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Hagedorn temperature in confining gauge theories from holography

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Abstract. We provide a short overview on recent results on the Hagedorn temperature of confining gauge theories having a dual holographic string description.

1 Introduction

A quantum system displays a Hagedorn behavior if at high energies E the density of states grows exponentially like the energy, $\rho(E) \sim e^{E/T_H}$, where T_H is called the Hagedorn temperature. The thermal partition function of such a system diverges when $T \to T_H^-$. Examples include both quantum field theories (QFT) like Yang-Mills or quenched QCD and string theory. The occurrence of such kind of behavior in the realm of strong interactions was actually first pointed out by Rolf Hagedorn [1], who argued for an asymptotically exponential hadron mass spectrum with a limiting temperature which hence took his name. Today we would say that this is expected to hold for the confining phase of $SU(N)$ Yang-Mills or quenched (large N) QCD: these theories (for $N > 2$) display first order deconfinement transitions at some critical temperature $T_c < T_H$ so that the Hagedorn regime is actually never reached on a stable branch.

Computing the Hagedorn temperature of a quantum field theory is not an easy issue, in general. For instance we do not have, yet, first principle computations of T_H in Yang Mills theories from lattice studies, although the latter provide clear indications and indirect hints on the Hagedorn temperature both at small [2] and large N [3].

The Hagedorn temperature for bosonic and fermionic strings in flat spacetime is exactly known. What about string theory on curved backgrounds? This is an interesting question in view of the holographic string/gauge theory correspondence, which, for instance, maps large N confining gauge theories at strong coupling, to weakly coupled string theories on curved gravity backgrounds with fluxes. As a corollary of such a correspondence, the gauge theory Hagedorn temperature corresponds to that of the dual string theory. Can we thus compute the Hagedorn temperature of strongly coupled large N confining gauge theories from the

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holographic dual description? At first sight, this appears as a daunting task since, apart from very special examples,[1] we do not know how to compute the partition function of string theory on curved backgrounds with fluxes. A possible way out is suggested by a flat space analysis that we are going to review in the following section. This will provide the basis for our study of the Hagedorn temperature in holographic confining gauge theories. Remarkably, in cases where complementary QFT results at strong coupling are available, we will be able to present non trivial cross-checks of our results.

2 Strings in flat spacetime

Consider type II superstrings in flat ten-dimensional spacetime. It is a textbook result that the number of particle species with mass less than M asymptotes to

$$d(M) \sim M^{-10} e^{2\pi\sqrt{2\alpha'}M}\,, \tag{1}$$

from which we can read the Hagedorn temperature

$$T_H = \sqrt{\frac{T_s}{4\pi}} = \frac{1}{2\pi\sqrt{2\alpha'}}\,, \tag{2}$$

where $T_s = (2\pi\alpha')^{-1}$ is the string tension. In fact, when $T \to T_H^-$, the torus (one-loop) partition function of string theory diverges, signaling the occurrence of a tachyonic mode in the spectrum. The latter can be directly detected by considering the mass shell condition for string theory on a background obtained from the ten-dimensional flat one by compactifying the (Euclidean) time direction on a circle of length $\beta = 1/T$. As it was shown in e.g. [5], strings winding the thermal circle have a tachyonic mode when the size of the circle is small enough. In fact, the ground state with winding number ± 1 has mass

$$m_W^2 = \frac{2}{\alpha'}\left(\frac{1}{8\pi^2\alpha' T^2} - 1\right)\,, \tag{3}$$

which actually becomes tachyonic when $T > T_H$. For T close to T_H this state corresponds to an almost massless complex scalar field χ (ofter referred to as the thermal scalar field) in target space, which can be accounted for in the low energy effective action as [5, 6]

$$S_\chi = \frac{1}{T}\int d^9x\left(\partial_i\chi^*\partial^i\chi + m_W^2\chi^*\chi\right)\,. \tag{4}$$

What we want to review here is how the extension of the above results to string theory on certain classes of curved backgrounds allows to obtain information about the Hagedorn temperature of the dual confining gauge theories in the planar limit at strong coupling [7–10]. To this aim, the interplay between the worldsheet perspective and the thermal scalar effective description turns out to play a crucial role.

3 Strings on confining backgrounds

In certain classes of theories, the $T > 0$ confining phase of $(q+1)$-dimensional QFTs at low energy, is holographically mapped to closed string theory on backgrounds with metric asymptotically going as

$$ds^2 \approx 2\pi\alpha' T_s\left(1 + \frac{r^2}{l^2}\right)(dt^2 + \eta_{ij}dx^i dx^j) + dr^2 + r^2 d\Omega_{d-1}^2 + ds_M^2\,,\quad (r \ll l) \tag{5}$$

[1] See [4] for recent results on the asymptotic density of states of string theory on exact pp-wave backgrounds.

where $i, j = 1, \cdots q$, $t \sim t + \beta$, r is the radial direction dual to the field theory renormalization group energy scale, l is the curvature radius (with the $r \sim 0$ limit mapped to the infra-red regime of the dual QFT), T_s is the confining string tension of the gauge theory and $\mathcal{M}$ is a transverse, typically compact, $(9 - d - q)$-dimensional space. Notice the presence of a flat Euclidean d-dimensional subspace in the background.

The backgrounds typically include non trivial Ramond-Ramond field strengths and may also feature a running dilaton field and a non-zero Kalb-Ramond field. Let us assume here that the latter has no legs along the time direction (the general case has been treated as well in [10]), and consider a closed string at $r = 0$ and at a point in $\mathcal{M}$, winding once along the time circle so that its embedding includes

$$t(\tau, \sigma) = \frac{\beta}{2\pi}\sigma + \xi^0(\tau, \sigma)\,, \tag{6}$$

where $\sigma \in [0, 2\pi]$ and τ are the string worldsheet coordinates.

Now, according to the results presented in [7–10], it is possible to obtain, at the quantum level, the spectrum of quadratic fluctuations of the worldsheet fields around that classical embedding, finding that the ground state of the winding string has a mass which becomes tachyonic when $T > T_H$, with T_H being the solution of

$$\frac{T_s}{2}\beta_H^2 = 2\pi[\Delta(\mu) + \delta\epsilon]\,, \tag{7}$$

where

$$\mu = \frac{\beta_H}{2\pi}\frac{\sqrt{2\pi\alpha' T_s}}{l}\,, \tag{8}$$

is the mass of the d scalar fields y^I, with $y^I y^I = r^2$, at $T = T_H$, and

$$\Delta(\mu) = 1 - \frac{d}{2}\mu + d\mu^2 \log 2 + O(\mu^4)\,, \tag{9}$$

is the zero-point energy of the worldsheet sigma model to second order in the mass parameter μ. The finiteness of $\Delta(\mu)$ is guaranteed by a mass-matching condition $\sum_b m_b^2 = \sum_f m_f^2$ involving the bosonic ($m_b = \mu$, with $b = 1, \cdots, d$)) and the fermionic (m_f, with $f = 1, \cdots, 8$) masses.

The $O(\mu)$ term in the expansion of $\Delta(\mu)$ arises from the contribution of the massive bosonic zero modes. As such it can also be captured by the thermal scalar effective approach, after suitably extending eq. (4) to the class of curved backgrounds considered here [11]. In general, within that framework, the Hagedorn temperature is obtained requiring that, neglecting the backreaction on the background, the linearized equation of motion for the thermal scalar admits a normalizable solution.

The $O(\mu^2)$ term in (9), instead, is due to the non-zero modes. The two-derivative effective approach misses such terms.[2] However, it provides another contribution, at the same order, which can be related to quartic terms in the bosonic zero modes in the worldsheet Hamiltonian. By construction, this contribution is not captured by the sigma model approach if one limits the analysis to quadratic order. The $\delta\epsilon$ piece in (7) precisely accounts for such a missing term (and higher order ones) which, being due to contributions of the bosonic zero modes, can be captured by the effective action for the thermal scalar field.

Solving eq. (7) we can determine T_H in a perturbative expansion in

$$\frac{\sqrt{\alpha'}}{l} \sim \lambda^{-k} \sim \frac{M_{gl}}{\sqrt{T_s}}\,, \quad k > 0\,, \quad \lambda \gg 1\,, \tag{10}$$

[2] Curvature corrections to the effective action can capture such kind of contributions [12]; however they come with coefficients which are not a-priori determined.

where λ is a 't Hooft-like coupling and M_{gl} is the glueball mass scale. To leading order we get [7]

$$T_H = \sqrt{\frac{T_s}{4\pi}}\,, \tag{11}$$

which is a simple generalization of the flat space relation (2), where T_s, the tension of the confining gauge theory, replaces the string tension in flat space.

To next-to-leading order (NLO) we get [8]

$$T_H = \sqrt{\frac{T_s}{4\pi}}\left[1 + \frac{d}{2\sqrt{2}}\frac{\sqrt{\alpha'}}{l}\right]. \tag{12}$$

This result coincides with the one obtained from the effective action for the winding string scalar mode [11].

As we have anticipated, we can go beyond NLO using an interplay of worldsheet and effective methods. The latter rely on a perturbative analysis of the equation of motion for the thermal scalar χ, whose low energy dynamics is captured by the generalization to curved space of the action in (4). In particular, the higher derivative corrections to the mass m_W^2 (see [12]) match with what we can single out from (7).

As an example, let us consider backgrounds involving a global AdS_{d+1} factor in the metric. Near the center of AdS, the latter can be written in the form (5), with $q = 0$, $l = R_{AdS}$ and $2\pi\alpha' T_s = 1$. The dual QFTs are conformal field theories (CFT) compactified on $S^{(d-1)}$ spheres. They are "confining" in the sense explained in [13], since they experience a first order transition between a low temperature phase with $O(1)$ degrees of freedom and a high temperature "deconfined" phase with $O(N^2)$ degrees of freedom. The critical temperature T_c for such a transition is set by the inverse radius of the $S^{(d-1)}$ sphere, hence, in turn, by R_{AdS}^{-1}. Setting $R_{AdS} = 1$, the Hagedorn temperature for such theories turns out to be given by [9, 14, 15]

$$T_H = \sqrt{\frac{g}{2\pi}} + \frac{d}{8} + \frac{d^2 + d - 8d\log 2}{32\sqrt{2}\pi^{3/2}\sqrt{g}} + \frac{4d^3 + 7d^2 - 2d}{1024\pi^2 g} + O(g^{-3/2})\,, \tag{13}$$

where, in the above mentioned units, $g = 1/4\pi\alpha'$ is proportional to some positive power of the 't Hooft coupling λ. Hence, T_H is parametrically larger than T_c. Remarkably, the NNLO coefficient of (13), can be entirely computed from a pure sigma model perspective including interaction terms in the worldsheet Hamiltonian which are quartic in the bosonic zero modes [10]. In principle, we could envisage the full result to be fixed without any ambiguity by a generalization of this method. Unfortunately, the latter is very demanding and the interplay with the effective approach turns out to be very convenient.

Notably, in two relevant cases, QFT results for T_H at strong coupling are available, via integrability and quantum spectral curve methods, which allow us to test the validity of the result in (13).

When $d = 4$ and the dual field theory is $\mathcal{N} = 4$ super-Yang-Mills on S^3, we have that $g = \lambda/4\pi$. Numerical quantum spectral curve results for T_H at strong coupling, as collected in [16] and, more recently in [14], show an impressive agreement with our analytic result (13) at NNNLO.

An analogous impressive agreement to NNNLO is found when $d = 3$ and the dual gauge theory is ABJM [17] on S^2. Here the numerical quantum spectral curve results at strong coupling have been found in [15].

4 Further extensions

The above holographic results on T_H have been extended in [18] in the following directions. From one side the effect of N_f quenched flavors on the Hagedorn temperature has been considered, finding that, in the scheme where the 't Hooft coupling is fixed, $T_H/\sqrt{T_s}$ decreases when N_f increases. Analogously, in the same scheme, $T_H/\sqrt{T_s}$ has been found to increase for increasing value of the Yang-Mills θ-angle.

Finally, embedding the string theory setup in M-theory, where the winding string is replaced by a double-winding $M2$-brane, it has been possible to compute, in certain cases, both non-perturbative instanton-like corrections to the value of T_H discussed above, and the value of T_H in a regime where the effective string coupling gets large, along the lines of what has been done in flat space in [19].

References

[1] R. Hagedorn, "Statistical thermodynamics of strong interactions at high energies. 3. Heavy-pair (quark) production rates," Nuovo Cim. Suppl. **6** (1968), 311-354 CERN-TH-751

[2] M. Caselle, A. Nada and M. Panero, "Hagedorn spectrum and thermodynamics of SU(2) and SU(3) Yang-Mills theories," JHEP **07**, 143 (2015) [erratum: JHEP **11**, 016 (2017)] https://doi.org/10.1007/JHEP07(2015)143

[3] B. Bringoltz and M. Teper, "In search of a Hagedorn transition in SU(N) lattice gauge theories at large-N," Phys. Rev. D **73**, 014517 (2006) https://doi.org/10.1103/PhysRevD.73.014517

[4] T. Canneti, "On the asymptotic density of states in solvable models of strings," [arXiv:2406.08405 [hep-th]]

[5] J. J. Atick and E. Witten, "The Hagedorn Transition and the Number of Degrees of Freedom of String Theory," Nucl. Phys. B **310** (1988), 291-334. https://doi.org/10.1016/0550-3213(88)90151-4

[6] G. T. Horowitz and J. Polchinski, "Selfgravitating fundamental strings," Phys. Rev. D **57** (1998), 2557-2563 https://doi.org/10.1103/PhysRevD.57.2557

[7] F. Bigazzi, T. Canneti and A. L. Cotrone, "On the Hagedorn temperature in holographic confining gauge theories," JHEP **01** (2023), 034 https://doi.org/10.1007/JHEP01(2023)034

[8] F. Bigazzi, T. Canneti and W. Mück, "Semiclassical quantization of the superstring and Hagedorn temperature," JHEP **08** (2023), 185 https://doi.org/10.1007/JHEP08(2023)185

[9] F. Bigazzi, T. Canneti and A. L. Cotrone, "Higher Order Corrections to the Hagedorn Temperature at Strong Coupling," JHEP **10** (2023), 056 https://doi.org/10.1007/JHEP10(2023)056

[10] F. Bigazzi, T. Canneti, F. Castellani, A. L. Cotrone and W. Mück, "Hagedorn temperature in holography: world-sheet and effective approaches," JHEP **09** (2024), 193 https://doi.org/10.1007/JHEP09(2024)193

[11] E. Y. Urbach, "The black hole/string transition in AdS_3 and confining backgrounds," JHEP **09** (2023), 156 https://doi.org/10.1007/JHEP09(2023)156

[12] T. Harmark, "Hagedorn temperature from the thermal scalar in AdS and pp-wave backgrounds," JHEP **06**, 140 (2024) https://doi.org/10.1007/JHEP06(2024)140

[13] E. Witten, "Anti-de Sitter space, thermal phase transition, and confinement in gauge theories," Adv. Theor. Math. Phys. **2** (1998), 505-532 https://doi.org/10.4310/ATMP.1998.v2.n3.a3

[14] S. Ekhammar, J. A. Minahan and C. Thull, "The asymptotic form of the Hagedorn temperature in planar super Yang-Mills," J. Phys. A **56** (2023) no.43, 435401 https://doi.org/10.1088/1751-8121/acf9d0

[15] S. Ekhammar, J. A. Minahan and C. Thull, "The ABJM Hagedorn Temperature from Integrability," JHEP **10** (2023), 066 https://doi.org/10.1007/JHEP10(2023)066

[16] T. Harmark and M. Wilhelm, "Solving the Hagedorn temperature of AdS_5/CFT_4 via the Quantum Spectral Curve: chemical potentials and deformations," JHEP **07** (2022), 136 https://doi.org/10.1007/JHEP07(2022)136

[17] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, "N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals," JHEP **10** (2008), 091 https://doi.org/10.1088/1126-6708/2008/10/091

[18] F. Bigazzi, T. Canneti, A. L. Cotrone and J. M. Penín, "Holographic Hagedorn Temperature: Strong Coupling, Flavor and θ-angle Effects," [arXiv:2407.07943 [hep-th]]

[19] J. G. Russo, "Free energy and critical temperature in eleven-dimensions," Nucl. Phys. B **602** (2001), 109-131 https://doi.org/10.1016/S0550-3213(01)00128-6