Hadronic contribution to the muon $g − 2$: Theoretical challenges with the MUonE experiment

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Abstract. After briefly reviewing the present status on the theoretical predictions for the hadronic contribution to the muon anomalous magnetic moment, the MUonE approach is introduced, with a discussion on the challenges for the needed theoretical predictions.

1 Introduction

The muon magnetic moment anomaly $a_\mu = (g_\mu − 2)/2$ is a fundamental observable in particle physics, which allows to test the structure of the Standard Model (SM) as a local Quantum Field Theory with unprecedented accuracy [1]. Its present experimental value [2], known with a precision of 0.19 ppm, stems from the average of the final result of the E821 experiment at the Brookhaven National Laboratory [3] with the Run−1 and Run−2/3 measurements at the Fermilab Muon $g − 2$ experiment [2, 4]. The SM prediction for a_μ, as compiled by the Muon $g − 2$ Theory Initiative in 2020 [5], yields a discrepancy of the order of 5.0σ with the up-to-date experimental result, which would be an evidence of New Physics beyond the SM, provided all sources of systematic error are under control. The latter include also the uncertainty of the SM theoretical prediction for a_μ, which is dominated by non-perturbative contributions, such as the Leading Order Hadronic Vacuum Polarization (HVP,LO) contribution $a_\mu^{\text{HVP,LO}}$ and the Hadronic Light-by-Light contribution a_μ^{LxL}, the latter being at present subleading with respect to the former [5]. Traditionally two independent methods are used to calculate $a_\mu^{\text{HVP,LO}}$, the data-driven approach based on dispersive relations and ab-initio Lattice QCD simulations.

In the data-driven method, thanks to their analyticity and unitarity properties, loop integrals containing insertions of hadronic vacuum polarization in the photon propagators can be calculated as dispersion integrals over the cross section of a virtual photon decaying into hadrons [1, 5]. In particular, $a_\mu^{\text{HVP,LO}}$ can be expressed as

$$a_\mu^{\text{HVP,LO}} = \frac{\alpha^2}{3\pi^2} \int_{m_\pi^2}^{\infty} \frac{K(s)}{s} R(s)\, ds \,, \qquad (1)$$

where $K(s)$ is a kernel function and $R(s)$ is the hadronic R−ratio defined as

$$R(s) = \frac{3s}{4\pi\alpha^2} \sigma^0(e^+ e^- \rightarrow \text{hadrons}(+\gamma)) \,. \qquad (2)$$

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The hadronic cross section $\sigma^0(s)$ in Eq. (2) should not include the effect of the vacuum polarization effect, to avoid double counting. In the integral of Eq. (1), in particular in the region below the charm and bottom resonances, where QCD perturbation theory becomes not reliable, the cross section entering $R(s)$ is taken from the measurements of the hadronic cross section at flavour factories, through a direct energy scan, where the collider center of mass (c.m.) energy is changed over an allowed energy window, or through the radiative return method, where the collider c.m. energy is fixed at the nominal value and the emission of an additional photon is exploited to explore a di-pion invariant mass window below the c.m. energy. The problem of calculating $a_\mu^{\text{HVP,LO}}$ is shifted to the combination of several measurements corresponding to several hadronic final states in the integral of Eq. (1). Different approaches for the calculation of the time-like dispersion integral are available in the literature [6–8]. Even if the estimated precision of the predictions are at the few 0.1% level and in mutual agreement, the accuracy of this method is complicated by the existence of several resonances in the low-energy cross section. Furthermore, the CMD-3 collaboration has recently published a new measurement of the $e^+e^- \to \pi^+\pi^-$ cross section [9, 10], which disagrees with the results of previous experiments [11–13], pointing out some tension among different estimates of the pion form factor [1].

The method based on LQCD amounts to numerically calculate [1, 5]

$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 \, f(Q^2) \left\{\Pi(Q^2) - \Pi(0)\right\} , \tag{3}$$

where the vacuum polarization function $\Pi(Q^2)$ is related to the Fourier transform of the electromagnetic current correlator

$$\Pi_{\mu,\nu}(Q) = \int d^4x \, e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0) \rangle = (Q_\mu Q_\nu - \delta_{\mu\nu} Q^2) \Pi(Q^2) , \tag{4}$$

J_μ is the current in terms of quark fields, $J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \cdots$, and $f(Q^2)$ is an appropriate kernel function. Different ways of performing the volume and momentum integration of the above Eqs. (3) and (4) with lattice regularization characterize different approaches to $a_\mu^{\text{HVP,LO}}$ by various LQCD collaborations with corresponding results. Until recently the theoretical uncertainties associated to the LQCD approaches by different collaborations were estimated at the few % level, not competitive with the time-like dispersive approach. During 2020 the BMW collaboration published the first LQCD result for $a_\mu^{\text{HVP,LO}}$ with estimated accuracy below the % level [14], with further improvement in [15], in strong tension with the data-driven determinations and in substantial agreement with the SM. All the present tensions among predictions based on different methods suggest possible inaccuracies in the SM prediction of a_μ, preventing a reliable comparison with the experimental value.

2 Calculating a_μ^{HVP} with space-like data in the MUonE experiment

A novel and independent data-driven approach to evaluate a_μ^{HVP} has been proposed in [16]. The method is based on the measurement of the hadronic contribution to the running of the QED coupling $\Delta\alpha(t)$ in the space-like region $t < 0$, where the LO HVP contribution is a smooth function [17] [2]:

$$a_\mu^{\text{HVP,LO}} = \frac{\alpha}{\pi} \int_0^1 dx \, (1 - x) \, \Delta\alpha_{\text{had}}[t(x)] , \tag{5}$$

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[1]We remark that the weight of the $e^+e^- \to \pi^+\pi^-$ process in the time-like dispersion integral is more than 70%.

[2]The kernel function $\frac{\alpha}{\pi}(1 - x)$ refers to the LO HVP contribution. Recently, also the Next-to-Leading Order (NLO) [18, 19] and the Next-to-Next-to-Leading Order (NNLO) [18] expressions for the kernel function have been computed.

where $\Delta\alpha_{\text{had}}(t)$ is the hadronic contribution to the running of the QED coupling, evaluated at the space-like squared four-momentum transfer:

$$t(x) = -\frac{x^2 m_\mu^2}{1-x} < 0\,. \tag{6}$$
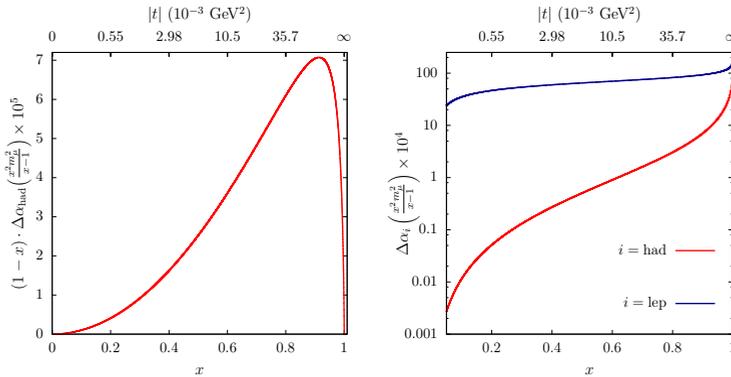
The running QED coupling is expressed as:

$$\alpha(t) = \frac{\alpha(0)}{1 - \Delta\alpha(t)}, \tag{7}$$

where $\alpha(0) = \alpha$ is the fine-structure constant, and

$$\Delta\alpha(t) = \Delta\alpha_{\text{lep}}(t) + \Delta\alpha_{\text{had}}(t) + \Delta\alpha_{\text{top}}(t)\,. \tag{8}$$

The hadronic contribution $\Delta\alpha_{\text{had}}(t)$ can be extracted from data of a scattering experiment by subtracting from $\Delta\alpha(t)$ the purely leptonic part $\Delta\alpha_{\text{lep}}(t)$, which can be calculated to very high precision in QED, and the top-quark contribution, which can be reliably calculated in perturbation theory.

Based on the method [16], the MUonE experiment has been proposed [20], with the aim of measuring $\Delta\alpha_{\text{had}}(t)$ from μe scattering at low energy, through the CERN M2 muon beam with energy $E_\mu \sim 160$ GeV off atomic electrons of a light target. The integrand of Eq. (5) is shown as a function of x in the left panel of Fig. 1 while the comparison between hadronic and leptonic contribution is shown on the right panel. In principle, this method could reach a

Figure 1. Left: the integrand $(1 - x)\Delta\alpha_{\text{had}}[t(x)] \times 10^5$ as a function of x and t. Right: $\Delta\alpha_{\text{had}}[t(x)] \times 10^4$ (red) and, for comparison, $\Delta\alpha_{\text{lep}}[t(x)] \times 10^4$ (blue), as a function of x and t (upper scale). From [20].

competitive precision below 0.5% on $a_\mu^{\text{HVP,LO}}$, provided the systematic errors are kept under control. After the original proposal, a Letter-of-Intent [21] has been submitted to the CERN SPS Committee in 2019. A detailed description of the proposed apparatus can be found in [21, 22].

In a simplified approach at Leading Order (LO) accuracy, the MUonE experimental data on $\frac{d\sigma}{dt}$ can be related to $\Delta\alpha_{\text{had}}(t)$ through the relation
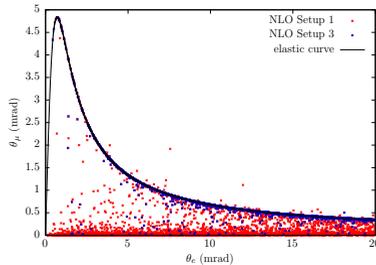
$$R_{\text{had}}^{LO}(t) = \frac{d\sigma^{LO}(\Delta\alpha_{\text{had}}(t) \neq 0)}{d\sigma^{LO}(\Delta\alpha_{\text{had}}(t) = 0)} \simeq 1 + 2\Delta\alpha_{\text{had}}(t)\,, \tag{9}$$

where the numerator of the ratio is intended to be taken from data, while the denominator should rely on Monte Carlo simulations. In the range $0 < |t| < |t_{max}| = 0.175$ GeV^2, the

hadronic contribution $\Delta\alpha_{\text{had}}(t)$ has a tiny variation from 0 to about 10^{-3} at the peak of the integrand of Eq. (5). A competitive determination of $a_\mu^{\text{HVP,LO}}$ requires a precision of $O(10^{-2})$ in the measurement of the hadronic running, which translates into an unprecedented precision of $O(10^{-5})$ in the shape of the differential cross section. Reaching this accuracy requires a huge statistics of data, in the order of few times 10^{12} events. More details on the MUonE data analysis strategies can be found in [21, 22]. Indipendent procedures to determine $a_\mu^{\text{HVP,LO}}$ with MUonE data has been proposed in [23–25].

3 Theoretical calculations for elastic muon-electron scattering

The MUonE precision goal requires a theoretical computation of the differential cross section for $\mu^\pm e^- \to \mu^\pm e^-$ with an accuracy of 10 ppm on the shape of differential cross sections. As a consequence, QED corrections have to be included with the highest available precision. In fact, just moving from LO to NLO accuracy in QED, the exclusive description of the events receives important corrections. For instance, the correlation between muon and electron scattering angles of the elastic process is spoiled, as can be observed in Fig. 2. A comprehensive

Figure 2. The correlation between the electron scattering angles θ_e and θ_μ for the process $\mu^+e^- \to \mu^+e^-$ at LO and NLO QED, for two different event selections. From [36].

analysis of the issues in the calculation of the radiative corrections to μe scattering at MUonE has been presented in [26]. In order to match the extreme precision requirements of MUonE, the calculation must include QED radiative corrections at least at NNLO. These have been computed, with different approximations, by two groups with indipendent methods for dealing with the infrared singularities [27–33]. Also NNLO hadronic contributions have been computed in [34, 35]. The calculations have been implemented in two fully differential Monte Carlo (MC) generators, Mesmer [27, 28, 36] and McMule [29, 37], which are currently under development. The LO Z-exchange contribution is relevant at the 10^{-5} scale on distributions while the NLO weak corrections have been evaluated in [36] and found to be below the 10^{-6} scale. As a general rule, the NNLO corrections are of the order of 10^{-4}, so they are not enough in view of the final MUonE precision. In this respect, a consistent matching of resummed multi-photon effects (described by the Parton Shower and/or the YFS techniques) with fixed order calculations at NLO and at NNLO accuracy will be required. Another challenging approach is the calculation of the corrections at the next order in QED, i.e. with the so-called N3LO accuracy, along the electronic line. In fact, for a sufficiently inclusive event selection, the radiation from the electronic line is by far the dominant component at NLO and NNLO. Important results for the needed building blocks have been already achieved [38–41].

4 Theoretical calculations for backgrounds

In addition to the calculation of radiative corrections to the elastic process, the impact of the possible background processes must be precisely evaluated. In this regard, the MESMER event generator has been extended to include the simulation of $\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$ with $\ell = \{e, \mu\}$ [28] and $\mu^\pm e^- \to \mu^\pm e^- \pi^0$ [42]. While the latter process has been found to be completely negligible, the former processes can be considered as part of the NNLO corrections to the signal. The real emission of an additional $e^+ e^-$ pair gives very large contributions, which can be mitigated by appropriate event selections, as shown in [28]. However, since the initial-state electrons are bound in a low-Z atomic target (beryllium-9 or carbon-12), the main source of experimental background is given by the muon-nucleus scattering with the emission of an addition leptonic pair. This process is particularly important because it can resemble an elastic event if one of the final leptons is not detected. Moreover, its cross section scales with the atomic number Z^2 while the muon-electron scattering scales with Z. Considering that the approximation contained in the GEANT4 toolkit [43] of muon scattering angle $\theta_\mu = 0$ is not acceptable for MUonE running conditions, a new LO exact calculation, in the approximation of infinite nuclear mass, has been carried out in [44] and implemented in the MC event generator MESMER. The results of the simulations show that, even considering appropriate event selections, this process is relevant on the 10^{-4} scale. In view of the final MUonE precision, NLO QED corrections to the process should be calculated, together with the contribution of the $e^+ e^-$ pair production through the muon scattering with two nuclei.

5 MUonE and New Physics

Since MUonE aims at a measurement of the SM hadronic contribution to the running of the QED coupling, the analysis should be robust against possible New Physics contamination. In fact, given the extreme target precision, it could be possible that the present bounds on New Physics do not guarantee such a requirement. The issue has been studied using the Effective Field Theory approach for heavy new degrees of freedom and explicit models for hypothetical light weekly coupled mediators [45–47]. Considering all the available bounds, all the analysis conclude that any possible contamination from not yet excluded New Physics candidates is very unlikely to give contributions at the 10^{-5} scale to MUonE observables. Only in the case of a model of vector unparticles with broken scale invariance, the present constraints on the couplings with leptons could allow a contamination in the MUonE measurements.

In addition to the above studies, other authors investigated the MUonE potential in exploring light New Physics models through the $2 \to 3$ processes $\mu^\pm e \to \mu^\pm e X$ [49, 50] and $\mu^\pm N \to \mu^\pm N X$ [51], i.e. through the real excitation of the new degree of freedom, finding room for improvement on the existing bounds on couplings and masses. This is very interesting and it would not interfere with the measurement of $\Delta\alpha(t)$. However, in order to understand the real discovery potential, further developments on the detector configuration would be necessary.

6 Summary

In the present contribution, after a brief account of the present status on the theoretical predictions for the muon $g - 2$, the space-like approach for the HVP calculation has been illustrated, with a discussion on the theoretical challenges and future developments required to successfully accomplish the MUonE physics program.

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