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Low energy description of single flavor baryons

Francesco Bigazzi[1,*], *Aldo L.* Cotrone[1,2,**], and *Andrea* Olzi[1,2,***]

[1]INFN, Sezione di Firenze, Via G. Sansone 1, I-50019 Sesto Fiorentino (Firenze), Italy
[2]Dipartimento di Fisica e Astronomia, Universitá di Firenze, Via G. Sansone 1, I-50019 Sesto Fiorentino (Firenze), Italy

Abstract. In single-flavored large N QCD, the standard low-energy description of baryons in terms of Skyrmions is no longer available. Recently it has been proposed that the correct low-energy description in this case is in terms of a pancake-shaped sheet, a quantum Hall droplet. We will describe how this proposal can be made concrete in Holographic QCD. We present the brane configuration describing the sheet and an approximate solution with the expected physical properties.

1 No Skyrmion with one flavor

The usual low-energy description of pions and baryons for N_f quark flavors is in terms of the pion matrix

$$U = \exp\left(i \sum_{a=1}^{N_f^2-1} \frac{\pi^a(x) T^a}{f_\pi} \right) , \tag{1}$$

where π^a are the pion fields, T_a the generators of $SU(N_f)$ and f_π is the pion decay constant. The (effective) Chiral Lagrangian is

$$\mathcal{L}_{\text{eff}} = -\frac{f_\pi^2}{4} Tr[\partial_\mu U \partial^\mu U^\dagger] + \frac{1}{32e^2} Tr\left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U\right]^2 , \tag{2}$$

where e is a coupling constant. The second piece, called the Skyrme term, allows for the description of baryons as solitons of this Lagrangian [1], e.g. for $N_f = 2$ one has the standard "Skyrmion"

$$U = \exp\left(i \frac{f(r)}{r} x^i \sigma^i \right) , \tag{3}$$

where x^i are the target space coordinates, $r^2 = \sum_i (x^i)^2$, σ^i are the Pauli matrices and $f(r)$ is a known function. It can be shown that if $f(0) = \pi k$ with $k \in \mathbb{Z}$, then the solution has winding number k for $\Pi_3(SU(N_f = 2)) = \mathbb{Z}$, where the three-sphere of Π_3 is in coordinate space. This is interpreted as the baryon number.

The problem with this description is that $\Pi_3(U(N_f = 1)) = 0$. In other words, the Skyrme term trivializes if $N_f = 1$. So, there are no Skyrmions with one flavor. Of course, baryons made by one quark flavor do exist, so the problem is with their correct description at low energies.

*e-mail: bigazzi@fi.infn.it
**e-mail: cotrone@fi.infn.it
***e-mail: andrea.olzi@unifi.it

2 A proposal for the solution

Recently Komargodski, following previous studies [2–6], proposed that in single-flavor large N QCD, baryons can be seen as charged sheets [7]. With a single flavor, the low energy Lagrangian is just the Lagrangian for the equivalent of the η' field (which is light when the number of colors N is large) [8]

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial\eta')^2 - \frac{1}{2}m\Lambda_{\text{QCD}}\cos(\eta') - \frac{1}{2}m_{WV}^2 \text{Min}_{k\in\mathbb{Z}}(\eta' + 2\pi k)^2 \,. \tag{4}$$

In this expression m is the quark mass, Λ_{QCD} the QCD dynamical scale, and m_{WV} the Witten-Veneziano mass. The potential has a cusp singularity behavior at $\eta' = \pi$, where extra light gluonic degrees of freedom are localized. This locus is called the "sheet". The η' has non-trivial monodromy through the sheet. The infinitely extended sheet is similar to the standard QCD domain-wall at $\theta_{YM} = \pi$, but with no associated charge, so it is unstable or, more precisely, metastable at large N. But, crucially, it can be stabilized by the baryonic charge.

In fact, the sheet (as the domain wall) can be argued to host a $U(1)_N$ Chern-Simons theory on its world-volume, so it is a "quantum Hall droplet". Then, if it has a boundary, it can have a chiral edge mode carrying baryonic charge, see figure 1. If this is the case, the charge forbids the complete decay of the "pancake-shaped" sheet, since this is the lightest particle which carries one unit of charge.

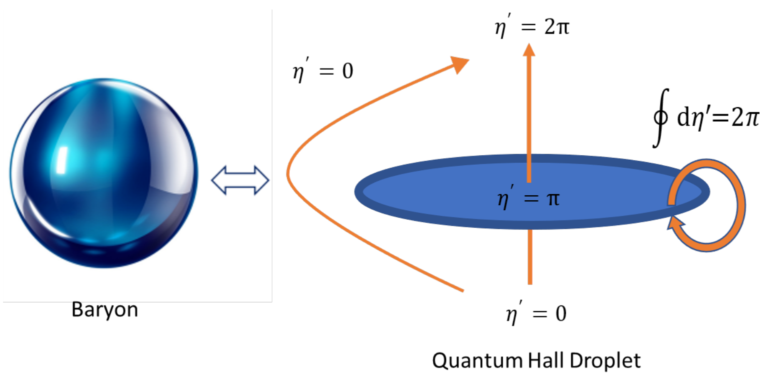

Figure 1. In the case of one flavor, the baryon is really a quantum Hall droplet of pancake shape. The η' field has a non-trivial monodromy across the droplet world-volume.

The crucial question is: can we test the proposal and compute properties of this baryon? In the next section we present some progress in this direction in a theory similar to QCD, known as "Holographic QCD" or the "Witten-Sakai-Sugimoto model".

3 Realization in Holographic QCD

Holographic QCD [9, 10] corresponds to a Type IIA background in string theory, generated by N D4-branes wrapped on a circle, with N_f D8/anti-D8-brane pairs in the probe approximation (that is, not backreacting on the background). At low energies, this configuration is dual to a $SU(N)$ non-supersymmetric four-dimensional Yang-Mills theory coupled to a tower of Kaluza-Klein modes in the adjoint representation and N_f chiral quark flavors. It is similar to (planar) QCD, showing confinement, mass gap, and chiral symmetry breaking. Its vacuum structure is the same as in QCD. Baryons in this theory can be described as non-Abelian instantonic configurations of the gauge field on the D8-branes [11], but only for $N_f \geq 2$. So, again: how do we describe the sheet, that is the baryon in a single-flavored holographic QCD?

In [12] it was realized that the behavior of η' through the sheet is reproduced if a D6-brane describes the latter (see also [13]). In fact, the η' corresponds to a fluctuating mode of the D8-brane gauge field. The action (4) can be derived as the low energy limit of the D8-brane action (following the steps in [14]), and its coupling with a D6-brane implies a 2π monodromy when the field configuration crosses the brane. Moreover, the D6-brane, upon reduction on a four-sphere of the background, hosts a $U(1)_N$ Chern-Simons theory on its world-volume [15].

So, the sheet is realized as a D6-brane in this theory. The quantum Hall droplet with a circular boundary corresponds to a D6-brane attached to the D8-brane, as depicted in figure 2. The D6-brane constitutes the gluonic core of the baryon. This is the equivalent of the string

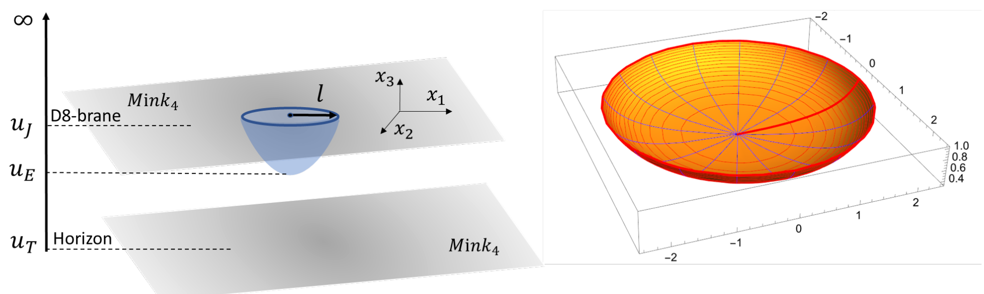

Figure 2. Left: a cartoon of the D6-brane describing the gluonic core of the baryon hanging from the D8-brane in the deconfined, chiral symmetry broken phase. Right: the actual solution for the D6-brane shape from solving the equations of motion coming from the Dirac-Born-Infeld action.

junction at the core of the baryon, where the chromoelectric flux tubes (strings) attached to the quarks meet.

In the following, we are going to describe the baryon in the simplest setting, corresponding to the deconfined phase of the quantum field theory but still in the chiral symmetry breaking phase - a possibility present in Holographic QCD [16]. The presence of a horizon in the background, corresponding to the deconfined phase, implies some simplifications in the equations [17].

We would like to describe the quantum Hall droplet. The profile of the D6-brane ending on the D8 brane in the background can be calculated numerically by solving the equations of motion coming from the Dirac-Born-Infeld action, which describes the low-energy physics of the D-branes. A representative profile is given by the plot on the right of figure 2. The profile is immersed in a three-dimensional space because of the holographic radial direction, keeping track of the energy scale - the dual quantum Hall droplet is two-dimensional.

Calculating the energy of this solution one finds out that it grows linearly for "large" radius l, and quadratically for small l. Nevertheless, the configuration cannot have an arbitrarily large radius in the deconfined phase: if l is too large, the bottom of the D6-brane falls into the horizon, transitioning to a cylinder-shaped configuration. The dual sheet is no longer a pancake, but an annulus. Physically, this is dual to a circular global string, or a so-called "vorton" in the charged case. Its energy grows linearly with l for every value of l. It can be shown that these two embeddings are separated by a first-order phase transition [17].

The D6-brane profile is not the end of the story. A D6-brane ending on a D8-brane is a magnetic source for the D8-brane gauge field F describing the η' and, crucially, the charge allowing for a stable configuration. The gauge field on the D8-brane describes the mesonic content of the baryon. So, the D8-brane gauge field is necessarily turned on. The profile of its components is again dictated by the DBI action, this time for the D8-brane. As customary, at leading order at low energies, the equations of motion reduce to electromagnetism (remember that $N_f = 1$). Since also the D8-brane is wrapped on a four-sphere of the background, the theory at low energies is five-dimensional. Electromagnetism in five dimensions admits a Chern-Simons coupling. Considering that the background is non-trivial, altogether one faces a problem in the five-dimensional Maxwell-Chern-Simons theory on curved space.

The equations for the charged sheet are a coupled system of non-linear partial differential equations. In fact, the symmetry of the problem dictates that the solution will depend on the radial direction ρ in the plane of the sheet, on the transverse direction in Minkowski space, which is denoted as x_3, and on the holographic radial direction, which we will call z. Needless to say, these equations are extremely hard to solve even numerically.

3.1 An approximate solution

An approximate solution, along the lines of [11], can be found if one assumes that the droplet is extremely small, i.e. its radius l scales as a negative power of the large 't Hooft coupling λ.[1]

The D6-brane intersects the D8-brane at its tip at $z = 0$, where it is a good approximation to consider the background as flat. In the flat space limit the symmetry in the x_3, z plane is restored, such that the dependence is just on the radius r in this plane. The set of equations "simplify" to

$$\begin{aligned}
&\rho\partial_r(rF_{tr}) + r\partial_\rho(\rho F_{t\rho}) - \tau(F_{\rho\psi}F_{r\theta} - F_{r\psi}F_{\rho\theta}) = 0\,,\\
&\frac{1}{\rho}\partial_r(rF_{r\psi}) + r\partial_\rho\left(\frac{1}{\rho}F_{\rho\psi}\right) - \tau/2(F_{tr}F_{\rho\theta} - F_{t\rho}F_{r\theta}) = 0\,,\\
&\frac{1}{r}\partial_\rho(\rho F_{\rho\theta}) + \rho\partial_r\left(\frac{1}{r}F_{r\theta}\right) - \tau/2(F_{t\rho}F_{r\psi} - F_{tr}F_{\rho\psi}) = 0\,,\\
&dF = -\sqrt{2}\delta(r)\delta(\rho - l)d\rho \wedge dr \wedge d\theta\,,
\end{aligned} \tag{5}$$

where $\tau \sim 1/\lambda$ is a constant and ϕ, θ are the angular coordinates in the plane of the sheet and the (x_3, z)−plane. The last equation is the violation of the Bianchi identities due to the circular source (the D6-brane boundary on the D8-brane).

Even the flat-space system (5) is not easily solved because the fields diverge close to the source as usual in electromagnetism. However, assuming that the droplet is extremely small, it admits an approximate solution with the desired properties:

- It is an equilibrium configuration, of size

$$l_{\rm eq} \sim \sqrt{\frac{\Lambda_{QCD}}{\lambda T^3_{\chi SB}}}\,, \tag{6}$$

  where Λ_{QCD} is the dynamical scale of the theory and $T_{\chi SB}$ is the temperature of the chiral symmetry breaking transition. As expected, $l_{\rm eq}$ does not depend on N and scales as a negative power of λ.

[1] More precisely, λ is a 't Hooft-like parameter proportional to the ratio between the confining string tension and the square of the Kaluza-Klein mass scale. The holographic description we are adopting is reliable when $\lambda \gg 1$.

- Its mass is

$$M \sim \frac{\lambda^2 N T_{\chi SB}^3}{\Lambda_{\rm QCD}^2}\,, \tag{7}$$

with the correct scaling with N for a baryon in the planar limit.

- It has a unit baryon number

$$n_B = \frac{1}{12\pi^2}\int d^4x\,(F_{\rho\phi}F_{r\theta} - F_{r\phi}F_{\rho\theta}) = 1\,. \tag{8}$$

- It has spin $N/2$ (for a single flavor all the quark spins are aligned).

Nevertheless, this solution is based on a scaling assumption about the fields and, most importantly, the cutoff close to the source.

4 Conclusions

In conclusion, we presented a (partial) concrete holographic realization of Komargodski's proposal for the low-energy description of single-flavor baryons and computed its basic properties. Calculating the corresponding numeric solution could corroborate and improve the approximated solution. We would also like to compute other observables associated with this baryon and extend the solution to the confined phase. Finally, it is worth mentioning that this construction has applications to axionic dark matter [18, 19], and that other studies related to the sheet appeared in [20–30].

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