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# Exploring type-I seesaw under $S_3$ modular symmetry

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**Abstract.** This work's novelty lies in using the simplest group i.e. $\Gamma(2) \simeq S_3$ modular symmetry implemented on the canonical seesaw to explain neutrino phenomenology. Here, we construct and classify models based on the doublet and singlet representations of supermultiplets under $S_3$ discrete symmetry along with their respective modular weights, allowing a mass matrix for the neutrino sector with minimal use of free parameters, namely models $A, B, C, D$. These modular symmetries become advantageous in avoiding the requirements of multiple flavon fields and the intricacies of vacuum alignments. In this way, we endeavor to clarify the effect and significance of modular $S_3$ symmetry, which is considered in explaining the neutrino phenomenology viable with the current observations. Additionally, we also shed some light on the neutrinoless double beta decay.

## 1 Introduction

The Standard Model (SM) initially described neutrinos as massless, but experimental evidence from neutrino oscillations shows they have small, non-zero masses, implying neutrino mixing and necessitating at least two massive neutrinos [1]. Neutrinos, unlike other SM fermions, cannot gain mass through the Higgs mechanism due to the absence of right-handed counterparts. However, the Weinberg operator provides a potential mass generation mechanism [2–4]. Various beyond Standard Model (BSM) scenarios, such as the seesaw model [5–9], radiative mass generation [10–13], and extra dimensions [14–18], have been proposed to explain neutrino masses and mixing. Symmetry-based approaches, including modular flavor symmetry, are used to enforce specific mixing patterns [19–27]. Modular symmetry, promoting Yukawa couplings and mass parameters to modular forms, has been applied to construct neutrino mass models with various symmetry groups[28–36], but the smallest group, $S_3$[37–39], has been less explored. This work focuses on applying modular $S_3$ symmetry to the supersymmetric type-I seesaw model, identifying four viable scenarios (models A, B, C, and D) compatible with current neutrino data and containing minimal free parameters.

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## 2 $S_3$ symmetry

Our investigation centers on the $S_3$ group, where $N = 2$. $S_3$ exhibits three irreducible representations: the doublet **2**, the singlet **1**, and the pseudo-singlet **1′**. The minimal even-weighted modular forms emerge at $k_I = 2$, with the modular coupling denoted as $Y_{\mathbf{2}}^{(2)} = (Y_1(\tau), Y_2(\tau))$, formulated using the Dedekind eta function $(\eta(\tau))$ [37]. For computational efficiency, we employ the $q$-expansion form [37]:

$$\begin{aligned} Y_1(\tau) &= \frac{1}{8} + 3q^2 + 3q^4 + 12q^6 + 3q^8 + \cdots, \\ Y_2(\tau) &= \sqrt{3}q(1 + 4q^2 + 6q^4 + 8q^6 + \cdots), \end{aligned} \tag{1}$$

where $q \equiv e^{2i\pi\tau/N} \equiv e^{i\pi\tau}$. More complex modular forms utilized in model construction are derived through the application of $S_3$ product rules, as elaborately expressed in [40].

| | Fields | $E_1^c$ | $E_2^c$ | $E_3^c$ | $L_1$ | $L_2$ | $L_3$ | $N_1^c$ | $N_2^c$ | $N_3^c$ | Free Parameters | NO | IO |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **MODEL A** | $S_3$ | 2 | | 1 | 2 | | 1 | 2 | | 1 | 7 | ✓ | ✓ |
| | $k_I$ | 1 | | -1 | 1 | | 1 | 1 | | 3 | | | |
| **MODEL B** | $S_3$ | 2 | | 1 | 2 | | 1 | 1 | 1 | 1′ | 7 | ✓ | ✓ |
| | $k_I$ | 0 | | 0 | 2 | | 0 | 0 | 2 | 0 | | | |
| **MODEL C** | $S_3$ | 1 | 1′ | 1 | 2 | | 1 | 2 | | 1 | 4 | ✗ | ✓ |
| | $k_I$ | 1 | 1 | -1 | 1 | | 1 | 1 | | 1 | | | |
| **MODEL D** | $S_3$ | 2 | | 1′ | 1 | 1′ | 1′ | 2 | | 1′ | 9 | ✓ | ✓ |
| | $k_I$ | 0 | | 0 | 2 | 2 | 0 | 0 | | 4 | | | |

Table 1: In this table, we depict the particle content of the different models and their charges under $S_3$ modular symmetry, where, $k_I$ is the modular weight. Also, the number of free real parameters, in addition of the complex modulus $\tau$, and the possible ordering of neutrino masses are provided for each model.

## 3 Model framework

### 3.1 Model A

The superpotential in the charged lepton sector consistent with modular $S_3$ symmetry is given by

$$\mathcal{W}_\ell = \alpha_\ell (E^c L)_2 Y_{\mathbf{2}}^{(2)} H_d + \beta_\ell (E^c Y_{\mathbf{2}}^{(2)})_1 L_3 H_d + \gamma_\ell E_3^c L_3 H_d, \tag{2}$$

This superpotential leads to a non-diagonal charged lepton mass matrix

$$M_\ell = \frac{v_d}{\sqrt{2}} \begin{pmatrix} -\alpha_\ell Y_1 & \alpha_\ell Y_2 & 0 \\ \alpha_\ell Y_2 & \alpha_\ell Y_1 & 0 \\ \beta_\ell Y_1 & \beta_\ell Y_2 & \gamma_\ell \end{pmatrix}. \tag{3}$$

The Yukawa couplings $\alpha_\ell$, $\beta_\ell$ and $\gamma_\ell$ are in general complex. However, one can perform arbitrary phase redefinition on $E_i^c$ to make them real.

For the neutral leptons, the superpotential is given by

$$\begin{aligned} \mathcal{W}_\nu = {} & \alpha_D (N^c L)_2 Y_{\mathbf{2}}^{(2)} H_u + \beta_D N_3^c (L Y_{\mathbf{2}}^{(4)})_1 H_u + \gamma_D (N^c Y_{\mathbf{2}}^{(2)}) L_3 H_u + \omega_D N_3^c L_3 Y_{\mathbf{1}}^{(4)} H_u \\ & + \alpha_R M (N^c N^c)_2 Y_{\mathbf{2}}^{(2)} + \beta_R M N_3^c (N^c Y_{\mathbf{2}}^{(4)})_1 + M N_3^c N_3^c Y_{\mathbf{1}}^{(6)}. \end{aligned} \tag{4}$$

The first three terms give rise to a Dirac matrix

$$M_D = \frac{\omega_D v_u}{\sqrt{2}} \begin{pmatrix} -\bar{\alpha}_D Y_1 & \bar{\alpha}_D Y_2 & \bar{\gamma}_D Y_1 \\ \bar{\alpha}_D Y_2 & \bar{\alpha}_D Y_1 & \bar{\gamma}_D Y_2 \\ \bar{\beta}_D (Y_{\mathbf{2}}^{(4)})_1 & \bar{\beta}_D (Y_{\mathbf{2}}^{(4)})_2 & Y_{\mathbf{1}}^{(4)} \end{pmatrix}, \tag{5}$$

where we introduce a shorthand notation $\bar{x} = x/\omega_D$. The last three terms of $\mathcal{W}_\nu$ give rise to the Majorana mass matrix for $N^c$,

$$M_R = M \begin{pmatrix} -2\alpha_R Y_1 & 2\alpha_R Y_2 & \beta_R (Y_{\mathbf{2}}^{(4)})_1 \\ 2\alpha_R Y_2 & 2\alpha_R Y_1 & \beta_R (Y_{\mathbf{2}}^{(4)})_2 \\ \beta_R (Y_{\mathbf{2}}^{(4)})_1 & \beta_R (Y_{\mathbf{2}}^{(4)})_2 & Y_{\mathbf{1}}^{(6)} \end{pmatrix}. \tag{6}$$

The above Dirac and Majorana mass matrices lead to the light neutrino mass matrix $M_\nu = M_D^T M_R^{-1} M_D$.

### 3.2 Model B

The superpotential for the charged leptons is given by

$$\mathcal{W}_\ell = \alpha_\ell (E^c L)_2 Y_{\mathbf{2}}^{(2)} H_d + \beta_\ell E_3^c (Y_{\mathbf{2}}^{(2)} L)_1 H_d + \gamma_\ell E_3^c L_3 H_d. \tag{7}$$

This leads to a charged lepton mass matrix

$$M_\ell = \frac{v_d}{\sqrt{2}} \begin{pmatrix} -\alpha_\ell Y_1 & \alpha_\ell Y_2 & 0 \\ \alpha_\ell Y_2 & \alpha_\ell Y_1 & 0 \\ \beta_\ell Y_1 & \beta_\ell Y_2 & \gamma_\ell \end{pmatrix}. \tag{8}$$

For the neutral leptons, the superpotential is given by

$$\begin{aligned} \mathcal{W}_\nu = {} & \alpha_D N_1^c (L Y_{\mathbf{2}}^{(2)})_1 H_u + \beta_D N_2^c (L Y_{\mathbf{2}}^{(4)})_1 H_u + \gamma_D N_3^c (L Y_{\mathbf{2}}^{(2)})_{1'} H_u + \omega_D N_1^c L_3 H_u \\ & + \alpha_R M N_1^c N_1^c + \beta_R M N_2^c N_2^c Y_{\mathbf{1}}^{(4)} + M N_3^c N_3^c. \end{aligned} \tag{9}$$

The first four terms in $\mathcal{W}_\nu$ lead to the Dirac mass matrix

$$M_D = \frac{\omega_D v_u}{\sqrt{2}} \begin{pmatrix} \bar{\alpha}_D Y_1 & \bar{\alpha}_D Y_2 & 1 \\ \bar{\beta}_D (Y_{\mathbf{2}}^{(4)})_1 & \bar{\beta}_D (Y_{\mathbf{2}}^{(4)})_2 & 0 \\ \bar{\gamma}_D Y_2 & -\bar{\gamma}_D Y_1 & 0 \end{pmatrix}, \tag{10}$$

where we have employed a short-hand notation $\bar{x} = x/\omega_D$. The last three terms of $\mathcal{W}_\nu$ are the Majorana masses for $N_i^c$ leads to a Majorana mass matrix as below

$$M_R = M \begin{pmatrix} \alpha_R & 0 & 0 \\ 0 & \beta_R Y_{\mathbf{1}}^{(4)} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{11}$$

### 3.3 MODEL C

The superpotential for the charged lepton sector is

$$\mathcal{W}_\ell = \alpha_\ell E_1^c (L Y_{\mathbf{2}}^{(2)})_1 H_d + \beta_\ell E_2^c (L_2 Y_{\mathbf{2}}^{(2)})_{1'} H_d + \gamma_\ell E_3^c L_3 H_d \tag{12}$$

This leads to a charged lepton mass matrix

$$M_\ell = \frac{v_d}{\sqrt{2}} \begin{pmatrix} \alpha_\ell Y_1 & \alpha_\ell Y_2 & 0 \\ \beta_\ell Y_2 & -\beta_\ell Y_1 & 0 \\ 0 & 0 & \gamma_\ell \end{pmatrix}. \tag{13}$$

In the neutral lepton sector, the superpotential is given by

$$\begin{aligned} \mathcal{W}_\nu = {} & \alpha_D \left[(N^c L)_2 Y_{\mathbf{2}}^{(2)}\right]_1 H_u + \beta_D (N^c Y_{\mathbf{2}}^{(2)})_1 L_3 H_u + \omega_D N_3^c (L Y_{\mathbf{2}}^{(2)})_1 H_u \\ & + M \left(\left[(N^c N^c)_2 Y_{\mathbf{2}}^{(2)}\right]_1 + \alpha_R N_3^c (N^c Y_{\mathbf{2}}^{(2)})_1\right). \end{aligned} \tag{14}$$

It leads to the Dirac mass matrix

$$M_D = \frac{\omega_D v_u}{\sqrt{2}} \begin{pmatrix} -\bar{\alpha}_D Y_1 & \bar{\alpha}_D Y_2 & \bar{\beta}_D Y_1 \\ \bar{\alpha}_D Y_2 & \bar{\alpha}_D Y_1 & \bar{\beta}_D Y_2 \\ Y_1 & Y_2 & 0 \end{pmatrix}, \tag{15}$$

and the Majorana mass matrix

$$M_R = M \begin{pmatrix} -Y_1 & Y_2 & \alpha_R Y_1 \\ Y_2 & Y_1 & \alpha_R Y_2 \\ \alpha_R Y_1 & \alpha_R Y_2 & 0 \end{pmatrix}. \tag{16}$$

### 3.4 Model D

The superpotential in the charged lepton sector can be written as

$$\mathcal{W}_\ell = \alpha_\ell (E^c Y_{\mathbf{2}}^{(2)})_1 L_1 H_d + \beta_\ell (E^c Y_{\mathbf{2}}^{(2)})_{1'} L_2 H_d + \gamma_\ell E_3^c L_3 H_d \tag{17}$$

The above superpotential leads to a charged lepton mass matrix

$$M_\ell = \frac{v_d}{\sqrt{2}} \begin{pmatrix} \alpha_\ell Y_1 & \beta_\ell Y_2 & 0 \\ \alpha_\ell Y_2 & -\beta_\ell Y_1 & 0 \\ 0 & 0 & \gamma_\ell \end{pmatrix}. \tag{18}$$

In the neutral lepton sector, the superpotential is given by

$$\begin{aligned} \mathcal{W}_\nu = {} & \omega_D (N^c Y_{\mathbf{2}}^{(2)})_1 L_1 H_u + \alpha_D (N^c Y_{\mathbf{2}}^{(2)})_{1'} L_2 H_u + \beta_D N_3^c L_1 Y_{\mathbf{1'}}^{(6)} H_u + \gamma_D N_3^c L_2 Y_{\mathbf{1}}^{(6)} H_u \\ & + \eta_D N_3^c L_3 Y_{\mathbf{1}}^{(4)} H_u + M \left[(N^c N^c)_1 + \alpha_R N_3^c (N^c Y_{\mathbf{2}}^{(4)})_{1'} + \beta_R N_3^c N_3^c Y_{\mathbf{1}}^{(8)}\right]. \end{aligned} \tag{19}$$

The first 5 terms in the superpotential give rise to a Dirac mass matrix

$$M_D = \frac{\omega_D v_u}{\sqrt{2}} \begin{pmatrix} Y_1 & \bar{\alpha}_D Y_2 & 0 \\ Y_2 & -\bar{\alpha}_D Y_1 & 0 \\ \bar{\beta}_D Y_{\mathbf{1'}}^{(6)} & \bar{\gamma}_D Y_{\mathbf{1}}^{(6)} & \bar{\eta}_D Y_{\mathbf{1}}^{(4)} \end{pmatrix}, \tag{20}$$

and the last 3 terms give the Majorana mass matrix

$$M_R = M \begin{pmatrix} 1 & 0 & \alpha_R (Y_{\mathbf{2}}^{(4)})_2 \\ 0 & 1 & -\alpha_R (Y_{\mathbf{2}}^{(4)})_1 \\ \alpha_R (Y_{\mathbf{2}}^{(4)})_2 & -\alpha_R (Y_{\mathbf{2}}^{(4)})_1 & \beta_R Y_{\mathbf{1}}^{(8)} \end{pmatrix}. \tag{21}$$

## 4 Results

In our analysis, the parameters $\bar{\alpha}_D$, $\bar{\beta}_D$, $\bar{\gamma}_D$, $\bar{\eta}_D$, $\alpha_R$, and $\beta_R$ are varied within the interval $[10^{-4}, 10^4]$. This range is selected to emulate the hierarchy observed between the electron and the top Yukawa couplings. A similar range has been extensively utilized in the literature, as seen in references such as [41–43]. The factor $\omega_D^2 v_u^2/M$, which determines the neutrino mass scale, is considered within the interval (0, 1 eV]. The modulus $\tau$ is varied within the fundamental domain defined by

$$\mathrm{Im}(\tau) > 0, \quad \left|\mathrm{Re}(\tau)\right| \leq \frac{1}{2}, \text{ and } |\tau| \geq 1. \tag{22}$$

For each parameter set, we first numerically diagonalize the charged lepton and neutrino mass matrices to determine the PMNS matrix $U$ and the mass splittings $\Delta m^2_{sol}$ and $\Delta m^2_{atm}$. The oscillation angles are then extracted from

$$\sin^2\theta_{13} = |U_{13}|^2, \quad \sin^2\theta_{12} = \frac{|U_{12}|^2}{1-|U_{13}|^2}, \quad \sin^2\theta_{23} = \frac{|U_{23}|^2}{1-|U_{13}|^2}, \tag{23}$$

and the Jarlskog invariant ($J_{CP}$) is determined from

$$J_{CP} = \mathrm{Im}\left(U_{12}U_{23}U_{13}^*U_{22}^*\right). \tag{24}$$

Furthermore, the effective electron neutrino mass and the neutrinoless double beta decay is determined by the eqns. (25) and (26).

$$m_{\nu_e}^{\mathrm{eff}} = \sqrt{\sum_i |U_{ei}|^2 m_{\nu_i}^2}. \tag{25}$$

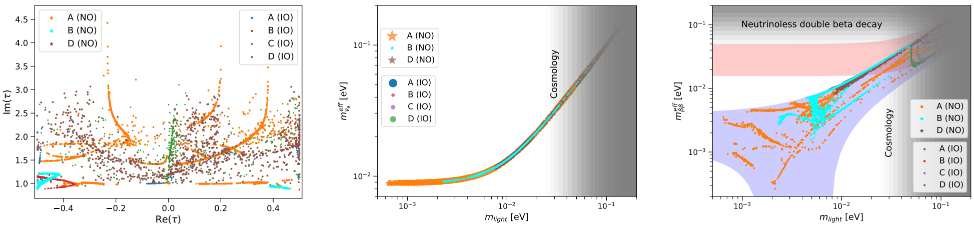
$$m_{\beta\beta}^{eff} = \left|\sum_i U_{ei}^2 m_{\nu_i}\right|. \tag{26}$$

Our analysis shows that Models A, B, and D can accommodate normal ordering (NO) and inverted ordering (IO) neutrino masses. However, Model C is only compatible with IO due to its minimal structure. Fig. 1a shows the modulus $\tau$ in each model, which produces neutrino oscillation parameters within 2-$\sigma$ of the best-fit values. Further, Fig. 1b represents the effective electron neutrino mass and its upper bound is provided by KATRIN experiment with $m_{\nu_e}^{eff} \lesssim 0.8$ eV at 90% CL [44]. Finally, Fig. 1c represents the range in the upper bounds reflects uncertainties in nuclear matrix element estimations, with strongest bound on $m_{\beta\beta}^{eff}$ is provided by the KamLAND-Zen experiment, with $m_{\beta\beta}^{eff} \leq 0.036$–$0.156$ eV at 90% CL [45]. With the most stringent estimation of the bound, the IO scenario of Model B and C are ruled out.

## 5 Conclusion

We have developed models of neutrino masses using $S_3$ modular symmetry within a type-I supersymmetric framework. By introducing three $SU(2)_L$ singlet chiral supermultiplets ($N^c$) for the type-I seesaw mechanism and constructing Yukawa couplings with $S_3$ modular forms, we identified four realizations of the $S_3$ symmetry. All are compatible with inverted ordering (IO), while Models A, B, and D also support normal ordering (NO).

A parameter scan was conducted to align these models with neutrino oscillation data at the 2-$\sigma$ level, assessing the lightest neutrino mass, effective electron neutrino mass, and




(a) (b) (c)

Figure 1: Panel 1a shows the modulus $\tau$ that results in viable neutrino oscillation parameters for Models A, B, C, and D. Panel 1b illustrates the effective electron neutrino mass relative to the lightest neutrino mass. Panel 1c displays the effective Majorana mass for $\nu_e$ as a function of the lightest neutrino mass with the horizontal shaded region is excluded by neutrinoless double beta decay experiments, while the vertical shaded regions in 1b & 1c panels are excluded by cosmological measurements.

effective Majorana mass of $\nu_e$. IO scenarios are tightly constrained by cosmological limits, particularly in Models B and C, while the effective electron neutrino mass remains below current experimental limits across all models.

Future experiments like the Simon Observatory [46] and LEGEND [47] phase-II are expected to probe the full parameter space of these models, with KATRIN and HOLMES covering parts of Model A's IO scenario. Additionally, distinguishing among Models A, B, C, and D is possible through measurements of neutrino Majorana phases, as the number of physical phases varies among them, influencing the relations between Dirac and Majorana phases.

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