Spectral features of structuring fractal objects

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Abstract. The spectral characteristics of various geometries aperiodic gratings approximants with fractal properties are considered. The equivalence of the Fibonacci structure approximants Fourier spectra formed by the projection and block methods is shown.

1 Introduction

The features of fractals individual types can be taken into account on the basis of their spectral properties analysis. Conventionally, fractals can be divided into four main groups. The first group includes fractals with a certain ratio explicitly self-similar elements both in the structure and Fourier spectrum. This relation determines the scaling coefficient ζ, which is an invariant of the considered systems. The first group fractals are characterized by the presence of one constant scaling coefficient ζ both in the structure and in the Fourier spectra. The first group fractals examples are Cantor's grating structures and their approximants [1].

The second group includes objects with internal symmetry of self-similarity (objects of Fibonacci, Period- doubling, Thue-Morse, etc. [1]). This group fractal-like structures are characterized by the presence of fractal Fourier spectra with one scaling coefficient value.

The third group fractals differ from the first two in that they have a difference in the scaling coefficients in the geometry of the fractal and its Fourier transform. This group includes the so-called star fractals, some Lindenmeyer-systems, for example, "snowflake" [2].

The fourth group includes fractal objects that do not have the fractality of the Fourier spectra. It includes dendritic structures and fractals built on the basis of using the properties of the Mandelbrot set [3-4].

Often the fractal objects construction is carried out using one-dimensional models of quasicrystals [1]. The use of such aperiodic structures has significantly expanded the element base of optical devices and improved a number of diagnostic methods. In particular, aimed at studying the properties of nanostructured materials [5]. Despite a significant number of publications devoted to the study of aperiodic structures various characteristics [1,6], the optical properties of their approximants [7-9] have not been fully investigated.

In the simplest case, approximants can be represented as block systems of the form: $A_l = \{S_l\}^p$. $S_l = \{A, B\}$ – approximant unit cell, l – the generation level of the used numerical aperiodic sequence (Fibonacci, etc.), A and B – sequence elements, p – the order of the approximant, determined by the number of elementary cells.

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The characteristics approximants study occupying an intermediate position between aperiodic and periodic systems has practical and theoretical interest. Replacing aperiodic systems with their approximants in some cases leads to obtaining structures with desired optical properties that are simpler to fabricate. The approximants optical characteristics similarity constructed by different methods indicates a high resistance to possible various changes introduced into their structure. Approximants of aperiodic structures have holographic properties. Each approximant carries information about the aperiodic structure on the basis of which it was built.

The work aim is to estimate the influence of the testing aperiodic second group fractal systems geometric features on the Fourier spectra both systems and their approximants. Aperiodic systems approximants are given by the block and projection methods.

This makes it possible to supplement the aperiodic structures and their approximants analysis the fractal properties stability to possible changes in their geometry at their structure formation stage.

2 Construction of approximants of fractal objects

The approximants construction was carried out on the basis of using the aperiodic numerical sequences properties $S_l = \{A, B\}$. Here, l is the generation level, A and B are the sequence elements. In aperiodic diffraction gratings, the values of elements A and B determine the distances between the slits.

The considered sequences S_l determine the elements alternation law in the primary structure. Binary representation $S_l = \{0,1\}$ is used to study the gratings and their approximants optical properties. As applied to grating structures, the position of units determines the distribution of scattering centers, and the position of zeros determines the free vacancies distribution.

As applied to the considered aperiodic structures, the approximants can be divided into two types according to their geometric construction method. The first approximants type is block: $A_l^{(1)} = A_l = \{S_l\}^p$.

The block method for constructing aperiodic systems is implemented using the substitution rules g, for example:

$$g(A) = A, B;\ \ g(B) = A;$$
$$g(A) = A, A, B; g(B) = A;$$
$$g(A) = A, A, A, B; g(B) = A. \quad (1)$$

Formula (1) is valid for the numerical sequences of Fibonacci, Silver mean and Bronze mean, respectively.

For construct aperiodic structures, formula (1) must be supplemented by specifying the initial levels S_0 or S_1. The second approximants type $\tilde{A}_{l'}^{(2)} = \{\tilde{S}_{l'}\}^p$is elementary cells sequence $\tilde{S}_{l'}$ with an aperiodic structure that can vary over a wide range without changing the principle of constructing the original numerical sequence. In this case, the elementary cells do not coincide with the generation levels of the aperiodic structure: $\tilde{S}_{l'} \neq S_l$. The index l’ is analogous l to determine the elementary cell complexity degree.

In particular, the second type includes approximants obtained by the projection method [10]. In articles, as a rule, the projection method for constructing aperiodic structures is considered in relation to Fibonacci systems and their approximants [10].

The projection approach is based on specifying a strip with a slope $\theta = arctg(1/\tau)$, where τ defines the structure. The strip is separated from the 2D periodic structure. The 2D grating points covered by this strip are projected onto a line that is parallel to the constructed strip with a width equal to the regular two-dimensional grating period. If $\tau = (1 + \sqrt{5})/2 \approx$

1.618 is true, then an aperiodic structure with the geometry of the Fibonacci system is constructed.

3 Simulation results

In Figure 1 there are strips selected in a 2D periodic grating at different tilt angles with a binary transmission function $F^o(A,B)=\{1,0\}$ using the graphical form [15], on the left.

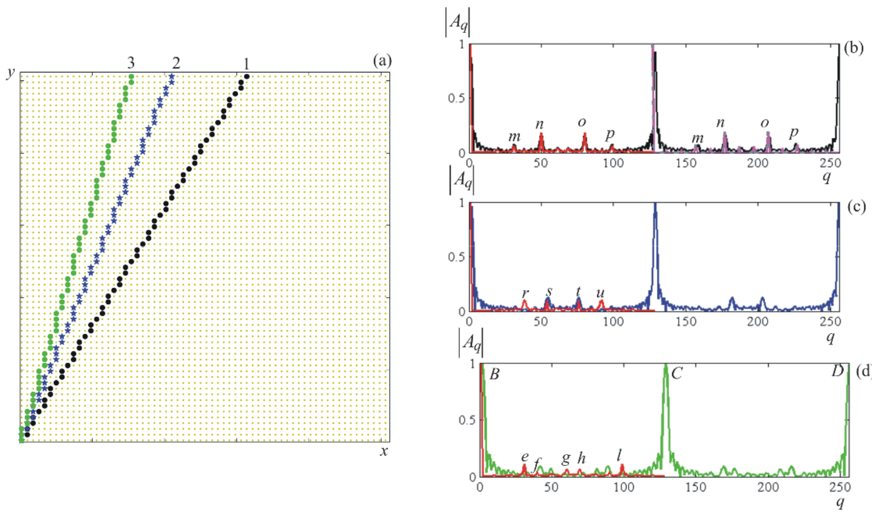

Fig. 1. Graphical representation (a) of grating structures (1-3) and amplitude distribution A_q (b)-(d) in the wave diffraction field on gratings with the Gold mean coefficient $\tau=(1+\sqrt{5})/2$ (1), Silver mean $\tau=1+\sqrt{2}$ (2) and Bronze mean $\tau=3.313$ (3). q is the spatial frequency. Red color shows on (b)-(d) spectra fragments corresponding to the block method 1-3. The dotted line in (b) shows the Fibonacci approximant amplitude distribution on $A_6^{(1)}$ with a unit cell of 13 elements $N_{cell}=13$. The spectral fragments local maxima are marked *mnop*, *rstu* and *efghl* for the Gold, Silver, and Bronze means, respectively.

Under normal structures illumination 1-3 (Figure 1, a), amplitude distribution projections $|A_q|$ $(q_x=128, q_y=q)$ are obtained, corresponding to different inclination angles of the structures selected on the graph (Figure 1, b-d). In Figure 1, b a fragment of the amplitude spectrum corresponds to the Fibonacci system with $\tau\approx 1.618$ (black color). The Fibonacci system spectrum fragment $|A_q|$ formed by formula (1), corresponding to the block construction method, is shown in red. From Figure 1, b it can be seen that the shape amplitude distribution coincides both for the grating obtained by the block method and for the grating formed by the projection method.

This is evidenced by the high correlation coefficient between the amplitude spectra fragments $|A_q|$ corresponding to the block and projection construction Fibonacci system methods ($K\approx 0{,}9$ on the interval *mp*). Spectrum fragment $|A_q|$ for the Fibonacci system approximant $A_l^{(1)}$ with a unit cell of 13 elements and a period p =10 is shown by a dotted line in Figure 1, b. In this case, the shape correlation coefficient between the amplitude spectra fragments $|A_q|\Big|_{q=1\ldots127}$ for the block implementation of the Fibonacci system and its approximant $A_l^{(1)}$ is $K\approx 0.96$. The large correlation coefficient value of the considered optical characteristics indicates the possibility of replacing the aperiodic system with its simpler periodic analogue, which is an approximant.

If $\tau \approx 2.414$ is satisfied, an aperiodic structure is obtained with a geometry characteristic of systems built on the Silver mean basis. Figure 1, c shows the amplitude spectrum fragment $|A_q|$ $(q_x = 128, q_y = q)$ of the system with $\tau \approx 2.414$ (blue color). The amplitude spectrum fragment $|A_q|$, obtained for the structure based on the Silver mean according to formula (1) with the initial generation level $S_0 = B$, is marked in red. The correlation coefficients between the amplitude fragments $[BC)$ of the spectra $|A_q|$ corresponding to the block and projection methods of specifying systems based on the silver and bronze means are much smaller values $K \approx 0.57$ and $K \approx 0.48$, respectively. This result indicates a significant influence of the considered systems initial construction with the geometry different from the Fibonacci systems geometry. If τ is an integer, then the projection method result is some periodic sequence. The such a sequence Fourier spectrum is characterized by a system of peaks *B, C, D*.

The best coincidence ($K \approx 0{,}99$) of the spectra $|A_q|$ corresponding to the block and projection methods of specifying aperiodic systems is achieved using the classical 1D projection Fibonacci systems representation.

The numerical simulation results showed that the Fourier transforms of the considered aperiodic systems have scaling properties and are fractal. Figure 2 shows the wavelet analysis results using the wavelet coefficients pattern of the Fibonacci structure approximant obtained by the projection method (Figure 2, a). The Fibonacci objects pattern characteristic structure *cd* is indicated by dotted lines. "Discrete" Meyer Wavelet is used for wavelet transform. The wavelet coefficients pattern (Figure 2, b) well demonstrates the Fibonacci spectrum hierarchical structure, which has self-similar fragments.

The scaling coefficients ζ for fragments of the amplitude spectra $|A_q|$ were determined by the self-similar elements' sizes in their structure ratio $\varsigma_1 = no/mn$; $\varsigma_2 = ru/st$; $\varsigma_3 = eg/ef$. So, for the systems in Figure 1, b - $\varsigma_1 \approx 1.6$, in Figure 1, c -$\varsigma_2 \approx 2.4$ and in Figure 1, d -$\varsigma_3 \approx 3.3$.

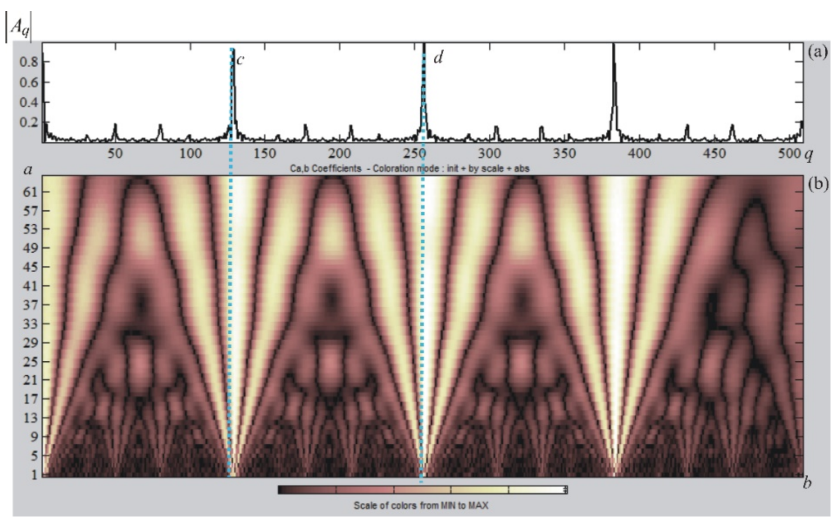

Fig. 2. The Fibonacci structure Fourier spectrum Wavelet analysis based on the "Discrete" Meyer Wavelet.

Note that the obtained scaling coefficients ζ_1, ζ_2 and ζ_3 the approximants optical characteristics, correspond to the previously obtained scaling coefficients of diffraction spectra $|A_q|$ for structures characterized by the coefficients of the Gold, Silver and Bronze means. The Thue-Morse, Period-doubling, and M-bonacci approximants grating systems [8] have similar scaling properties. The results presented in Figure 1 also point to the structural similarity in individual fragments shape (patterns *no* and *mn*) of the considered aperiodic systems spectral characteristics.

4 Conclusion

The research carried out indicates the Fibonacci structures and their approximants optical characteristics equivalence obtained by the projection and block methods. The aperiodic structures and approximants optical characteristics under condition $\tau \neq (1+\sqrt{5})/2$ obtained by various methods have certain differences. The considered questions about the influence the aperiodic structures approximants geometric construction on their optical properties are the general theoretical fractal optics problem heart solving. This problem consists in determining and analyzing general patterns between the objects structural features and the of their characteristics fractality. Its solution for different geometry approximants makes it possible to develop common criteria for identifying a wide aperiodic objects class.

References

1. E.L. Albuquerque, M.G. Cottam, Phys. Rep. **376**, 225- 337 (2003).
2. T. DeJong, Crop Science **62**, 2091-2106 (2022).
3. B. Mandelbrot, *The fractal geometry of nature* (WH freeman, New York, US, 1982).
4. J.R. Nicolás-Carlock, J.L. Carrillo-Estrada, V. Dossetti, Scientific reports **6**, 19505 (2016).
5. M. Paulsen, L.T. Neustock, S. Jahns, J. Adam, M. Gerken, Opt. Quant. Electron. **49**, 107 (2017).
6. E. Macia, Rep. Prog. Phys. **69**, 397 (2006).
7. L.D. Negro, *Optics of Aperiodic Structures – Fundamentals and Device Applications* (UK, Taylor & Francis Group: CRC Press, Abingdon, 2014).
8. P.V. Korolenko, P.A. Logachev, Yu.V. Ryzhikova, Phys. Wave Phenom. **23**, 46-51 (2015).
9. S. Olsson, E. Broitman, M. Garbrecht, J. Birch, L. Hultman, F. Eriksson, J. Mater. Res. **31**, 232-240 (2016).
10. A.P. Tsai, Sci. Technol. Adv. Mater. **9**, 013008 (2008).