

How the hadron gas affects $X(3872)$ and $\psi(2S)$ production in heavy ion collisions

Luciano M. Abreu^{1,*}, Fernando S. Navarra^{1,**}, and Hildeson P.L. Vieira^{1,***}

¹Instituto de Física, Universidade de São Paulo, Rua do Matão, 1371, CEP 05508-090, São Paulo, SP, Brazil

Abstract. The $X(3872)$ to $\psi(2S)$ yield ratio ($N_X/N_{\psi(2S)}$) has been recently measured in pp , pPb and $PbPb$ collisions by the LHCb and CMS Collaborations at the LHC. It was found that this ratio grows with the system size and this growth was attributed to the formation of quark gluon plasma and/or of a hot hadron gas. Here we focus on the effects of the hadron gas on the abundance of $X(3872)$ and $\psi(2S)$. The interaction of charmonium states with the light mesons in the gas can be studied with an effective Lagrangian formalism. One can calculate the relevant cross sections and use them as input in rate equations. With this formalism one can follow the time evolution of the $X(3872)$ and $\psi(2S)$ abundances until the final freeze-out of the gas. The initial conditions for this evolution are given by the coalescence model, which depends on the spatial configuration of the states and yields quite different numbers for compact tetraquarks and extended meson molecules. It has been found that the interactions in the hadron gas yield a suppression in most of the states and, most importantly, preserve the difference between the number of tetraquarks and the number of meson molecules, formed at the beginning of the hadronization. We discuss the predictions made for the ratio $N_X/N_{\psi(2S)}$ in $PbPb$ collisions at $\sqrt{s_{NN}} = 5.02$ TeV to be measured by the ALICE Collaboration in the Run 3. The existing calculations suggest that the molecular configuration generates a ratio compatible with the CMS data, whereas the ratio obtained with the tetraquark configuration is 50 times smaller.

1 Introduction

In the last two decades, several newly discovered hadrons have exhibited properties that conflict with quark model predictions, leading them to be classified as nonconventional states. For a recent review, see [1]. The characteristics of these states are still the focus of heated debate, making the study of exotic quarkonium spectroscopy a prominent research area. The central question is: what is their internal structure? These states could be weakly bound hadronic molecules, compact multi-quark systems, kinematic cusps, excited conventional hadrons, glueballs, hybrids, or even a mix of these possibilities [1]. To date, there is no definitive answer. A quintessential example is the $X(3872)$, the first and most well-known exotic state. Its true nature remains controversial. The two most widely explored configurations in the literature are an open-charm meson weakly bound state and a compact tetraquark.

* e-mail: luciano.abreu@ufba.br

** e-mail: navarra@if.usp.br

*** e-mail: hildeson.paulo@ufba.br

A new phase in the exploration of exotic charmonium states began with the first observation of $X(3872)$ in relativistic heavy-ion collisions, as recently reported by the CMS Collaboration [2]. The data were given in terms of the ratio $\mathcal{R} = N_{X(3872)}/N_{\psi(2S)}$. These two particles have similar masses and the same decay channel. In their ratio some experimental uncertainties are cancelled. The comparison of the ratios measured in pp, pPb and Pb-Pb collisions [3] shows a growth with the system size, which may be seen as an evidence of a medium effect.

In nucleus-nucleus collisions we expect to observe the effects of three media: the color glass condensate (CGC), a dense system of partons formed at the very initial state of the collisions, the quark gluon plasma (QGP) formed later, after the thermalization, and the hadron gas (HG), formed at the late stage of the collisions, after the expansion and cooling of the QGP and the phase conversion from deconfined quarks to hadrons [4]. The interpretation of the LHC data must be done cautiously because of the different p_T ranges measured by the different experiments. The bulk of particles with low and moderate transverse momenta were formed at the end of the QGP and lived in the hadron gas until freeze-out. They carry information about both media. On the other hand, particles with high p_T , and in particular those measured by the CMS Collaboration, were produced in the very beginning of the collision, before thermalization, and hence they carry information about the CGC. Information from the QGP and HG will come from the ALICE data, which will include particles with low p_T .

Looking at Fig. 1a we observe that \mathcal{R} grows with the system size. This behavior can be attributed either to a suppression of $\psi(2S)$ or to an enhancement of $X(3872)$ or to both effects. Assuming that the QGP and HG play a role, this suppression/enhancement can come from the hadronization process (at the end of the QGP), from the interactions between $X(3872)$ and $\psi(2S)$ with the light mesons of the hadron gas and from the hadron gas time evolution. This latter aspect was shown to only amplify the effects of the former two.

The hadron gas lives for about 10 fm and freezes out generating the observed particles. Conventional and exotic hadrons formed at the end of the mixed phase can interact with the (mostly light) particles in the hadron gas and their multiplicities may experience modifications due to production and absorption processes. In the case of exotic hadrons the final multiplicity will depend on the interaction cross sections, which, in turn, depend on the spatial configuration of the states. Meson molecules are larger, and therefore have greater cross sections and stronger interaction with the hadronic medium than compact tetraquarks. We will discuss this issue in the next section.

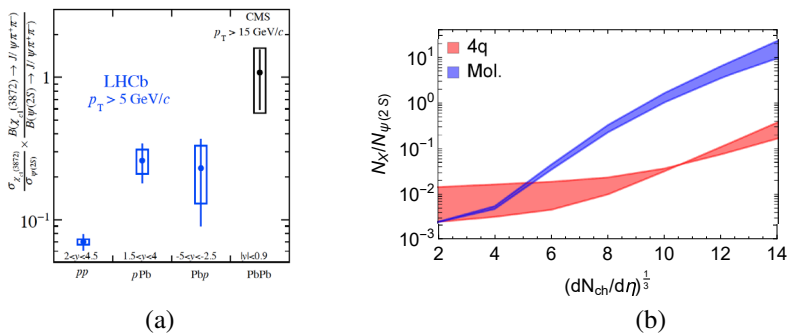


Figure 1. a) Ratio \mathcal{R} observed at the LHCb and CMS. Figure taken from Ref. [3]. b) Ratio \mathcal{R} as a function of the system size (represented by $(dN/d\eta)^{1/3}$) calculated in [8].

2 Particle interactions in the hadronic medium

In [5] we improved the effective Lagrangian formalism introduced in [6] to study the interactions of the $X(3872)$ with the π 's and ρ 's of the hadron gas. In the case of the $J/\psi - \pi$ interactions it was possible to check that our cross sections were in agreement with those obtained previously with QCD sum rules [7]. More recently, in [8], we introduced the analogous formalism for the interactions of the $\psi(2S)$. In both cases, we employed form factors to take into account the finite size of hadrons. In the case of the $\psi(2S)$, due to the similarities with the J/ψ , we assumed that the form factors are given by the same parametrizations used for the J/ψ , which had already been computed via QCD sum rules [9]. The only difference is in the coupling constant. It is well known that the $\psi(2S)$ is larger than the fundamental charmonium state by a factor of about 2 and hence, based on geometrical arguments, we expect the $\psi(2S)$ to have cross sections 4 times bigger than the J/ψ . Following this reasoning, in [8] the form factor parametrizations obtained for the J/ψ were used for the $\psi(2S)$ vertices with the coupling constants varying in the range $[g, 2g]$.

With the effective Lagrangians one can compute the vacuum cross sections and then take the average over the momenta of the incoming particles, which follow the Bose-Einstein distribution. In [8] it was found that the resulting "thermally averaged cross sections" show a remarkably weak dependence on the temperature. Moreover, it was found that, for the studied particles, the absorption cross sections were larger than the production ones. However, as it will be seen in the next sections this does not automatically imply a suppression in the final yields.

3 Time evolution

With the thermally-averaged cross sections discussed in the previous section one can solve the rate equation and compute the time evolution of the hadron abundances. The rate equation is written as:

$$\frac{dN_h(\tau)}{d\tau} = \sum_{a,b,c} [\langle \sigma_{ab \rightarrow ch} v_{ab} \rangle n_a(\tau) N_b(\tau) - \langle \sigma_{ch \rightarrow ab} v_{ch} \rangle n_c(\tau) N_h(\tau)], \quad (1)$$

where $N_h(\tau)$ represents the multiplicity of the state of type h ($h = \psi(2S), X(3872)$); $n_i(\tau)$ and $N_i(\tau)$ denote the density and the number of the meson of type i at a given time τ , respectively. The particles of the hadron gas are open charm mesons (a, b) and pions and ρ 's (c). They are assumed to be in equilibrium, implying that in the Maxwell-Boltzmann approximation $n_i(\tau)$ becomes

$$n_i(\tau) = \frac{1}{2\pi^2} \gamma_i g_i m_i^2 T(\tau) K_2\left(\frac{m_i}{T(\tau)}\right), \quad (2)$$

where γ_i , g_i and m_i is the fugacity, degeneracy factor and mass of the particle of type i , respectively. The multiplicity $N_i(\tau)$ is then obtained by multiplying $n_i(\tau)$ by the volume $V(\tau)$. The hadron gas expansion is often modeled [10] by the boost invariant Bjorken flow with an accelerated transverse expansion. The volume and temperature profiles as a function of the proper time τ are as follows [10]:

$$\begin{aligned} V(\tau) &= \pi \left[R_C + v_C (\tau - \tau_C) + \frac{a_C}{2} (\tau - \tau_C)^2 \right]^2 \tau c, \\ T(\tau) &= T_C - (T_H - T_F) \left(\frac{\tau - \tau_H}{\tau_F - \tau_H} \right)^{\frac{4}{5}}, \end{aligned} \quad (3)$$

where R_C, v_C, a_C and T_C are the transverse size, transverse velocity, transverse acceleration and temperature at the time τ_C , respectively; $T_H(T_F)$ is the temperature at the hadronization (kinetic freeze-out) time $\tau_H(\tau_F)$. The parameters in Eq. (3) are fixed for a hadronic medium formed in central Pb Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV [10].

Having fixed all these parameters, one can solve Eq. (1), once the initial conditions are chosen, as will be discussed in the next section.

4 Initial conditions

The conversion of quarks from the QGP into hadrons is a non-perturbative process and it is not well understood. There are two models which are widely used to describe this process: the coalescence model (CM) [10] and the statistical hadronization model (SHM) [11]. According to the coalescence model, the probability of producing multi-quark hadrons from quarks in the QGP is given by the overlap of the Wigner function of the produced hadron with the phase-space distribution of the constituents in the medium. The Wigner function is obtained from the wave function of the state. Hence the CM is sensitive to the spatial structure of the state and yields different results for compact (tetraquark) and extended (molecules) objects. From the coalescence model one obtains the following multiplicities for $\psi(2S)$ and $X(3872)$ at the end of the mixed phase [8, 10]:

$$N_{\psi(2S)}(\tau_H) \approx 1.8 \times 10^{-4}, \quad N_X^{(4q)}(\tau_H) \approx 1.8 \times 10^{-4}, \quad N_X^{(Mol)}(\tau_H) \approx 1.1 \times 10^{-2}, \quad (4)$$

As it can be seen, the coalescence mechanism generates initial conditions in which molecules are more abundant than compact tetraquarks by a factor of about 50, reflecting the fact that forming a loosely bound state is easier than a compact tetraquark. With these initial condi-

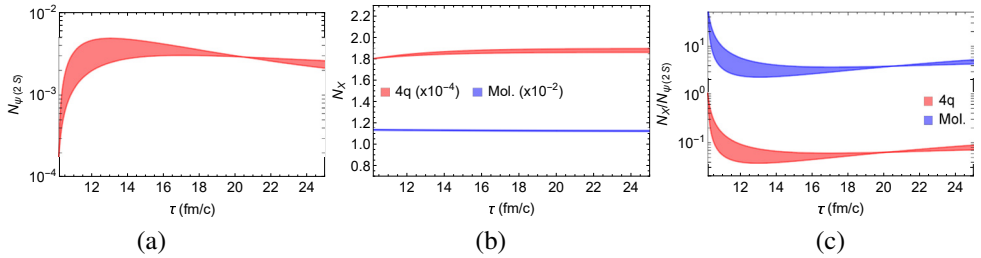


Figure 2. Multiplicity of $\psi(2S)$ (a), of $X(3872)$ (b) and their ratio (c) as a function of the proper time in central Pb Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV.

tions, Eq. (1) can be solved, as will be discussed in the next section.

5 Multiplicities and system size dependence

5.1 The time evolution of the ratio \mathcal{R}

In Fig. 2a one can see the time evolution of the $\psi(2S)$ multiplicity, which, from the beginning to the end of the hadron gas phase, increases by about one order of magnitude. This means that the gain terms in the evolution equation (1) play a dominant role at early times and

higher temperatures. Indeed, at the beginning of the hadron gas phase the number of $\psi(2S)$'s is very small and so is the second term in the right hand side of Eq. (1). As a result, the derivative $dN_h/d\tau$ becomes large and positive leading to a strong growth of $N_{\psi(2S)}$. As time evolves, the gain and loss terms become of the same order and the $\psi(2S)$ multiplicity suffers just a slight reduction. Fig. 2b shows a very important result, namely that the abundance of both multiquarks states, tetraquarks and molecules, is nearly conserved during the hadron gas phase. In Fig. 2c one can see the time evolution of \mathcal{R} . As expected from the discussion above, one observes a strong variation of this ratio in the early times and after that a relative stabilization. Interestingly, the molecular configuration generates a ratio greater than 1, which seems compatible with the value obtained by the CMS Collaboration for Pb Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV within the rapidity and transverse momentum ranges $|y| < 1.6$ and $15 < p_T < 50$ GeV. Since the data have been collected at high-transverse momenta and in a specific rapidity range whereas the results from [8] have been computed considering the full range in p_T and y , a direct comparison between theoretical results and data is not possible yet.

5.2 The system size dependence

The size of the source can be related to a measurable quantity, the charged-particle pseudorapidity density at mid-rapidity, $[dN_{ch}/d\eta(|\eta| < 0.5)]$. This quantity can be related to the freeze-out temperature by means of the empirical formula [12, 13]:

$$T_F = T_{F0} e^{-b\mathcal{N}}, \quad (5)$$

where $T_{F0} = 132.5$ MeV, $b = 0.02$, and $\mathcal{N} \equiv [dN_{ch}/d\eta(|\eta| < 0.5)]^{1/3}$. Assuming that the hadron gas undergoes a Bjorken cooling, i.e. $T = T_h (\tau_h/\tau)^{1/3}$, then the freeze-out time τ_F can be written in terms of \mathcal{N} as:

$$\tau_F = \tau_H \left(\frac{T_H}{T_{F0}} \right)^3 e^{3b\mathcal{N}}. \quad (6)$$

A larger source generates a bigger \mathcal{N} , which, from the equation above, implies a bigger τ_F , i.e. a longer hadron gas phase. As a consequence, the use of Eq. (6) in (1) will give rise to a dependence of the multiplicity N_h on the size of the source. As shown in Ref. [14], similar empirical formulas relating \mathcal{N} with the volume of the system (V), charm quark number (N_c) and light quark number (N_q) can also be obtained. They are [14]:

$$V = 2.82 \mathcal{N}^3 \quad N_c = 7.9 \times 10^{-5} \mathcal{N}^{4.8} \quad N_u = N_d = 0.37 \mathcal{N}^3. \quad (7)$$

In these estimates it is assumed that the charm quark number and the number of D mesons (N_D) are proportional. Hence, the use of Eq. (7) in the formula of the coalescence model to estimate the dependence of the initial conditions with \mathcal{N} (as done in [14]) together with (6) in (1), generates a dependence of N_h on \mathcal{N} . Using the above expressions to compute $N_{X(3872)}$ and $N_{\psi(2S)}$, it is possible to obtain their ratio, \mathcal{R} , which is shown in Fig. 1b. In the figure one can see that the ratio \mathcal{R} grows with \mathcal{N} , in qualitative agreement with the data shown in Fig. 1a. From the results shown in Fig. 2 one can conclude that \mathcal{R} grows mostly because $\psi(2S)$ is suppressed. Unfortunately, as mentioned before, a direct comparison is not yet possible. Another limitation of this approach is that for very small systems it loses validity, as it becomes unlikely to produce quark gluon plasma and even a hadron gas, which would live long enough to produce the effects discussed here. Therefore, the curves shown on the left part of the plot shown in Fig. 1b, i.e. in the region $2 < \mathcal{N} < 4$, are at the limit of their validity.

6 Final remarks

Taken together, the results in Figs. 1, 2, suggest that in central Pb Pb collisions at $\sqrt{sNN} = 5.02$ TeV the ratio \mathcal{R} for minimum bias events is:

$$\frac{N_X}{N_{\psi(2S)}} \simeq 5 \quad \text{for molecules} \qquad \frac{N_X}{N_{\psi(2S)}} \simeq 0.1 \quad \text{for tetraquarks} \quad (8)$$

This result obtained in [8] is a prediction to be tested at the run 3 of the ALICE Collaboration, where it will be possible to measure $X(3872)$ and $\psi(2S)$ at low transverse momentum, i.e. $2 < p_T < 8$ GeV. In this kinematical region the hydrodynamical approach discussed here should be applicable. With these forthcoming data, it will be possible, for the first time, to investigate the medium effects on $X(3872)$, on $\psi(2S)$ and on their ratio and check the predictions made in [8]. Moreover, in view of the numbers above one may have a good chance to discriminate between molecules and tetraquarks. From the figures one can conclude that is easier to identify the different configurations in larger systems and in more central collisions.

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