Exclusive $c \to u\gamma$ transitions of B_c meson

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Abstract. We study the rare decays of the B_c meson induced by the flavour changing neutral current $c \to u\gamma$ transition. In the Standard Model they are strongly suppressed by the Glashow-Iliopoulos-Maiani mechanism, therefore they are sensitive to new physics. The difficulty is to get rid of long-distance contributions. We study such effects in radiative B_c transitions both to B^* and to the axial-vector B_1' meson.

1 Introduction

The decays induced by the flavour changing neutral current $c \to u$ transitions play an important role in the search for new physics (NP) phenomena [1]. In the Standard Model (SM) the relevant weak Hamiltonian involves small Wilson coefficients resulting from the efficient GIM cancellation [2]: the corresponding hadronic amplitudes are highly suppressed for $c \to u\gamma$, $c \to u\ell^+\ell^-$ and $c \to u\nu\bar{\nu}$ [3]. Studies devoted to $B_c^+ \to B^{*+}\gamma$ concluded that in this channel the long-distance (LD) contributions do not have the overwhelming size as in the case of D, D_s and Λ_c decays [4]. We reconsider the issue, extending the analysis to the $B_c^+ \to B_1'^+\gamma$ mode, with $B_1'^+$ the lightest axial-vector beauty meson. We find that for B_c decays to spin-1 positive-parity B_u excitations, the LD contributions are comparable to the B^{*+} case. However, a hadronic suppression in the short-distance (SD) amplitude reduces the role of this channel for searching NP.

2 Effective weak Hamiltonian

To describe the $c \to u\gamma$ transition we consider the effective weak Hamiltonian [3]

$$\mathcal{H}_{eff} = 4\frac{G_F}{\sqrt{2}}\left[\sum_{q=d,s} V_{cq}^* V_{uq}\left(C_1 O_1^{(q)} + C_2 O_2^{(q)}\right) + \sum_{i=3}^{6} C_1 O_i + \sum_{i=7}^{8}\left(C_i O_i + C_i' O_i'\right)\right] \quad (1)$$

expressed in terms of current-current operators $O_{1,2}^{(q)}$, QCD penguins operators $O_{3,\dots 6}$ and electromagnetic and gluon dipole operators $O_{7,8}^{(\prime)}$. The effective coefficient C_7^{eff} including two-loop QCD matrix elements of the operators $O_{3,\dots,6}$, is in the range [3]

$$C_7^{\text{eff}} \in [-0.00151 - i0.00556|_s + i0.00005|_{CKM}, -0.00088 - i0.00327|_s + i0.00002|_{CKM}], \quad (2)$$

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where the subscripts indicate the contributions to the imaginary part from the strong phases and the phases of the CKM matrix. C_7' turns out to be small, $C_7' \sim m_u/m_c$, therefore the operator $\mathcal{O}_7'$ can be neglected in SM. Eq. (2) shows the GIM suppression in the SM for the dipole $\mathcal{O}_7$, since $C_7 \sim 10^{-3}$. This coefficient can be significantly enhanced in a SM extension (BSM) scenario. General bounds can be established [5]:

$$|C_7|, |C_7'| \lesssim 0.5. \tag{3}$$

The amplitude of the transition $B_c(p) \to A(p', \epsilon)\gamma(q, \lambda)$, where A is a 1^+ state is given by

$$\mathcal{A}(B_c(p) \to A(p', \epsilon)\gamma(q, \lambda)) = \left\{A_{PC}\left[p \cdot q g^{\alpha\beta} - q^\alpha p^\beta\right] + iA_{PV}\varepsilon^{\alpha\beta\mu\nu} p_\mu q_\nu\right\} \epsilon^*_\alpha \lambda^*_\beta. \tag{4}$$

3 Short-distance amplitude

The SD amplitude comes from the $\mathcal{O}_7'$ and $\mathcal{O}_{1,2}$ operators Fig. 1. The weak annihilation (WA) topology is doubly Cabibbo-suppressed, therefore it can be ignored in the evaluation of the SD.

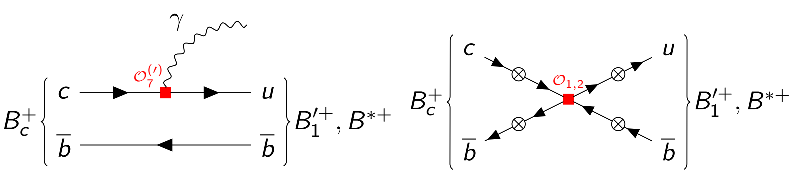

Figure 1. Electromagnetic dipole (left) and weak annihilation amplitudes (right) for $B_c \to B_1'(B^*)\gamma$. The crosses in the WA diagram correspond to the photon emission.

Focusing on the positive parity channel, the hadronic matrix element is parametrized as

$$\begin{aligned}\langle B_1'(p', \epsilon)|\, \bar{u}\sigma_{\mu\nu}c\, |B_c(p)\rangle = &\frac{\epsilon^* \cdot q}{\left(m_{B_c} + m_{B_1'}\right)^2}\left(p_\mu p'_\nu - p_\nu p'_\mu\right) T_0'(q^2) \\ &+ \left(p_\mu \epsilon^*_\nu - p_\nu \epsilon^*_\mu\right) T_1'(q^2) + \left(p'_\mu \epsilon^*_\nu - p'_\nu \epsilon^*_\mu\right) T_2'(q^2),\end{aligned} \tag{5}$$

with $q = p - p'$, λ and ϵ the photon and B_1' polarization vectors. Since the $\bar{u}\sigma_{\mu\nu}\gamma_5 c$ matrix element is obtained using $\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$, with $\varepsilon^{0123} = -1$, the parity-conserving and parity-violating SD amplitudes in Eq. (4) are given by

$$A^{SD}_{PC} = i\frac{G_F}{(2\pi)^{3/2}} m_c \alpha^{1/2}(C_7^{eff} + C_7')(T_1'(0) + T_2'(0)), \tag{6}$$

$$A^{SD}_{PV} = -i\frac{G_F}{(2\pi)^{3/2}} m_c \alpha^{1/2}(C_7^{eff} - C_7')(T_1'(0) + T_2'(0)). \tag{7}$$

To determine the form factors T_i we use an approach based on the heavy quark spin symmetry [6–9]. For a generic Dirac matrix Γ, the matrix elements of quark currents for negative and positive parity doublets, invariant under rotations of the heavy quark spins, can be obtained using the trace formalism [10]:

$$\langle B_1'(v, k)|\, \bar{u}\Gamma c\, |B_c(v)\rangle = -\mathrm{Tr}\left[\bar{S}\,\Omega'\,(v, a_0 k)\,\Gamma H\right], \tag{8}$$

where S and H are the positive and negative parity spin doublet fields representation of the states (B_c, B_c^*) and (B_0, B_1'). The form factor Ω' encoding the nonperturbative dynamics is written as [6, 7]

$$\Omega'(v, a_0 k) = \Omega_1'(v, a_0 k) + \not{k} a_0 \Omega_2'(v, a_0 k). \tag{9}$$

The dimensionful parameter a_0 represents the length scale involved in the process: we use the same value for both matrix elements. Since $m_c \ll m_b$, the b quark remain almost unaffected in the B_c transition, and the velocity of the final meson is nearly unchanged. Using Eq. (8) we obtain for B_1'

$$\langle B_1'(v,k;\epsilon)|\,\bar{u}\sigma_{\mu\nu}c\,|B_c(v)\rangle = -i\sqrt{\frac{m_{B_c}}{m_{B_1'}}}\left[\Omega_1'\left(k_\mu\epsilon_\nu^* - k_\nu\epsilon_\mu^*\right) + 2a_0\Omega_2' m_{B_1'}\epsilon^*\cdot v\left(k_\mu v_\nu - k_\nu v_\mu\right)\right]. \tag{10}$$

The form factors $\Omega_{1,2}'$ can be obtained using a determination of matrix elements in a non perturbative approach. In [11] using a covariant light-front approach the form factors for the $B_c \to (B_{sJ}, B_{dJ})$ decays in SM have been determined. We can use these form factors assuming isospin symmetry.

The heavy quark spin symmetry produces relations allowing to determine the tensor form factors. We obtain the finite mass form factors in term of Ω_1' and $a_0\ \Omega_2'$:

$$T_0'(q^2) = 2i\frac{(m_{B_c} + m_{B_1'})^2\sqrt{m_{B_1'}}}{m_{B_c}^{3/2}} a_0\,\Omega_2', \tag{11}$$

$$T_1'(q^2) = -\frac{m_{B_1'}}{m_{B_c}} T_2'(q^2), \tag{12}$$

$$T_2'(q^2) = -i\sqrt{\frac{m_{B_c}}{m_{B_1'}}}\Omega_1'. \tag{13}$$

Eq. (12) together with Eqs. (6)-(7) shows that the SD contribution for the positive-parity final state is suppressed due to a cancellation between two terms.

4 Long-distance amplitude

LD contributions correspond to processes involving intermediate hadrons. There are two kind of terms. The first one comes from the weak annihilation (WA) amplitude in Fig. 2 (top panel) with the photon radiated by any quark. The second type of contributions are pole terms induced by Eq. (1) with intermediate neutral vector mesons, as in Fig. 2 (bottom panel). For $B_c \to B_1'\gamma$ the intermediate ρ^0, ω and ϕ mesons are far from the kinematical range, therefore their contribution is suppressed respect to the case for $B_c \to B_u^*\gamma$. Let us focusing on B_1'.

4.1 WA with intermediate hadrons

The first weak annihilation diagram in Fig. 2 involves two terms. In the first one the B_c has weak transition to B, B_0 and B^*, coupled to B_1' and a photon:

$$\mathcal{A}^{B_c\to B_1'\gamma} = \mathcal{A}^{B_c\to B_{\rm res}}\frac{i}{\tilde{p}^2 - m_{B_{\rm res}}^2}\mathcal{A}^{B_{\rm res}\to B_1'\gamma}. \tag{14}$$

$B_{\rm res}$ is one of the off-shell states B, B_0 and B^*, the momentum $\tilde{p}$ is such that $\tilde{p}^2 = m_{B_c}^2$. $\mathcal{A}^{B_c\to B_{\rm res}}$ can be computed from the Hamiltonian (1) using factorization:

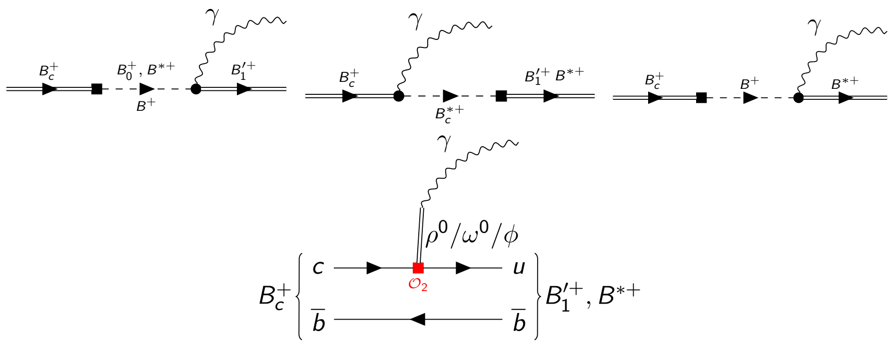

Figure 2. LD contribution to $B_c \to B_1'\gamma$ and B^*. Top panel: Weak annihilation contribution, where the box represent the insertion of a weak operator. Bottom panel: long-distance pole contribution for the B_1' and B^* cases.

$$\mathcal{A}^{B_c\to B_{\rm res}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} a_1 \langle B_{\rm res}| \bar{u}\gamma^\mu (1-\gamma_5) b |0\rangle \langle 0| \bar{b}\gamma_\mu (1-\gamma_5) c |B_c\rangle \tag{15}$$

where $a_1 = C_1 + \frac{C_2}{3}$. The matrix elements in the amplitude involves the decay constants of mesons in the process, f_{B_c},$f_{B_{\rm res}}$. The matrix elements of two mesons and the photon can be written in terms of effective couplings $g_{1,2,3}$, which can be determined by light-cone QCD sum rules (LCSR) in HQET [12]. Using Eq. (14), the LD contributions are given by

$$\mathcal{A}(B_c \to B \to B_1'\gamma) = i\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} a_1 f_{B_c} f_B \frac{m_{B_c}^2}{m_{B_c}^2 - m_B^2} e g_1 \left[(\lambda^* \cdot \epsilon^*)(p'\cdot q) - (\lambda^*\cdot p')(\epsilon^*\cdot q) \right], \tag{16}$$

$$\mathcal{A}(B_c \to B^* \to B_1'\gamma) = 0, \tag{17}$$

$$\mathcal{A}(B_c \to B_0 \to B_1'\gamma) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} a_1 f_{B_c} f_{B_0} \frac{m_{B_c}^2}{m_{B_c}^2 - m_{B_0}^2} e g_3 \varepsilon_{\alpha\beta\sigma\tau} \lambda^{*\alpha} \epsilon^{*\beta} \tilde{p}^\sigma q^\tau. \tag{18}$$

For the contribution of the second diagram in Fig. 2 the radiative emission from the B_c to the B_c^* is followed by the annihilation to B_1'. The photon emission amplitude can be parametrized as

$$\mathcal{A}^{B_c\to B_c^*\gamma} = \langle B_c^*(\tilde{p},\eta)\gamma(q,\lambda)|B_c(p)\rangle = ieg_4 \varepsilon_{\alpha\beta\sigma\tau}\lambda^{*\alpha}\eta^{*\beta}\tilde{p}^\sigma q^\tau, \tag{19}$$

with g_4 obtained from the prediction $\Gamma(B_c^* \to B_c\gamma) = 33\,\text{eV}$ [13]:

$$g_4^2 = \frac{24 m_{B_c^*}^3 \Gamma(B_c^* \to B_c\gamma)}{\alpha\left(m_{B_c^*}^2 - m_{B_c}^2\right)^3}. \tag{20}$$

For the transition to B_1' the amplitude reads:

$$\mathcal{A}^{B_c^*\to B_1'} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} a_1 \langle 0| \bar{b}\gamma_\mu (1-\gamma_5) c |B_c^*\rangle \langle B_1'| \bar{u}\gamma^\mu (1-\gamma_5) b |0\rangle, \tag{21}$$

with the decay constant $f_{B_c^*}$ and $f_{B'_1}$ parametrizing the matrix elements. The amplitude for the whole process is

$$\mathcal{A}(B_c \to \gamma B_c^* \to B'_1) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} a_1 f_{B_1} f_{B_c^*} \frac{m_{B_1} m_{B_c^*}}{m_{B_1}^2 - m_{B_c^*}^2} e g_4 \varepsilon_{\alpha\beta\sigma\tau} \lambda^{*\alpha} \eta^{*\beta} \tilde{p}^\sigma q^\tau. \tag{22}$$

4.2 Pole contribution

Even though the pole contributions should be suppressed, we examine their impact on the B'_1 channel.

The amplitude for $B_c \to B'_1 V \to B'_1 \gamma$, where V is one of the resonances, reads

$$\mathcal{A} = \mathcal{A}^{B_c \to B'_1 V} \frac{i}{q^2 - m_V^2 + i m_V \Gamma_V} \mathcal{A}^{V \to \gamma}, \tag{23}$$

where

$$\mathcal{A}^{B_c \to B'_1 V} = i \frac{G_F}{\sqrt{2}} a_2 V_{cD}^* V_{uD} m_V f_V \epsilon_V^{*\mu} \langle B'_1(p', \epsilon) | \bar{u} \gamma_\mu (1 - \gamma_5) c | B_c(p) \rangle, \tag{24}$$

$$\mathcal{A}^{V \to \gamma} = -ie \tilde{Q}_D m_V f_V \epsilon_V^\mu \lambda_\mu^*. \tag{25}$$

D refers to the kind of down quark in the process. For $D = d$ we have $\tilde{Q}_d = \frac{Q_d}{\sqrt{2}}$, for $D = s$ we have $\tilde{Q}_s = Q_s$. To evaluate the full amplitude in Eq. (23), we consider that from gauge invariance the longitudinal helicity amplitude must be discarded [14]. This condition is translated into a constraint on the form factor $V'_0(0) = 0$.

5 $B_c \to B_u^* \gamma$

To test NP in the radiative decay of B_c through the rare transition $c \to u\gamma$, the process $B_c \to B^* \gamma$ is the first one to study. The SD amplitude is given in Eqs. (6)-(7), after switching the parity conserving with parity violating expression and with suitable substitutions, $m_{B'_1} \to m_{B^*}$ and appropriate tensor form factors parametrizing the matrix element. The LD contributions for this channel are depicted in Fig. 2. Their evaluation follows the same steps as for the LD case of the B'_1 channel. The LD WA contribution with the B^+ resonance is evaluated following [4]. All matrix elements are parametrized as in [15], with the form factors for the $B_c \to B_d$ transition constructed using lattice QCD results and heavy quark spin symmetry. Isospin symmetry is also used.

6 Results

In Fig. 3 we compare the effect of the LD contributions and of the SD one in the B^* and B'_1 modes.
The LD contributions dominate in the small region of $|C_7^{eff}|$, which corresponds to the SM prediction for the Wilson coefficient. If some NP effect increases C_7^{eff}, it would be better observed in the 1^- channel.

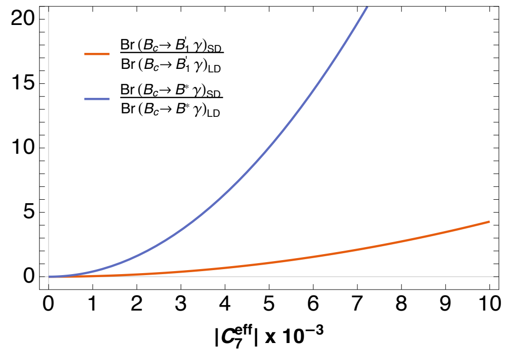

Figure 3. Comparison between the ratio due to SD and LD contributions to the rates of channels with B_1' and B^* as final states.

7 Conclusions

Motivated by previous analysis on the rare transition $c \to u\gamma$ in the B_c decays to B^*, we focus on the B_1' channel to study if it is less polluted by LD contributions.

From a model independent analysis based on heavy quark spin symmetry, we found the relation in Eq. (12) between the tensor form factor entering in the SD amplitude for the B_1' channel. Their combination in the amplitude produces a suppression of the SD contribution to the process. Therefore, while LD contributions affect more the B^* channel, the hadronic suppression in the B_1' case makes it less suitable for accessing NP.

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