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Recent developments in HQET

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Abstract. In this talk we review recent perturbative and non-perturbative developments in Heavy Quark Effective Theory (HQET).

1 Introduction

Heavy Quark Effective Theory (HQET) is an effective field theory for heavy quarks. It is most useful for cases where the quark mass m_Q is much larger than the QCD scale, namely, $m_Q \gg \Lambda_{\rm QCD}$. Using well-known transformations, see, e.g., [1, 2], we can obtain from the QCD Lagrangian the following Lagrangian:

$$\mathcal{L} = \bar{h}_v iv\cdot D h_v + \bar{h}_v i\not{D}_\perp \frac{1}{2m_Q + iv\cdot D} i\not{D}_\perp h_v\,, \tag{1}$$

where h_v is the heavy quark field and $D^\mu_\perp = D^\mu - (v\cdot D)v^\mu$. For $v = (1,\vec{0})$, $D^\mu_\perp = \vec{D}$. Expanding in powers of $iv\cdot D/2m_Q$ gives the dimension-five HQET Lagrangian:

$$\mathcal{L}_{\rm HQET} = \bar{h}_v iv\cdot D h_v - c_2 \bar{h}_v \frac{D_\perp^2}{2m_Q} h_v - c_F \bar{h}_v \frac{\sigma_{\alpha\beta}G^{\alpha\beta}}{4m_Q} h_v + O\left(\frac{1}{m_Q^2}\right). \tag{2}$$

Expanding to higher orders in $1/m_Q$ would give the HQET Lagrangian with tree-level Wilson coefficients. Starting at dimension seven there are other operators with $O(\alpha_s)$ Wilson coefficients, see [3], where the Lagrangian up to dimension seven was presented. The dimension-eight Lagrangian was presented in [4] and [5]. Ref. [4] also presented a method to easily construct local HQET operators of, in principle, any dimension.

Using HQET, observables can be written schematically as a series

$$\text{Observable} = \sum_{n=0}^{\infty}\sum_j c_n^j(\mu)\frac{\langle\, O_n^j(\mu)\,\rangle}{m_Q^n}\,, \tag{3}$$

where $\langle\, O_n^j(\mu)\,\rangle \sim \Lambda^n_{\rm QCD}$ and $\mu \sim m_Q$. The Wilson coefficients $c_n^j(\mu)$ are perturbative and the matrix elements $\langle\, O_n^j(\mu)\,\rangle$ are non-perturbative. Since $\alpha_s(\mu)$ becomes smaller for large μ and $\Lambda_{\rm QCD}/m_Q$ is small, we expect to achieve good precision with just a few terms. To improve the precision we can calculate $c_n^j(\mu)$ to higher powers in α_s and/or include $\langle\, O_n^j(\mu)\,\rangle$ with larger n, assuming we can extract them from data or calculate them using Lattice QCD. What $\langle\, O_n^j(\mu)\,\rangle$ do we usually encounter?

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Strong interaction operators are made of quarks and gluons. These can be local, e.g., $\bar{q}(0)\cdots q(0)$, or non-local, e.g., $\bar{q}(0)\cdots q(tn)$, where n is a light-cone vector ($n^2 = 0$). The general matrix element is of the form, $\langle f(p_f)|O_n^j(\mu)|i(p_i)\rangle$, where $O_n^j(\mu)$ can be local or non-local and p_i, p_f can be independent or not. Let us look at the most common options, ordered by increased complexity.

A local operator between the vacuum and a state gives rise to a number: decay constant, e.g., $\langle 0|\bar{q}\gamma^\mu\gamma_5 h_v|P(v)\rangle = -i\sqrt{m_P} f_P v^\mu$. Decay constants will appear in section 2. A diagonal matrix element of a local operator gives rise to a number: HQET parameter, e.g., $\langle\bar{B}|\bar{b}\,\vec{\boldsymbol{D}}^{\boldsymbol{2}}\,b|\bar{B}\rangle = 2M_B\mu_\pi^2$. HQET parameters will appear in section 2. A non-diagonal matrix element of a local operator gives rise to a function: form factor, e.g., $\langle D(p_f)|\bar{c}\gamma^\mu b|\bar{B}(p_i)\rangle = f_+(q^2)(p_i+p_f)^\mu + f_-(q^2)(p_i-p_f)^\mu$, where $p_f - p_i = q$. Form factors will appear in section 4. A non-local operator between the vacuum and a state gives rise to a function, e.g., LCDA (Light-Cone Distribution Amplitude): $\langle H_v|\bar{h}_v(0)\not{n}\gamma_5\,[0,tn]\,q_s(tn)|0\rangle = -iF(\mu)\int_0^\infty d\omega\, e^{i\omega t}\phi_+(\omega,\mu)$. LCDAs will appear in section 3. A diagonal matrix element of a non-local operator gives rise to a function, e.g., shape function: $S(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle\bar{B}(v)|\bar{b}(0)\,[0,tn]b(tn)|\bar{B}(v)\rangle/4\pi M_B$. Finally, non-diagonal matrix element of a non-local operator, e.g. $\langle K^{(*)}(p_f)|\bar{s}_L(0)\gamma^\rho\cdots\tilde{G}_{\alpha\beta}b_L(tn)|B(p_i)\rangle$, gives rise to a non-local form factor, see [6]. We will not discuss shape functions or non-local form factors here.

In the following we review recent perturbative and non-perturbative developments in HQET. Recent is defined to be the period of Spring 2022-Spring 2024. To allow for a broad overview, only some aspects of the papers are highlighted. More details can be found in the original papers. The rest of the talk is structured as follows. Section 2 discusses recent perturbative developments. Section 3 discusses recent developments related to LCDAs. Section 4 discuses recent developments related to form factors. Section 5 discusses other recent developments. We conclude in section 6.

2 Recent developments in HQET: Perturbative

As mentioned in the introduction, we can improve theoretical predictions by calculating c_n^j to higher orders in α_s. Moreno calculated $c_3^{\rm Darwin}$ to $O(\alpha_s)$ for $\bar{B}\to X_u\,\ell\,\bar{\nu}_\ell$ total rate and leptonic invariant mass in [7]. In some cases the "technology" improved to $O(\alpha_s^4)$. For example, Takaura used the four-loop relation between the pole and $\overline{\rm MS}$ masses to extracted HQET parameters from B and D meson masses [8]. Lee and Pikelner calculated the four-loop HQET propagator in [9]. This four-loop calculation was used by Grozin to find the four-loop HQET heavy to light anomalous dimension [10]. Looking at [10] in more detail, the anomalous dimension can be used to calculate the ratio of the B meson and D meson decay constants f_B/f_D. Including the four-loop calculation Ref. [10] finds

$$\begin{aligned}\frac{f_B}{f_D} &= \sqrt{\frac{m_D}{m_B}}\left(\frac{\alpha_s^{(4)}(m_c)}{\alpha_s^{(4)}(m_b)}\right)^{-\frac{\tilde{\gamma}_{j0}}{2\beta_0^{(4)}}}\left\{1+\cdots\alpha_s+\cdots\alpha_s^2+\cdots\alpha_s^3+[\sim 1\mathrm{GeV}]\left(\frac{1}{m_c}-\frac{1}{m_b}\right)+\cdots\right\}\\ &= 0.669\cdot(1+0.039+0.029+0.032+[\sim 0.46])\,,\end{aligned}\tag{4}$$

where the last term includes an estimate of power corrections. Ref. [10] comments on this equation that "Convergence of the perturbative series is questionable [...]". Numerically, without power corrections $f_B/f_D = 0.736$, and with power corrections $f_B/f_D = 1.04$. Comparing these to the lattice QCD value, $f_B/f_D = 0.896\pm 0.009$, Ref. [10] concludes that "The effect of the (poorly known) $1/m_{c,b}$ correction is large."

3 Recent developments in HQET: Non-local matrix elements

Non local matrix elements arise in many processes, e.g., the proton parton distribution function in hard QCD processes. In B decays such as $B \to K^*\gamma$ we encounter the B-meson LCDA. It is the Fourier transform of $\langle B|\bar{b}(0)\dots[0,tn]q_s(tn)|0\rangle$. B-meson LCDA also arises when the B meson is in the *final* state, see, e.g., in the decay $W, Z \to B + \gamma$ [11]. Such processes were recently considered in [12] ($W^\pm \to B \pm \gamma$) and [13] ($W^+ \to B^+\ell^+\ell^-$).

Beneke, Finauri, Vos, and Wei considered the process $W^\pm \to B \pm \gamma$ in [12] . It has three scales: hard scale $Q \gg$ heavy quark scale $m_Q \gg$ QCD scale $\Lambda_{\rm QCD}$. In [12] the QCD LCDA:

$$\langle H(p_H)|\bar{Q}(0)\not{n}\gamma^5[0,tn]q(tn)|0\rangle = -if_H n\cdot p_H \int_0^1 du\, e^{iutn\cdot p_H}\phi(u;\mu)\,, \tag{5}$$

was matched to a perturbative function convoluted with HQET LCDA:

$$\langle H_v|\bar{h}_v(0)\not{n}\gamma^5[0,tn]q_s(tn)|0\rangle = -iF_{\rm stat}(\mu)\, n\cdot v \int_0^\infty d\omega\, e^{i\omega tn\cdot v}\varphi_+(\omega;\mu)\,. \tag{6}$$

Such a factorization allows to resum large logs between $\Lambda_{\rm QCD}$ and m_Q and m_Q and the hard scale Q. This paper also consider the evolution of the LCDA. Starting with HQET LCDA at soft scale $\mu_s = 1$ GeV, it is evolved in HQET to the matching scale μ and matched to $\phi(u)$. It is then evolved in QCD to the hard scale m_W, see figure 6 of [12]. The branching ratio they find is [12]

$$\mathrm{Br}(W \to B\gamma) = (2.58 \pm 0.21_{\rm in}\,{}^{+0.05}_{-0.08}{}_{\mu_h}\,{}^{+0.05}_{-0.08}{}_{\mu_b}\,{}^{+0.18}_{-0.13}{}_{\delta}\,{}^{+0.61}_{-0.34}{}_{\beta}\,{}^{+2.95}_{-0.98}{}_{\lambda_B})\cdot 10^{-12}\,. \tag{7}$$

The uncertainty from the low-scale HQET LCDA parameters λ_B, β is large.

Ishaq, Zafar, Rehman, and Ahmed considered the process $W^+ \to B^+\ell^+\ell^-$ in [13]. Using the scale hierarchy $m_W \sim m_b \gg \Lambda_{\rm QCD}$ they factorize the amplitude as

$$\mathcal{M} = e\,\bar{\ell}\gamma^\mu\ell \int_0^\infty d\omega\, T_\mu(\omega, m_b, q^2, \mu_F)\Phi_B^+(\omega,\mu_F) + O\left(m_b^{-1}\right), \tag{8}$$

where T_μ is the perturbative hard-scattering kernel. Calculating T_μ at $O(\alpha_s)$ they find "[...] the scale dependence in the NLO decay rates [...] gets largely reduced, particularly for relatively large μ_F." See figure 5 of [13]. The theoretical prediction for the branching ratio is $\sim 10^{-11}$ for electrons and muons. It is sensitive to the hadronic parameter λ_B defined as

$$\frac{1}{\lambda_B} \equiv \int_0^\infty \frac{d\omega}{\omega}\phi_B^+(\omega)\,. \tag{9}$$

See figure 6 of [13]. Ref. [13] uses $\lambda_B = 0.35 \pm 0.15$ GeV. Searching for $B^+ \to \ell^+\nu_\ell\gamma$ Belle got $\lambda_B = 0.36^{+0.25}_{-0.09}$ GeV [14]. Ref. [13] concludes that observing $W^+ \to B^+\ell^+\ell^-$ at the LHC could help constrain λ_B.

4 Recent developments in HQET: Local non-diagonal matrix elements

In the SM, $B \to D$ transitions are described by two form factors, and $B \to D^*$ transitions are described by four form factors. At leading power in the heavy quark symmetry *all* of these form factors are described by one universal Isgur-Wise function ξ. Including $1/m_c$ & $1/m_b$ power corrections there are three additional functions and the number grows rapidly at higher powers. Including $1/m_c^2$ corrections, there are additional 20 functions and including $1/m_c^2$ & $1/m_b^2$ there are additional 32 functions, see Table 1 of [15].

Bernlochner, Ligeti, Papucci, Prim, Robinson, and Xiong, suggested in [16] supplemental power-counting that reduces these numbers. The QCD Lagrangian before the $1/m_Q$ expansion is given in equation (1). The postulated power counting is in powers of $i\not{D}_\perp$: currents involve one $i\not{D}_\perp$, Lagrangian insertions involves two $i\not{D}_\perp$'s. Many subleading contributions arise from Lagrangian insertions. Ref. [16] conjectures that terms entering at third order or higher should be suppressed and calls this residual chiral (RC) expansion. Under RC expansion including $1/m_c^2$ corrections, there is one additional function and including $1/m_c^2$ & $1/m_b^2$ there are additional 3 additional functions, see Table 1 of [15]. Even more recently, Bernlochner, Papucci, and Robinson, applied the same method to $\Lambda_b \to \Lambda_c l \nu$ decay [17].

Turning to phenomenology recall that

$$R(D^{(*)}) \equiv \text{Br}(\bar{B} \to D^{(*)}\, \tau\, \bar{\nu}_\tau)/\text{Br}(\bar{B} \to D^{(*)}\, \ell\, \bar{\nu}_\ell), \quad \ell = e, \mu \tag{10}$$

Ref. [16] obtained $R(D) = 0.288(4)$. This should be compared to HFLAV 2024 Standard Model (SM) prediction of $R(D) = 0.298(4)$, and value from experiment of $R(D) = 0.342(26)$ [18]. Ref. [16] obtained $R(D^*) = 0.249(3)$. This should be compared to HFLAV 2024 SM prediction of $R(D^*) = 0.254(5)$, and value from experiment of $R(D^*) = 0.287(12)$ [18]. Clearly the tension between experiment and theory remains for both theoretical predictions.

5 Recent developments in HQET: Other topics

Manzari and Robinson suggested a new theoretical framework for heavy quark resonances in [19]. The new framework uses on-shell recursion techniques to express resonant amplitude as a product of on-shell sub-amplitudes. In figure 5 of [19] the authors present a toy example calculation in this framework and a fixed-width Breit-Wigner side by side with Belle data for D_2^* resonance. From the figure the preliminary results of Ref. [19] seem promising.

Garg and Upadhyay studied F-wave B mesons in HQET in [20]. F-wave B mesons have angular momentum $L = 3$. Adding $L = 3$ to the heavy-quark spin $s_Q = 1/2$ gives 7/2 and 5/2 angular momenta. Adding the light quark spin 1/2 to 5/2 gives a $J = 2$ and $J = 3$ doublet. Adding the light quark spin 1/2 to 7/2 gives a $J = 3$ and $J = 4$ doublet. Thus the spectrum contains two doublets. Ref. [20] used information from, e.g., D mesons, to calculate B meson properties. For example, in table 2 of [20] the authors compare the calculated masses to two quark models.

Vishwakarma and Upadhyay presented an analysis of 2S singly-heavy baryons in HQET in [21]. They used information from measured 2S baryons: $\Xi_c(2970)$ and $\Lambda_b(6070)$, and HQET to calculate 2S baryon properties. For example, in table 7 of [21] the authors compare the calculated masses to two quark models.

6 Conclusions

Arguably, this year marks the 35th anniversary of HQET, based on the date of the publication of "Weak Decays of Heavy Mesons in the Static Quark Approximation," by Isgur and Wise [22]. This paper includes the line "This logarithm can be displayed explicitly by going over to an effective theory where the heavy quark is treated as a static color source." Note the words "effective theory".

HQET is a mature field where some perturbative corrections are known to fourth order, and some non-perturbative power corrections are known to fourth order. In this talk we reviewed recent developments in HQET both on the perturbative side and the non-perturbative side. Theoretical progress in the field mirrors the experimental progress. It is clear from several of the papers discussed in this talk that non-perturbative effects often dominate the uncertainties. Controlling them might require further developments in HQET.

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