Soft Dipole Resonances in ^{8}He and ^{8}C

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Abstract. Soft dipole resonances in ^{8}He and ^{8}C are investigated by applying the complex scaling method to the $\alpha+N+N+N+N$ five-body model for the isospin $T=2$ system. The 1^- excited resonant states are calculated at $(E_x,\Gamma)=(14.0,\ 21.1)$ MeV and $(12.7,\ 24.0)$ MeV for ^{8}He and ^{8}C, respectively. From the analyses of the configuration mixing and the spatial properties, the soft dipole resonances of two nuclei are understood to be the vibrational states of valence nucleons and the α core. They are also seen to have a good isospin symmetry. In contrast, the ground states of two nuclei show different spatial properties indicating the symmetry breaking due to the Coulomb repulsion of valence protons in ${}^{8}_{6}C_2$.

1 Introduction

Our main interest is to understand the characteristic properties of structures and excitations in unstable nuclei very far from the stability line. For this aim, we have studied light neutron-rich and proton-rich nuclei above the stable ^{4}He (α)-core nucleus using a ^{4}He+kN model, where k is the number of valence nucleons $N=p$ (proton) and n (neutron). In the previous work [1], energy spectra of ^{5}He, ^{6}He, ^{7}He, and ^{8}He isotopes and ^{5}Li, ^{6}Be, ^{7}B, and ^{8}C isotones were successfully investigated for $N=n$ and p with $k=1,2,3,4$, respectively.

In Refs. [2–4], the soft dipole resonances are investigated for mirror nuclei ^{8}He and ^{8}C with four valence neutrons/protons above α-core. The possibility of the soft dipole resonances has been proposed at the early stage of studies of unstable nuclear physics as a new collective motion with the dipole oscillation of excess neutrons against the core nucleus [5, 6]. Ikeda gives the schematic image of the soft dipole resonance in two-neutorn halo nuclei consisting of the core nucleus+n+n system as shown in Fig 1.

There have been many studies to search for this exotic resonance in neutron halo nuclei experimentally and theoretically. It is very interesting to see the possibility of the soft dipole resonances in ^{8}He and ^{8}C, where we assume that the core nucleus ^{4}He (α) is kept in the ground state. The number of excess nucleons is the same as the mass number of the core nucleus, and the ratio of proton and neutron numbers is 2:6 (^{8}He) and 6:2 (^{8}C), respectively. There are several problems to be solved for soft dipole resonances; 1) Do the soft dipole resonances exist in ^{8}He and ^{8}C? 2) Are the soft dipole resonant states stable under the coupling with many open channels? 3) Does the isospin symmetry of ^{8}He and ^{8}C hold? 4) Whether the excess four-neutrons or -protons form a cluster?

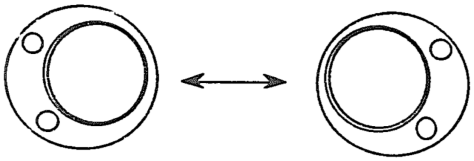

Figure 1. Schematic picture of the soft dipole resonance in two-neutron halo nuclei with the core+n+n system proposed by K. Ikeda [6].

In this report, we will review the results obtained in Refs. [2–4], and discuss these problems on the soft dipole resonances in ^{8}He and ^{8}C.

2 Method

Here, we briefly explain our method to solve the many-body resonances and to analyze the properties. A detailed explanation is given in Refs. [1–4]. We use the cluster orbital shell model (COSM) [7] for the wave function and the Hamiltonian to describe the five-body cluster system $\alpha+N+N+N+N$.

The coordinate system of COSM is shown in Fig. 2, where the vector $\boldsymbol{R}$ is a relative coordinate from the center of mass of α to the center of mass of the four nucleons. We adopt the $(0s)^4$ configuration of the harmonic oscillator (HO) wave function for the α core, where the HO range

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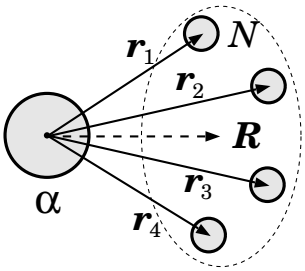

Figure 2. Coordinate system of α+N+N+N+N in COSM. The vector $\boldsymbol{R}$ is a relative coordinate from the center of mass of α to the center of mass of the four nucleons.

parameter of the $0s$ orbit is 1.4 fm to reproduce the observed charge radius. The advantage of the COSM is that we can reproduce the threshold energies of the subsystems of the $A = 8$ systems including the unbound nuclei, such as ^{5}He and ^{7}He. This aspect is important to describe the open channels for nucleon emissions, and then we can treat the many-body decaying phenomena of nuclei. For the α-nucleon interaction $v_{\alpha N}$ and the nucleon-nucleon interaction v_{NN} in the five-body system, we use the KKNN potential [8] with a folding Coulomb potential and the Minnesota potential [9] with the point Coulomb potential, respectively. These interactions well reproduce the observed low-energy phenomena of α-nucleon and nucleon-nucleon systems.

The antisymmetrization between the α-core nucleons and excess nucleons is treated by the orthogonality condition model (OCM) [10]. The relative wave functions between the α core and four excess nucleons are expanded by using the Gaussian basis functions. We employ at most 14 Gaussian basis functions for each relative motion up to about 50 fm, while the orbital angular momentum is $\ell < 3$.

To obtain many-body resonances of the $\alpha + N + N + N + N$ system, we use the complex scaling method (CSM) [11–15]. By applying a simple scaling transformation, $\boldsymbol{r}_i \to \boldsymbol{r}_i \exp(i\theta)$, we can obtain resonant states as the eigenstates with regularized asymptotics satisfying the outgoing boundary condition, and the continuum states are discretized and orthogonal to the resonant solutions. By seeing the eigenvalues obtained on the complex energy plane for different values of the scaling parameter θ, it can be confirmed that a bound state has a constant negative energy below the particle threshold, and a resonant state has a constant complex energy with an argument smaller than 2θ. Also, the continuum states are easily identifiable because they have the energies that lie on a straight line of the angle 2θ starting from the threshold of nucleon emission on the complex energy plane.

To see the properties of the resonant states, we calculate various physical quantities, such as r.m.s. radius, transition strength, sum-rule value *etc*, by using the wave functions obtained in the CSM. In these calculations, we often use the so-called extended completeness relation (ECR)[16–18]. In the case of the $\alpha + n + n + n + n$ system of ^{8}He, the ECR is presented as

$$\begin{aligned} 1 &= \sum_{\nu} |\Psi^J_{\theta,\nu}\rangle\langle\tilde{\Psi}^J_{\theta,\nu}| \\ &= \{\text{bound state of } ^8\text{He}\} + \{\text{resonances of } ^8\text{He}\} \\ &\quad +\{\text{two-body continua of } ^7\text{He}^{(*)} + n\} \\ &\quad +\{\text{three-body continua of } ^6\text{He}^{(*)} + n + n\} \\ &\quad +\{\text{four-body continua of } ^5\text{He}^{(*)} + n + n + n\} \\ &\quad +\{\text{five-body continua of } ^4\text{He}+n + n + n + n\}, \end{aligned} \tag{1}$$

where the index ν represents the eigenstate of the complex-scaled Hamiltonian with the scaling angle θ and $\{\Psi^J_{\theta,\nu}, \tilde{\Psi}^J_{\theta,\nu}\}$ form a set of biorthogonal bases. This ECR is useful for seeing what states of the system are contributing to the physical quantity.

3 Results

In Fig. 3, we show the energy eigenvalues of ^{8}He (1^-) on the complex energy plane, measured from the ground state energy [3]. We use the scaling angle $\theta = 26°$, which gives the stationary point of the 1^- resonance energy in the complex energy plane. The discretized continuum states are obtained almost along the 2θ lines, starting from the energy eigenvalues of the subsystems of $\alpha+n+n+n+n$. We confirm the resonance pole at $(E_x,\ \Gamma) = (14.02,\ 21.12)$ MeV and that this resonance exhausts about half of the strength of the electric-dipole transition from the ground state [2, 3]. In addition, the dipole transition density of this resonance is spatially confined in the α-n distance [2]. Hence we regard this resonance as the soft dipole resonance (SDR).

In Fig. 4, we show the electric dipole strength function of ^{8}He [2]. In the strength (red-dotted line), the sharp peak at 2.5 MeV comes from the ^{7}He+n(s-wave) two-body component, and the soft dipole resonance makes a broad bump structure at around 14 MeV in the distribution due

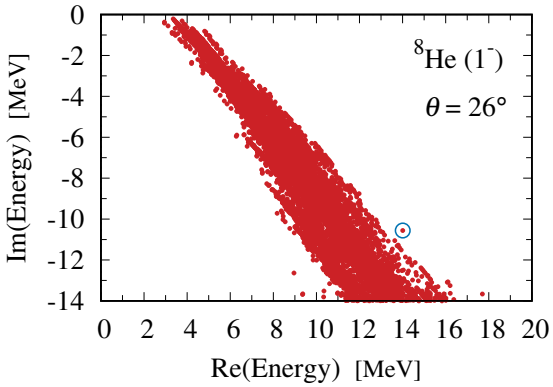

Figure 3. ^{8}He 1^- eigenstates obtained by using the CSM with $\theta = 26°$ in the complex energy plane measured from the ground state energy. The blue circle indicates the eigenenergy of the soft dipole resonance (SDR).

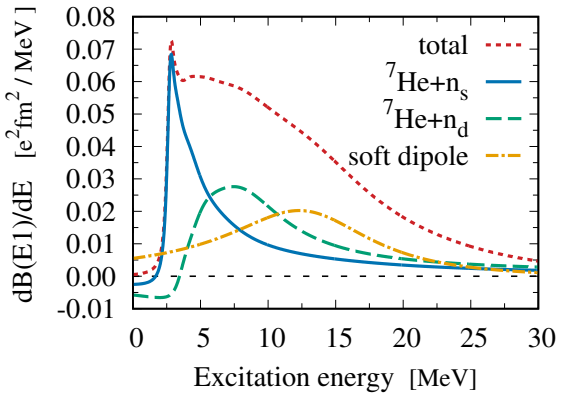

Figure 4. Electric dipole strength of ^{8}He using the complex scaling as a function of the excitation energy E with a red dotted line, taken from Ref. [2]. The decomposed components are ^{7}He(3/2$^-$)+n(s-wave) with a blue solid line, ^{7}He(3/2$^-$)+n(d-wave) with a green dashed line, and soft dipole resonance with an orange dash-dotted line.

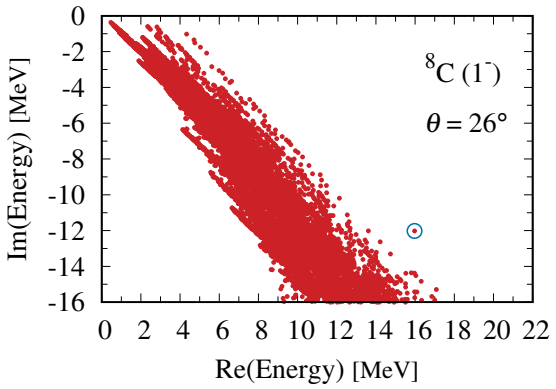

Figure 5. ^{8}C 1^- eigenstates obtained by using the CSM with $\theta = 26°$ in the complex energy plane measured from the $\alpha + p + p + p + p$ threshold energy. The blue circle indicates the eigenenergy of the soft dipole resonance (SDR).

to its large decay width of 21 MeV. We hope the experimental observation of the dipole strength via the Coulomb breakup reactions [19].

In Fig. 5, we also show the energy eigenvalues of ^{8}C measured from the threshold energy of $\alpha+p+p+p+p$ using the same value of the scaling angle θ as used for ^{8}He. As well as the discretized continuum states, we confirm the resonance pole at $(E_r,\ \Gamma) = (15.97,\ 24.04)$ MeV with the excitation energy of $E_x = 12.65$ MeV, which corresponds to a mirror state of the SDR in ^{8}He. The excitation energy is lower than that of ^{8}He by 1.4 MeV and the decay width is larger than that of ^{8}He by 2.9 MeV.

In Fig. 6, the excitation energy spectra of ^{8}He and ^{8}C are compared using their resonance energies including the positive-parity states [1] It is found that he level spacing is smaller in ^{8}C than that in ^{8}He including SDRs. This is an indication of the isospin-symmetry breaking from the Thomas-Ehrman effect of the Coulomb interaction.

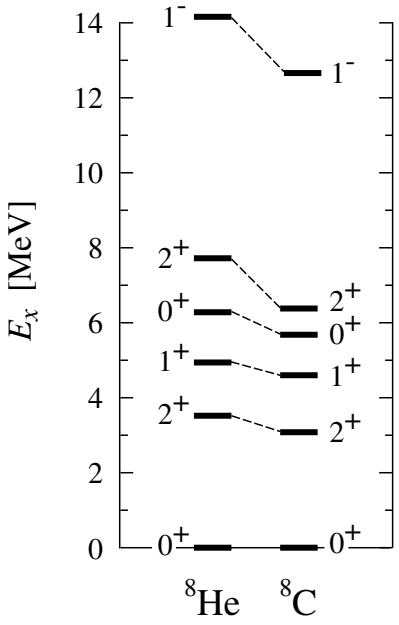

Figure 6. Comparison of the excitation energy spectra between ^{8}He an ^{8}C in units of MeV.

Table 1. Dominant parts of the squared amplitudes of the nucleon configurations in the ground 0^+ states of ^{8}He and ^{8}C [20].

Configuration	^{8}He(0^+_{gr})	^{8}C(0^+_{gr})
$(p_{3/2})^4$	0.860	0.878 − 0.005i
$(p_{3/2})^2(p_{1/2})^2$	0.069	0.057 + 0.001i
$(p_{3/2})^2(d_{5/2})^2$	0.042	0.037 + 0.000i

Table 2. Dominant parts of the squared amplitudes of the nucleon configurations in the soft dipole resonances of ^{8}He and ^{8}C [4].

Configuration	^{8}He(1^-_{SDR})	^{8}C(1^-_{SDR})
$(p_{3/2})^3(d_{5/2})$	0.573 − 0.186i	0.666 − 0.165i
$(p_{3/2})^2(p_{1/2})(d_{3/2})$	0.197 + 0.118i	0.138 + 0.115i
$(p_{3/2})^3(d_{3/2})$	0.149 + 0.120i	0.087 + 0.150i

For the comparison of ^{8}He and ^{8}C, we show dominant configurations of the four valence nucleons in the ground 0^+ and resonant 1^- states of these mirror nuclei in Tables 1 and 2. They show a similar tendency in the mixing property of the nucleon configurations in the mirror nuclei. These results in Table 2 indicate the good isospin symmetry for SDR.

In Table 3, we list the various radii and the relative distances for ^{8}He and ^{8}C. The value of $\alpha-N$ is the mean relative distance between α and valence nucleon. The value of $4N$ is the root-mean-square radius of $4N$ measured from the center of mass of $4N$. The value of $\alpha-4N$ is the relative distance between the center of mass of the α core and the center of mass of $4N$. Note that, for resonances, the radius becomes complex numbers in general, and we recently proposed a general scheme to interpret the complex expectation values for resonances [21], in which the real

Table 3. Various radii and relative distances of the ground 0^+ states and the soft dipole resonances (SDR) in ^{8}He and ^{8}C. Units are fm.

	^{8}He		^{8}C	
	$0^+_{\rm gr}$	$1^-_{\rm SDR}$	$0^+_{\rm gr}$	$1^-_{\rm SDR}$
Matter	2.53	3.11+0.86i	2.81−0.08i	3.09+1.27i
Proton	1.81	1.97+0.29i	3.06−0.10i	3.39+1.46i
Neutron	2.73	3.41+0.99i	1.90−0.01i	1.94+0.39i
α-N	3.56	4.58+1.42i	4.05−0.12i	4.54+2.05i
α-4N	2.05	2.67+0.84i	2.36−0.03i	2.62+1.15i
4N	2.91	3.72+1.14i	3.29−0.13i	3.71+1.70i

part can be related to the physical interpretation. According to this scheme, we discuss the real part of the complex radii in the present analysis.

In Table 3, for ground states of ^{8}He and ^{8}C, the matter radius of ^{8}C is larger than that of ^{8}He due to the Coulomb repulsion of valence protons in ^{8}C. The α-4p distance of ^{8}C is also larger than the α-4n distance in ^{8}He. The mean radii of 4p in ^{8}C is larger than that of 4n in ^{8}He by about 0.4 fm. These differences indicate the breaking of the isospin symmetry in the ground states of mirror nuclei. For SDR, it is found their matter radii show similar values and this tendency can also be confirmed in other values such as the radii of 4n and 4p. These results indicate the good isospin symmetry in SDR of ^{8}He and ^{8}C.

4 Discussions

We discuss the excitation energies E_x of SDRs in ^{8}He and ^{8}C because they are more than 10 MeV and not small. We interpret these resonances as the collective dipole oscillation of four valence nucleons (4N) against the α core from the analysis of the transition density [2]. Hence it is meaningful to consider the relative motion between the α core and 4N. In the ground states of two nuclei, the relative distances $\sqrt{\langle \boldsymbol{R}^2 \rangle}$ between the α core and 4N are 2.05 fm for ^{8}He and 2.36 fm for ^{8}C, respectively, where $\boldsymbol{R}$ is defined in Fig. 2. When we assume the lowest $(0p)^4$ configuration of 4N in ^{8}He and ^{8}C using the HO basis states, this configuration gives the quanta $N_R = 2$ in the $\alpha - 4N$ relative motion, while 4N has the lowest internal quanta of two from the Pauli principle. We derive $\hbar\omega_R$ of the $\alpha - 4N$ relative motion using the length parameter b_R determined from $\langle \boldsymbol{R}^2 \rangle$ and the reduced mass μ_R as follows:

$$\begin{aligned} \hbar\omega_R &= \frac{\hbar^2}{\mu_R b_R^2}, \\ \langle \boldsymbol{R}^2 \rangle &= \left(\frac{3}{2} + N_R\right) b_R^2. \end{aligned} \tag{2}$$

We refer the value of $\hbar\omega_R$ to estimate the excitation energy of the relative motion. For ^{8}He, this relation gives $\hbar\omega_R$ = 17.2 MeV while the calculation gives 14.0 MeV. For ^{8}C, $\hbar\omega_R$ = 13.0 MeV while the calculation gives 12.7 MeV. From these comparisons, the estimated values of $\hbar\omega_R$ can simulate well the excitation energies of the SDRs obtained in two nuclei. These results support the picture of the excitation of the relative motion in SDRs, namely, the collective excitations of 4N. The calculated excitation energies are slightly lower than the estimated ones, which indicates a deviation from the simple α + 4N two-body picture.

From Table 3, we see that the radius 3.72 (3.71) fm of 4n (4p) and the distance 2.67 (2.62) fm between α core and the center-of-mass of 4n (4p) for the SDR states in ^{8}He (^{8}C), respectively. Since the radius of the α core is 1.48 fm, it can be seen that the α core exists and vibrates within the 4n (4p) expanse. This result supports the picture proposed by Ikeda [6] for the excitation mode of the soft dipole resonance, as shown in Fig. 1. The present results also show that this picture holds not only for neutron-rich ^{8}He but also for proton-rich ^{8}C.

The number of protons and neutrons is 2 and 6 in ^{8}He and 6 and 2 in ^{8}C, and despite the large difference, the isospin symmetry holds well. From Tables 1 and 2, the results for the ground states of ^{8}He and ^{8}C show slight differences, but for the SDR states they are very similar.

5 Summary

We investigated the soft dipole resonance (SDR) in neutron-rich ^{8}He and proton-rich ^{8}C nuclei, which is a collective oscillation of four valence neutrons/protons against the α core. We also discussed the isospin symmetry in two nuclei. We use the five-body cluster model and many-body resonances are described with the complex scaling. We obtained the 1^- resonance in two nuclei with the similar excitation energy of around 13 MeV showing very broad decay width, and their structures are similar to each other such as the spatial properties. These results indicate a good isospin symmetry in the SDR of two nuclei with collective excitations of multineutrons and multiprotons, while the ground states of two nuclei show different properties due to the Coulomb repulsion in ^{8}C, leading to the symmetry breaking. In conclusion, the isospin symmetry is depending on the states of ^{8}He and ^{8}C.

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