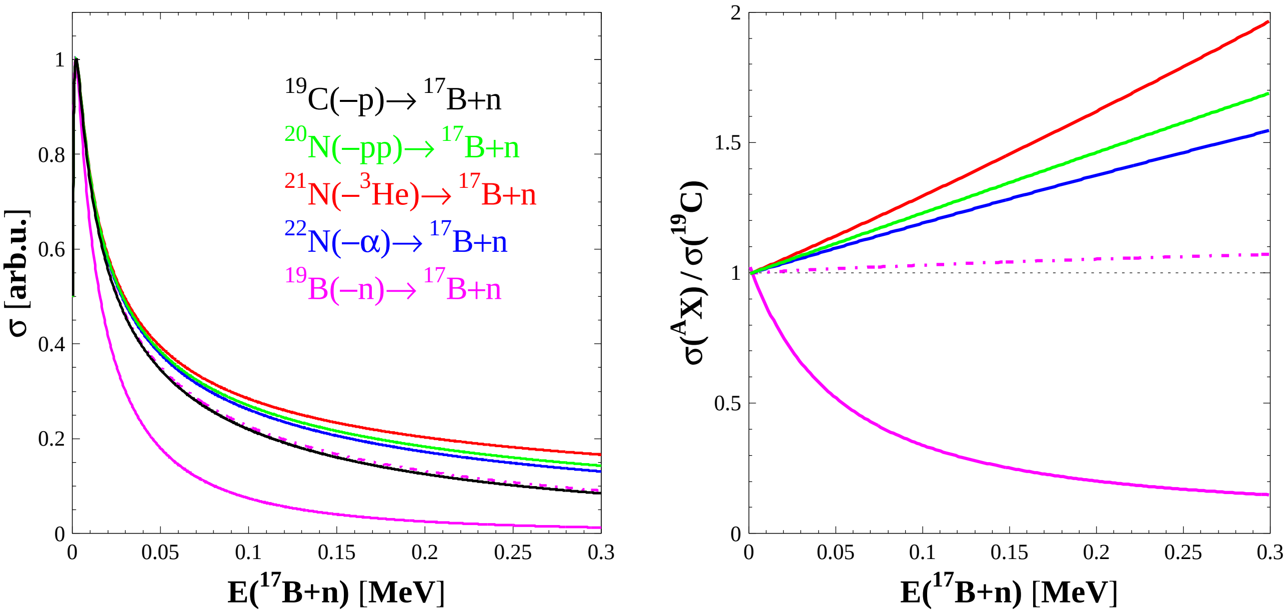

Figure 3. On the left, the n-^{17}B energy spectrum for fast few-nucleon removal from different beams using Eq. (12) and $(a_s, r_e) = (-100, 5)$ fm. On the right, the ratio of cross-sections between each channel and the ^{19}C($-p$) one. The purple dash-dotted line corresponds to the ^{19}B($-n$) channel but using the upper limit of the initial $S_n = 90\pm^{560}_{90}$ keV given in the AME [7].

7 Some analytical results

On the left panel of Fig. 2 we see the results of the different formulas for the known n-n case. The black line corresponds to Eq. (2), the hypothetical case in which we would be able to run a n+n scattering experiment. In practice, we can only form the n+n system in the final state of a reaction, and the blue and red solid lines show what we should expect if we form it respectively by removing a proton from ^{3}H or an α particle from ^{6}He, using Eq. (8). The spectrum is thus shifted to higher energies, depending on the more or less strong binding, i.e. more or less confined neutron wave function, of the initial state. In the two latter cases, the dotted lines show the effect of taking into account the internal part of the wave functions within a square well, using Eq. (12).

On the right panel of Fig. 2 we see the results of the asymptotic Eq. (8) for the still unknown n-^{17}B case, formed by removing a proton from ^{19}C. The red line corresponds to the upper limit of a_s (−50 fm) given in Ref. [4]. Compared to the n-n case, which corresponds to an already large a_s, the spectrum is shifted significantly more towards zero energy, even for this upper limit. The blue line corresponds to their best fit (−100 fm) [4], and the purple line to one order of magnitude higher (−1000 fm). Clearly, experimental setups with higher acceptance and resolution are needed in order to probe the precise value of the scattering length in such a near-threshold regime.

Moreover, in the case of very large values of a_s one should consider r_e, the second term of Eq. (3), even at relatively low energies. In fact, for large a_s the first term of the expansion becomes smaller, and as a consequence the relative importance of the second term may become significant even at energies close to 1 MeV. The blue dotted line on the right panel of Fig. 2 corresponds to $a_s = -100$ fm but with a reasonable effective range of $r_e = 5$ fm. Even at few hundreds of keV, the shape of the spectrum begins to change. Therefore, in order to determine a_s in the hundreds of fm regime, in addition to a high experimental acceptance and resolution one should not neglect the inclusion of r_e in the fit.

8 Multi-channel determination

In Fig. 2 we have seen the n-^{17}B energy spectrum for fast proton removal from ^{19}C and different values of (a_s, r_e). In order to test the formalism, we have fixed the scattering parameters to a given example value $(-100, 5)$ fm and explored the sensitivity of the cross-section to the initial state of the reaction using Eq. (12). On the left panel of Fig. 3, the black line is the analogue of the blue dotted line in Fig. 2 obtained using Eq. (8), and the other solid lines correspond to the fast removal of few nucleons from slightly heavier beams, leading also to ^{17}B+n.

There seems to be a strong, measurable dependence of the lineshape on the neutron binding in the initial state (taken from the Atomic Mass Evaluation [7]). The dependence on the beam is more explicit on the right panel, corresponding to the ratio between each cross-section (normalized at its maximum) and the reference one, ^{19}C($-p$), ranging from about 6 times smaller for ^{19}B($-n$) to some 2 times larger for ^{21}N($-^{3}$He). Starting with proton removal from ^{19}C (S_n = 580 keV), the cross-section increases with the removal of an extra proton from ^{20}N (S_n = 2160 keV) and then of an extra neutron from ^{21}N (S_n = 4610 keV), finally decreasing with the removal of an extra neutron from ^{22}N (S_n = 1540 keV), following the evolution of the S_n values. These ratios for different few-nucleon removal channels should therefore provide a sensitive test of the formalism, that would then be applicable to the extraction of the scattering parameters of the n-^{17}B system.

Finally, the ^{19}B($-n$) channel exhibits a peculiar behavior. Even if ^{19}C is a neutron-halo nucleus due to its low S_n, the neutron binding in ^{19}B is even lower, only 90 keV, which should thus lead to the strongest change in lineshape. However, the present error bar, $S_n = 90\pm^{560}_{90}$ keV [7], is still much larger than that of the other nuclei in Fig. 3. Its upper limit would be 650 keV, slightly more bound than ^{19}C, as we can see from the dash-dotted lines on both panels of Fig. 3. According to this formalism, measuring the n-^{17}B energy spectrum from ^{19}C($-p$) and ^{19}B($-n$) could improve our knowledge of the S_n of ^{19}B, and thus of its mass, without the need to measure it.

9 Summary and perspectives

Neutron scattering off nuclei is an intriguing process, resulting from a complex and delicate balance between nuclear attraction and Pauli repulsion. Even within a small range of similar nuclei, the scattering of neutrons can lead to a wide range of phenomena, oscillating between bound states, a variety of resonant states, or even net repulsion. In particular, it is potentially possible to form strongly resonant states very close to threshold, characterized by scattering lengths of tens, hundreds or even thousands of fm, but such a case is yet to be measured.

This threshold character makes the prediction of scattering lengths almost impossible. In order to explore the most exotic neutron-nucleus combinations, scattering experiments must be replaced by high-energy few-nucleon removal reactions, able to produce any neutron-nucleus system in the final state at low relative energies. The high energy of the reaction allows the use of the sudden approximation, in which the cross-section is calculated as the overlap integral between the wave functions of the initial bound state and of the final scattering one.

We have proposed analytical formulas to describe the energy spectra using the asymptotic expression of those wave functions, and also the analytical ones from a square-well potential. The influence on the measured cross-sections of the different parameters characterizing the initial and final states has been displayed through a particular case, the n-^{17}B system. With an upper limit of $a_s < -50$ fm obtained by proton removal from ^{19}C [4], the determination of this parameter, together with the corresponding effective range r_e, might lead to the observation of the first nuclear system with universal properties.

In order to determine these scattering parameters, an experimental campaign has been undertaken at RIKEN using several beams close to ^{19}C (see for example Ref. [8]), all leading to a ^{17}B+n final state. The high resolution and acceptance of the setup, together with the variety of initial states used, should allow us to measure for the first time the value of (a_s, r_e) for the n-^{17}B system. Besides the intrinsic universal properties of ^{18}B, it would be interesting to study their effect on the three-body structure of ^{19}B, with the eventual formation of Efimov trimers [6].

The unambiguous determination of (a_s, r_e) from the few MeV of the n-^{17}B energy spectrum could also open the way to the exploration of the third term of Eq. (3), associated with the shape of the potential, at slightly higher relative energies. Moreover, the suggested sensitivity of the neutron-nucleus scattering spectrum to the binding of the neutron in the initial state might be used backwards to constrain this binding energy for cases in which it is poorly known, for example ^{19}B [7]. The analysis of the different reaction channels is in progress [9].

References

[1] W.F. Hornyak, *Nuclear structure* (Academic Press, New York, 1975), pp. 63-73.

[2] P. Naidon and S. Endo, *Efimov physics: a review*, Rep. Prog. Phys. **80**, 056001 (2017).

[3] J. Carbonell *et al.*, *Low-energy neutron scattering on light nuclei and ^{19}B as a ^{17}B-n-n three-body system in the unitary limit*, SciPost Phys. Proc. **3**, 008 (2020).

[4] A. Spyrou *et al.*, *First evidence for a virtual ^{18}B ground state*, Phys. Lett. B **683**, 129 (2010).

[5] K.J. Cook *et al.*, *Halo structure of the neutron-dripline nucleus ^{19}B*, Phys. Rev. Lett. **124**, 212503 (2020).

[6] E. Hiyama *et al.*, *Modeling ^{19}B as a ^{17}B-n-n three-body system in the unitary limit*, Phys. Rev. C **100**, 011603R (2019).

[7] M. Wang *et al.*, *The AME 2020 atomic mass evaluation*, Chinese Phys. C **45**, 030003 (2021).

[8] S. Leblond *et al.*, *First observation of ^{20}B and ^{21}B*, Phys. Rev. Lett. **121**, 262502 (2018).

[9] E. Oliveira, Ph.D. thesis, LPC-Caen, Normandie Université, in progress.