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Approach to automating numerical calculation of the state of heat transfer processes in closed devices

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Abstract. In recent decades, changes in temperature and humidity have been used in many fields of technology: chemical technology, metallurgy, oil refining, mechanical engineering, biology and space research. One of the important issues in space exploration is the creation of closed facilities for the study of underwater processes. For large-scale systems, it is necessary to maintain a certain temperature and humidity regime that allows people inside the apparatus to live and work. The work examines heat exchange processes occurring in closed-loop hardware systems. It is assumed that the considered network is filled with refrigerant (liquid, gas, air, etc.) and consists of flow line, structural elements, volumes, valve, regulator and mixer - distributor. In such networks, systems of differential equations (ordinary and special derivatives), as well as an algorithm for automatic construction of mathematical models of heat transfer described by numerical schemes have been developed. A special methodology and algorithm have been developed for solving this type of problems. During the solution, two methods were used and the obtained results were compared. During the solution, obvious and non-obvious schemes were used, and the most stable scheme was chosen among them.

1 Introduction

The work examines the heat exchange processes occurring in closed-loop apparatus (CA) systems. It is assumed that the network under consideration is filled with a coolant (liquid, gas, air, etc.) and consists of a flow line, structural elements and volumes. The set of network elements may also include automation elements: valves, regulators, mixer-separators, sensors, etc.

The main elements of the thermal circuit are:

- heat exchange units (liquid-liquid and gas-liquid heat exchangers, refrigeration-drying units, radiation heat exchangers, coils for direct cooling or heating of the unit);
- connecting pipelines and air ducts;
- structural elements;
- coolant flow regulators;
- fittings (valves) and hatches;

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- liquid and air heaters, electric heaters;
- elements of instrumentation equipment (sensors and automation elements)
- mixer-separators.

The heat exchange process in this network occurs by applying thermal disturbances from the outside to individual elements. The temperature of thermal control objects is determined either based on their direct thermal contact, or based on the fact that this contact is provided through a certain intermediate thermal connection. Thermal loads from elements in thermal contact with coolants depend on the temperature of the coolant in the volume.

The network consists of a set of circuits connected to each other. Each circuit can contain several branches. A branch is understood as a sequence of coolant volumes that does not contain mixer-separators. The branch is characterized by the same flow rate along its entire length.

A complex mathematical model of heat transfer processes in such networks is considered in [1-5] and represents generalized heat transfer equations of a system of ordinary differential equations, partial differential equations and algebraic equations (if the network involves elements of automation have been created), the solutions of which are associated with certain difficulties.

The task is as follows. Let all network elements have a certain temperature field at the initial moment of time. It is required to develop an algorithm for automatically constructing mathematical models of heat transfer in such networks, described by systems of differential equations (ordinary and partial derivatives), as well as numerical schemes. Their decisions are associated with certain difficulties.

Analysis of the mathematical description of thermal regimes in complex devices shows that this process can be described by generalized heat transfer equations, the number of which is quite large [5, 6].

In this regard, it is necessary to find a suitable finite-difference approximation of the system and solve the resulting system of equations using the most effective methods.

Below is one approach to solving the problem. The ideological side of this approach to the system lies in the division of the object into elements, the mathematical models of the processes of which are relatively simple and easily formed at the input-output level.

We have considered and analyzed a number of numerical methods in relation to the problem of heat transfer in a closed apparatus.

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Below is one approach to solving the problem. The ideological side of this approach to the system lies in the division of the object into elements, the mathematical models of the processes of which are relatively simple and easily formed at the input-output level [7-9].

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2 Problem statement

Let us consider mathematical models of the heat exchange process for individual elements of this network in Figure 1.

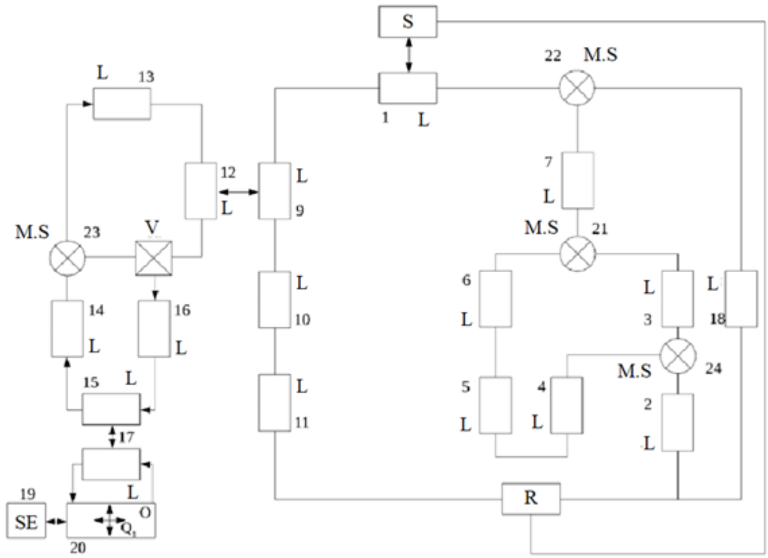

Fig. 1. Heating network: L – current line, O – volume, SE – structural element, M.S – mixer-separator, S- regulator.

1. The mathematical model of the heat exchange process in the k-th streamline has the form:

$$C_{kPk}\left(\frac{\partial T_k}{\partial t}+U_k\frac{\partial T_k}{\partial V_k}\right)=q_k \tag{1}$$

under initial and boundary conditions:

$$T_k(V_k,0)=C,(C=const)$$
$$T_k(0,t)=T_{k-1}(L_{k-1},t), \tag{2}$$
$$q_k=\frac{Q_k}{L_k\frac{V_k}{U_k}},0\le|V_k|\le L_k \tag{3}$$

where $q_k -$ heat pipe per unit time per unit volume of coolant; U_k – coolant flow rate; $C_k\rho_k$– density and heat capacity of the coolant, respectively; T_k– temperature of the *k*-th element [5, 10].

2. The mathematical model of heat transfer processes in volumes is described in the following formulation of the problem. Consider

$$C_{kPk}V_k\frac{dT_k}{dt}=C_{kPk}\left[\sum_{jax}\left(U_{jax}T_k-T_k\sum_{jaux}U_{jaux}\right)\right]+\sum_j Q_j+\sum_k Q_k \tag{4}$$

with the initial condition:

$$T_k(t)|_{t=0}=C,(C=const) \tag{5}$$

where

$$Q_k=\sum_{l=1}^{2}F_{l,k}\,(T_l-T_k) \tag{6}$$

3. The equation describing the change in heat content of a structural element or instrumentation equipment has the following form:

$$C_k\frac{dT_k}{dt}=F_{l,k}(T_k-T_l)+\sum_l Q_1+\sum_k Q_k, \tag{7}$$

with the initial condition:

$$T_k(t)|_{t=0}=C,(C=const), \tag{8}$$

where C_k and T_k are the temperature and heat capacity of the k-th element, respectively; T_l– temperature of the l –th element in thermal contact with the kth element; $F_{l,k}$– heat transfer parameter from the coolant to the element in contact with the coolant; Q_k – heat transferred to the element from adjacent elements or the environment; Q_l – internal heat release of the element.

It is required to solve a complex mathematical problem and determine a numerical algorithm for solving the problem. To solve system (1) – (8), we used the straight lines method and the grid method [5, 7].

3 Materials and methods

Unlike the grid method, in the straight lines method, partial derivatives are approximated not with respect to all, but only with respect to some differential-difference equations with a smaller number of continuous independent variables [5-8].

This method has two schemes - transverse and longitudinal. We have considered longitudinal schemes of the straight lines method for solving the ZA heat transfer problem.

Let us consider a section of a heating network. Let us denote V_k the current coordinate of the *k*-th streamline; L_k=$|V_k|$ – length of the *k* current line; c_k – heat capacity of the coolant; ρ_k – coolant density; T_k is the desired temperature of the *k* streamline $(T_k = T_k(V_k, t))$, and $0 \leq |V_k| \leq L_k$ is specified.

Let $T_{k-1}(L_{k-1,}t)$ be the temperature of the output of the $(k-1)$-th streamline at any time, i.e. $T_{k-1} = T_{k-1}(L_{k-1,}t)$, which is supplied to the input of the k-th current line. Let V_f and T_f, respectively, be the length and temperature (at the moment) in the f-th streamline, through which the external thermal effect is carried out on the k streamline with the heat transfer coefficient , $F_{(f,k)}$ U_k - coolant flow rate , Ω—set of streamlines, q_k-heat supply per unit time per unit volume of coolant. Then the mathematical model of the heat exchange process in the *k*-th streamline will have the form:

$$\frac{\partial T_k}{\partial t} = -U_k \frac{\partial T_k}{\partial V_k} + \frac{1}{p_k c_k} q_k, \tag{9}$$

where

$$k \in \Omega, q_k = F_{(f,k)}\left(T_f - T_{kbx}\right)V_k^{-1}, 0 \leq V_k \leq L_k \tag{10}$$

with the initial conditions:

$$T_k(V_k, 0) = C, (C = const) \tag{11}$$

boundary conditions:

$$T_k(0,t) = T_{k-1}(L_{k-1}, t), V_k \underset{x}{=} 0, T_{kb} = T_{kob}$$
$$V_k^j = jh, j = \overline{0, N_k}, N_k h = L_{k,}$$

When constructing computational schemes using the straight lines method, the differentiation operation is approximated along the straight line V_k. To do this, straight lines are first drawn in the source region (V_k,t)

$$V_k^j = jh, j = 0, \bar{N}_k, N_k h = L_k.$$

and on each of the internal lines $V_k = L_k^j, (j = 1, \bar{N}_{k-1})$, derivatives $\frac{\partial T_k}{\partial V_k}$ we approximate through the values of $T_k$to the neighboring lines:

$$\frac{\partial T_k^j(t)}{\partial V_k} = \frac{T_k^{j+1}(t) - T_k^{j-1}(t)}{2h},\ j = \overline{1, N_{k-1}}. \tag{12}$$

Then equation (1) will take the form:

$$\frac{dT_k^j(t)}{dt} = -U_k \frac{T_k^{j+1}(t) - T_k^{j-1}(t)}{2h} + \frac{1}{c_k p_k} q_k, \tag{13}$$

$$q_k \underset{ax}{=} F_{(f.k)}\left(T_f - T_k\right)V^{-1}, 0 \leq V \leq L. \tag{14}$$

Initial conditions:

$$T_k\left(V_k^j, 0\right) = C, j = \overline{1, N_k}. \tag{15}$$

Moreover

$$T_k(0,t) = T_{k-1}(0,t) =,\ V_k = 0, \tag{16}$$

where $T_k(0,t) = T_{k-1}(L_{k-1}, t)$ is the end point of the previous element.

Thus, the system of partial differential equations will be reduced to a system of ordinary first-order differential equations with initial conditions, i.e. to the Cauchy problem [5].

The heat transfer equation for structural elements and volumes in the heating network themselves are ordinary differential equations [7, 11].

Thus, a complex mathematical model of heat transfer when using the method of straight lines is reduced to solving systems of ordinary differential equations.

1) For current line:

$$\frac{dT_k^j(t)}{dt} = -U_k \frac{T_k^{j+1}(t) - T_k^{j-1}(t)}{2h} + \frac{1}{c_k p_k} q_k, \left(j = \overline{1, N_k}\right), \tag{17}$$

where $q_k \underset{ax}{=} F_{(f.k)}\left(T_f - {}_k T_k\right)V^{-1}, 0 \leq V \leq L,$

The initial conditions are:

$$T_k^j\left(V_{k,0}\right) = C, j = \overline{1, N_k}. \tag{18}$$

2) For volumes:

$$\frac{dT_k}{dt} = \frac{1}{c_k p_k V_k} q_k^{"}, \tag{19}$$

where

$$q_k^{"} = \sum_{k_{bx}} (U_k T_k)_{bx} - T_k \left(\sum_{k_{vix}} U_k\right) + q_k' c_k V_k,$$
$$q_k' = \sum_i Q_i + \sum_m Q_m, \tag{20}$$
$$T_k(t)|_{t=0} = C(C = const),$$

where $\sum_i Q_i$- is the heat released into the coolant according to the program, regardless of its temperature in the volume;

$\sum_m Q_m$ connection between the kth structural element and the m-th network element.

3) For structural elements:

$$c_k \frac{dT_k}{dt} = F_{(l,k)}(T_k - T_l) + \sum_l Q_l + \sum_k Q_k, \tag{21}$$

where Q_i- is the internal heat release of the element, Q_k- is the heat transferred to the element from adjacent elements.

Initial conditions:

$$T_k(t)|_{t=0} = C(C = const), \tag{22}$$

Let us apply the Euler method to the system of differential equations (17) – (22). In this case, the streamline equations (17) will take the form:

$$T_k^j(t + l) = T_k^j(t) + l\left[\frac{1}{c_k p_k} - U_k \frac{T_k^{j+1}(t) - T_k^j(t)}{h}\right], \tag{23}$$

with initial conditions

$$T_k^j(t)|_{t=0} = C(C = const), \tag{24}$$

for volumes

$$T_k^j(t+l) = T_k^j(t) + lF(t, c_k, p_k, V_k, T_k, T_{k-1}, Q_k), \tag{25}$$

with initial conditions (21), for structural elements

$$T_k^j(t+l) = T_k^j(t) + lF_1\big(t, c_k, V_k, T_k^{j+1}, T_k^j, F_k, Q_k\big), \tag{26}$$

with initial conditions (22).

As an example, we will solve a problem that implements a heat network consisting of a flow line, a volume, a structural element, as well as automation elements (valves, sensors, regulators and mixer dividers). The network consists of three circuits, 17 current lines, 3 mixers, 1 valve, separator, volume, structural element, regulator (RR) and sensor. To the right side of the streamline equation

$$c_i p_i V_i \left(\frac{\partial T_i(V_i,t)}{\partial t} + U_k \frac{\partial T_i(V_i,t)}{\partial V_i}\right) = \sum_i Q_i^{vix} + \sum_k Q_k^{bh}, \tag{27}$$

includes the term Q(t), which can be specified as a cyclogram (a tabulated function with a non-uniform time step), a function, or as a constant.

Using the cyclogram given in the table, we approximate Q = Q(t) using the formula:

$$Q(t) = \frac{(t-t_i)Q_i(t) - (t_{i+1}-t_i)Q_{i+1}(t)}{(t_i - t_{i-1})}, \tag{28}$$

at any given time. The temperature of mixer-separators is determined depending on their operating modes (mixer or separator mode).

By analyzing the flow rates entering the separator-mixer, according to formula (28), its mode is determined, then, by selecting formulas from (28) corresponding to this operating mode, we determine its temperature.

For this problem, the method of straight lines was implemented, a system of ordinary differential equations of the streamline, volume and structural element was solved using a modified Euler method.

Grid method. To do this, we introduce a uniform grid in V and t:

$$\omega_h = \{V_k^j = jh, j = \overline{0, N_k}; h = L_k/N_k\},$$

$$\omega_l = \{t_j = il, i = 0,1,2,3,\dots\},$$

where N_k- is the number of V_k nodes.

On the grid $\Omega_{hl} = \omega_h \times \omega_l$ we apply explicit schemes to equations (18) of the streamline, volume and structural element: left corner, four-point approximation and Lax scheme [5]. The results of solving the example using the above schemes are given in Table 1. We assume that counting time (8 sec) is equal to 10800, initial state of the element is equal to 283, position of the shifter and regulator is equal to 1, sensor temperature is equal to 275.5, valve position is Dis.

In order to be able to choose a time step regardless of the volume step, we also considered an implicit approximation scheme $\binom{**}{*}$, (left corner), which is absolutely stable.

$$\frac{T_{i+1,j}^k - T_{i,j}^k}{l} + U_k \frac{T_{i+1,j+1}^k - T_{i+1}^k}{h} = F(t, c_k, p_k, V_k, T^k, T^{k+1}, F_k, Q_k) \tag{29}$$

$$\left(\frac{1}{l} - \frac{U_k}{h}\right) T_{i+1,j}^k + \frac{U_k}{h} T_{i+1,j+1}^k = F(t, c_k, p_k, V_k, T^k, T^{k-1}, F_k, Q_k) + \frac{1}{8} T_{i,j}^k, \left(i = 1,2,\dots, j = \overline{1,N}\right), \tag{30}$$

where

$$F(t, c_k, p_k, V_k, T^k, T^{k-1}, F_k, Q_k) = \frac{\sum_{k'} Q_{k'} + \sum_l Q_l^{boz}}{c_k p_k V_k}, k \in \Omega, \tag{31}$$

total $\sum_l Q_l$is the sum of all internal sources of the *l* element.

Table 1. Explicit left corner scheme.

Item no.	Temperature values at the beginning of the element	Temperature values in the middle of the element	Temperature values at the end of the element
1	297.16	297.20	297.25
2	297.17	297.64	297.94
3	297.16	297.20	297.24
4	297.15	297.20	297.17
5	297.23	297.22	297.15
6	297.23	297.24	297.23
7	297.25	297.25	297.16
8	297.24	297.23	297.23
9	297.17	297.20	297.16
10	297.93	297.24	297.93
11	297.93	297.86	297.79
12	297.99	298.00	298.00
13	298.01	298.00	298.00
14	298.00	298.00	298.005
15	298.01	298.01	298.00
16	298.01	298.005	298.01
17	298.00	298.00	298.01
18	297.94	298.01	298.01
19	298.01	298.005	298.005

The initial and boundary conditions are:

$$T_{i,0}^{k} = C(C = const), i = \overline{0, N_k},$$
$$T_{0,j}^{k} = T_{N_{k-1,j}}^{k-1}, j = 1,2, \dots \quad (32)$$

The system obtained using the implicit approximation scheme (30)–(32) was solved by the Gauss method and a modified sweep method developed for solving systems of first-order differential equations. The left side of system (29)–(32) represents a bidiagonal band matrix in the form:

The last row of the matrix contains the boundary conditions. This structure of the last row of the matrix G is due to the fact that the network under consideration is closed. Solving a system with such a matrix of coefficients using standard methods is associated with great difficulties. Indeed, the matrix is not completely bidiagonal and therefore the known sweep method [10] does not give the desired results. In this regard, we modified the cyclic sweep method [5], developed for solving systems of first-order ordinary differential equations.

The essence of the modification of the straight lines method is to find the sweep coefficients in relation to our problem.

For this equation (31) – (32) we reduce it to the form:

$$G = \left\| \begin{matrix} \frac{1}{l} - \frac{U_k}{h}\frac{U_k}{h} 0 \dots 00 \\ 0 \frac{1}{l} - \frac{U_k}{h}\frac{U_k}{h} \dots 00 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ 10000 - 1 \end{matrix} \right\|$$

$$A_{j+1} T_{i+1,j}^{k} + T_{i+1,j+1}^{k} = B_{j+1}, j = \overline{1, N}; i = 1,2, \dots, \quad (33)$$

$$A_{j+1} = \left(\frac{1}{j} - \frac{U_k}{h}\right) / \left(\frac{U_k}{l}\right);$$

where

$$B_{j+1} = \left[F(t, c_k, p_k, V_k, T^k, T^{k-1}, F_k, Q_k) + \frac{1}{l} T^k_{i,j}\right] / \left(\frac{U_k}{l}\right), \quad (34)$$

$$F(t, c_k, p_k, V_k, T^k, T^{k-1}, F_k, Q_k) = \frac{\sum_{k'} Q_{k'} + \sum_l Q_l^{boz}}{c_k p_k V_k}.$$

Boundary and initial conditions have the form:

$$T^k_{i,0} = C (C = const), i = \overline{0, N_k}, j = 1,2,$$
$$T^k_{0,j} = T^{k-1}_{N,j}, \quad (35)$$

where 1, $T^{k-1}_{i,j}$-temperature of the previous $(k-1)$-th line

We will look for solutions to systems of linear algebraic equations in the following form:

$$T^k_{i+1,j} = P_{j+1} T^k_{i+1,j+1} + q_{j+1}, j = 1,2, \dots, j = \overline{1, N}, \quad (36)$$

where P_{i-1} and q_{i-1} are the running coefficients [12].

Now, taking into account the boundary conditions, we determine these coefficients. Substituting value (34) into (36), we obtain:

$$T^k_{i+1,j+1} = A_{j+1}\left(P_j T_{i+1,N} + q_j\right) + B_{j+1}. \quad (37)$$

Taking into account formula (37) and boundary conditions from formulas (34), (35), we obtain formulas for calculating the coefficients of cyclic sweep:

$$P_1 = 1, q_1 = 0,$$
$$P_{j+1} = A_{j+1} P_j, q_{j+1} = A_{j+1} q_j + B_{j+1}, (j = 1,2, \dots,). \quad (38)$$

From formula (38) with, $j = N, N-1$, we obtain, respectively:

$$T^k_{i+1,j+1} = A_{N+1} T^k_{i+1,N} + q_N,$$
$$T_{i+1,N} = P_N T^k_{i+1,N+1} + q_N. \quad (39)$$

From here we have:

$$T^k_{i+1,N+1} = (B_{n+1} + A_{N+1} q_N)/(1 - A_{N+1} P_{N-1}), (i = 1,2, \dots).$$

The temperature value $T^k_{i+1,j}-$ is $j = N, N-1, \dots, 0$ determined similarly by formula (33). Despite this advantage, using an implicit scheme in this problem is not very profitable. The result is shown in Table 2. We assume that counting time (8 sec) is equal to 10800, initial state of the element is equal to 283, position of the shifter and regulator is equal to 1, sensor temperature is equal to 275.5, valve position is Dis.

Now let us consider one of the approaches that allows you to choose the most optimal steps Δt and ΔV. As is known, in the explicit scheme the time step Δt - is strongly related to the step ΔV. Let us try to increase the integration step while maintaining the courant stability condition.

Let the network C consist of circuits$\{K_j\}_{j=1}^{N_i}$ and in each j-th circuit there are branches $\{B_{ji}\}_{i=1}^{T_j}$ and each branch consists of a set of elements $\{\Lambda_{jik}\}_{k=1}^{P}$ of the "stream line" type with different lengths $\{V_{jik}\}_{k=1}^{P}$ and coolant velocities $\{U_{ji}\}_{i=1}$. It is necessary to select the optimal integration approximation steps $\{\Delta t_{ji}\}_{i=1} * \{\Delta V_{jik}\}$ for systems that are a heat exchange model of a given network.

Table 2. Implicit left corner scheme.

Item no.	Temperature values at the beginning of the element	Temperature values in the middle of the element	Temperature values at the end of the element
1	296.86	297.02	297.23
2	297.27	297.35	297.59
3	297.14	297.32	297.35
4	297.17	297.25	297.27

5	297.23	297.20	297.17
6	297.14	297.24	297.23
7	297.23	297.24	297.14
8	297.24	297.24	297.23
9	297.17	297.24	297.16
10	297.93	297.24	297.91
11	297.91	297.86	297.79
12	297.99	298.00	298.00
13	298.01	298.00	298.00
14	298.00	298.00	298.01
15	298.01	298.005	298.01
16	298.01	298.005	298.01
17	298.01	298.005	298.01
18	297.59	297.64	297.84
19	298.01	298.005	298.01

4 Results and discussion

For such a selection, the following algorithm is proposed.

1. Let's define those network elements that can be considered as points or not considered at all;
2. Define $\{\Delta t_{jik}\}$ for all j, i, k, where $\Delta t_{jik} = |V_{jik}|/U_{ji}$;
3. Define $\Delta t_l^* = max\{\Delta t_{jik}\}$;
4. Find the step length values by volume $\Delta V_{jik}^* = U_{ji}\Delta t_{jik}$;
5. Let us take as a volume step any value ΔV_{jik}equal to or greater than ΔV_{jik}^* $(\Delta \bar{V}_{jik} \geq \Delta_{jik}^*)$, for any i, j, k;
6. Finite-difference approximation of equations for a given element is carried out in steps

ΔV_{jik} (by volume), Δt_{jik} (by time and the resulting system is solved until $\Delta t_{jik} \leq \Delta t_C^*$);

7. The calculation continues until the specified real time set T_{back}, where j —is the circuit number, i —is the branch number, k —is the element number.

Results of numerical calculations. The results of the heat transfer problem for the network shown in Figure 1 are given in Table 1. As one would expect, the best results were obtained by the grid method with the "left corner" approximation template (explicit scheme).

However, for the method to be stable, the Courant condition must be met: $U_k \frac{\Delta l}{\Delta t} \leq 1$, i.e. the choice of time step Δt depends on the volume step ΔV and at small Δt the problem was solved for quite a long time.

It is known that when using an implicit scheme, the stability condition has the form $U_k \frac{\Delta l}{\Delta t} \geq 1$:, which means that it is possible to select volume steps independently of each other.

5 Conclusion

An algorithm has been developed to automate the construction of discrete analogues of mathematical models (17), (19), (23).

In this case, it is necessary to know the initial temperatures of each element, the own structural dimensions of the network elements, the values of heat transfer coefficients between elements, the value of volumetric flow rates, etc. Based on these data, connection tables are built, which are the main -a new information base for the automatic generation of finite-difference schemes for models (17), (19), (23).

To take into account various thermal connections in a complex mathematical model, the algorithm provides a special coding system that allows you to identify the types of elements and their relative position (adjacency to each other).

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