Modeling roll contact curves

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Abstract. Roller devices equipped with deformable coatings are crucial for various mechanical operations in material processing. Understanding the contact lines between these rollers and deformable coatings is essential for revealing physical phenomena in roller devices. This study derives analytical expressions for contact curves based on the physical phenomena in the deformation zone. It has been established that the factor that most influences the mathematical model of the contact curve of the rolls is the ratio of the deformation rates of the roll coating to the deformation rate of the processed material during compression.

1 Introduction

To perform various mechanical operations for processing materials with roller devices, rollers equipped with deformable coatings are used. Research into the contact lines of rollers with deformable coatings is also of great importance for revealing physical phenomena in roller devices [1-2].

Physical phenomena in roller machines are similar to physical phenomena when a wheel rolls. The contact line of the wheel is expressed by various analytical expressions [3,4]. The most common theory is based on the theory of a rolling rigid wheel with a larger diameter than the deformable one being replaced [5-7].

In works [8-9], devoted to the study of roller devices with deformable rollers, physical phenomena in the deformation zone are studied with the choice of analytical expressions for the roller contact curves. Therefore, the results of these works do not accurately reveal the phenomenon of contact stress distribution.

In this article, the authors modeled the contact line of a roller module, in the case where the deformation of the contacting parts in the contact area occurs along the line of action of theelementary normal force.

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2 Analytical solutions of the problem posed

At the contact points, elementary normal forces act on the material being processed from the rollers in the direction of $n-n$, therefore, the deformation of the contacting bodies also occurs in the direction of $n-n$.

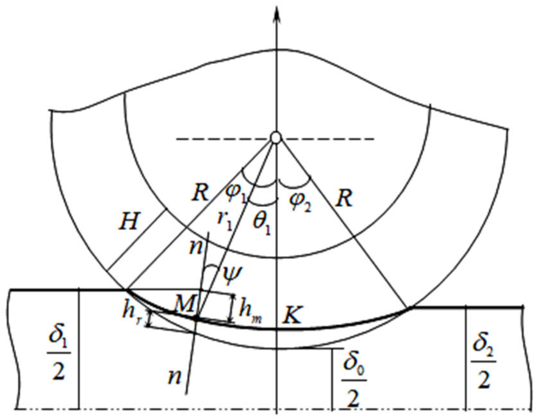

Fig. 1. Scheme of contact interaction in a two-roll module.

According to Figure 1, at point M , the thickness of the processed material and the roller coating change as:

$$h_{1M} = \frac{r_1 \cos\theta_1 - R\cos\varphi_1}{\cos(\theta_1 - \psi_1)}, \quad h_{1r} = \frac{R\cos\theta_1 - r_1\cos\theta_1}{\cos(\theta_1 - \psi_1)}, \tag{1}$$

where ψ – is the angle between directions $n-n$ and $r-r$.

We present the following notations

$$k_1 = \frac{\dfrac{dh_{1r}}{dt}}{\dfrac{dh_{1M}}{dt}}. \tag{2}$$

Then, considering expression (1), we obtain

$$\frac{d}{dt}\left(\frac{R\cos\theta_1 - r_1\cos\theta_1}{\cos(\theta_1 - \psi_1)}\right) = k_1 \frac{d}{dt}\left(\frac{r_1\cos\theta_1 - R\cos\varphi_1}{\cos(\theta_1 - \psi_1)}\right)$$

or

$$\frac{d}{d\theta_1}\left(\frac{R\cos\theta_1 - r_1\cos\theta_1}{\cos(\theta_1 - \psi_1)}\right) = k_1 \frac{d}{d\theta_1}\left(\frac{r_1\cos\theta_1 - R\cos\varphi_1}{\cos(\theta_1 - \psi_1)}\right), \tag{3}$$

3 Results

After transforming equality (3), we get

$$-tg(\theta_1-\psi_1)\frac{d}{d\theta_1}(\theta_1-\psi_1)=\frac{(1+k_1)\frac{d}{d\theta_1}(r_1\cos\theta_1)+R\sin\theta_1}{(1+k_1)r_1\cos\theta_1-R(\cos\theta_1+k_1\cos\varphi_1)}$$

or after integration

$$\ln C\cos(\theta_1-\psi_1)=\int\frac{(1+k_1)\frac{d}{d\theta_1}(r_1\cos\theta_1)+R\sin\theta_1}{(1+k_1)r_1\cos\theta_1-R(\cos\theta_1+k_1\cos\varphi_1)}d\theta_1, \tag{4}$$

in this case, we find the constant C . according to the condition:

$$r_1=R\text{, for }\theta_1=-\varphi_1. \tag{5}$$

Integration of the right side of equality (4) depends on the expression k_1, which in turn is a function of time, hence the angle θ_1.

In the compression zone, the processed material interacts with the roller coating for a short time. Therefore, k_1 can be considered constant.

Let us rewrite equalities (4) for the case $k_1=const$

$$\ln C\cos(\theta_1-\psi_1)=\int\frac{d((1+k_1)(r_1\cos\theta_1)-R(\cos\theta_1+k_1\cos\varphi_1)}{(1+k_1)r_1\cos\theta_1-R(\cos\theta_1+k_1\cos\varphi_1)}.$$

After integration, we obtain:

$$C\cos(\theta_1-\psi_1)=(1+k_1)r_1\cos\theta_1-R(\cos\theta_1+k_1\cos\varphi_1). \tag{6}$$

Considering condition (5), it follows from equality (6) that $C=0$.

Then from equality (6) we obtain

$$r_1=\frac{R}{1+k_1}\left(1+k_1\frac{\cos\varphi_1}{\cos\theta_1}\right),\quad -\varphi_1\le\theta_1\le 0. \tag{7}$$

By analogy with (7), for the recovery zone, we obtain:

$$r_2=\frac{R}{1+k_2}\left(1+k_2\frac{\cos\varphi_2}{\cos\theta_2}\right),\quad 0\le\theta_2\le\varphi_2\ , \tag{8}$$

where φ_2- is the angle that determines the end point of the contact curve, k_2- is the ratio of the strain rate of the roller coating to the strain rate of the processed material under recovery.

Generalizing formulas (7) and (8), we obtain

$$r=\begin{cases}\dfrac{R}{1+k_1}\left(1+k_1\dfrac{\cos\varphi_1}{\cos\theta}\right), & -\varphi_1\le\theta\le 0,\\ \dfrac{R}{1+k_2}\left(1+k_2\dfrac{\cos\varphi_2}{\cos\theta}\right), & 0\le\theta\le\varphi_2.\end{cases} \tag{9}$$

Let us analyze the contact lines of the roller module in cases when the thickness of the contacting bodies at the contact points occurs along the axis Oy , and when - along the radius r .

In the first case, this case $\psi_1=0$.

Then from (6) we get

$$C=(1+k_1)r_1\cos\theta_1-R(\cos\theta_1+k_1\cos\varphi_1).$$

Applying condition (5) we obtain a formula coinciding with formula (7).

Let the change in the thickness of the contacting bodies at each point of the contact zone occur along the radius r. In this case $\psi_1 = 0$.

Then from (6) we get

$$C\cos\theta_1 = (1+k_1)r_1\cos\theta_1 - R(\cos\theta_1 + k_1\cos\varphi_1).$$

Applying condition (5) we also obtain a formula coinciding with formula (7).

Thus, the mathematical models of the roll contact curve in cases where the deformation of the contacting bodies at each point of the contact zone occurs along the axis Oy, along the radius r and in the direction of the elementary normal force, coincide.

The above asserts that the mathematical model of the roll contact curve does not depend on the direction of deformation of the interacting bodies.

According to Figure 1, system (9) must be fulfilled under the condition:

$$r(0) = \frac{R}{1+k_1}(1+k_1\cos\varphi_1) = \frac{R}{1+k_2}(1+k_2\cos\varphi_2).$$

From here we have

$$k_2 = \frac{k_1(1-\cos\varphi_1)}{(1-\cos\varphi_2) - k_1(1-\cos\varphi_1 - (1-\cos\varphi_2))}. \tag{10}$$

From Figure 1, it follows that

$$R\cos\varphi_1 + \frac{\delta_1}{2} = R + \frac{\delta_0}{2}, \qquad R\cos\varphi_2 + \frac{\delta_2}{2} = R + \frac{\delta_0}{2},$$

where $\delta_0 -$ is the distance between the rollers.

From here we have

$$1-\cos\varphi_1 = \frac{\delta_1 - \delta_0}{2}, \quad 1-\cos\varphi_2 = \frac{\delta_2 - \delta_0}{2} \tag{11}$$

or to a first approximation

$$\varphi_1 = \sqrt{\frac{\delta_1 - \delta_0}{R}}, \qquad \varphi_2 = \sqrt{\frac{\delta_2 - \delta_0}{R}}. \tag{12}$$

Considering expressions (11), from equality (10), we find

$$k_2 = \frac{k_1(\delta_1 - \delta_0)}{\delta_2 - \delta_0 - k_1(\delta_1 - \delta_2)}. \tag{13}$$

From system (9) it follows that the factor influencing the mathematical model of the roll contact curve is the ratio of the roll coating deformation rates to the deformation rate of the processed material during compression. In this case, $k_1 \geq 0$.

In the limiting case, i.e. at $k_1 = 0$, $\frac{dh_{r1}}{dt} = 0$, therefore, $h_{1r} = const$, i.e. the roll coating is not deformed.

When $k_1 = 0$ from equality (13) we have that $k_2 = 0$.

Then from system (9), it follows that

$$r = R, \quad -\varphi_1 \leq \theta \leq \varphi_2. \tag{14}$$

In this case, the angle φ_2 depending on the nature of the deformation of the material being processed can take values from zero to φ_1.

Various materials are processed in the roller module. They can be absolutely elastic, elastic-viscous and plastic [8].

In the first case, the processed material, when leaving the roll tip, will completely restore its shape, therefore $\delta_2 = \delta_1$, therefore from expression (12) it follows that $\varphi_2 = \varphi_1$. Then from equality (14) we obtain

$$r = R, \quad -\varphi_1 \le \theta \le \varphi_1 \ . \tag{15}$$

In the second case, the processed material, when leaving the roll tip, will partially restore its shape, therefore $\delta_2 < \delta_1$, consequently, $\varphi_2 < \varphi_1$. Then from equality (14) we have

$$r = R, \quad -\varphi_1 \le \theta \le \varphi_2, \ \varphi_2 < \varphi_1 . \tag{16}$$

In the third case, there will be no reverse deformation of the processed material, therefore $\delta_2 = \delta_0$, consequently, $\varphi_2 = 0$. Then from equality (14) it follows

$$r = R, \quad \varphi_1 \le \theta \le 0 \ . \tag{17}$$

For values of $k_1 > 0$, the input part of the contact curve is described by the first equation of system (9), i.e. equation (7). The equation describing the output part depends on the nature of the deformation of the material being processed.

If the material being processed is absolutely elastic, then $\delta_2 = \delta_1$, therefore, $\varphi_2 = \varphi_1$ and $k_2 = k_1$ (according to expressions (12) and (13)). Then from equation (8) it follows

$$r_2 = \frac{R}{1+k_1}\left(1 + k_1 \frac{\cos\varphi_1}{\cos\theta_2}\right), \quad 0 \le \theta_2 \le \varphi_1 . \tag{18}$$

If the material being processed is elastic-viscous, then $\varphi_2 < \varphi_1$, therefore, $k_2 \ne k_1$. Then k_2 it is determined by expression (13) and the output part of the contact curve is described by the equation

$$r_2 = \frac{R}{1+k_2}\left(1 + k_2 \frac{\cos\varphi_2}{\cos\theta_2}\right), \quad 0 \le \theta_2 \le \varphi_2 \ . \tag{19}$$

If the material being processed is plastic, then $k_2 \to \infty$.

From equation (8) we obtain

$$r_2 = \frac{R\cos\varphi_2}{\cos\theta_2}, \quad 0 \le \theta_2 \le \varphi_2 \ . \tag{20}$$

k_1 has a positive and finite value. Therefore, according to equality (13), the condition must be satisfied

$$\delta_2 - \delta_0 - k_1(\delta_1 - \delta_2) = 0$$

Then from equality (12), we obtain:

$$\varphi_2 = \sqrt{\frac{k_1(\delta_1 - \delta_0)}{(1+k_1)R}}. \tag{21}$$

Thus, the rolling contact curve for a positive value of the parameter k_1 consists of two parts: the input part, described by formula (7), and the output part, described by one of formulas (18), (19) and (20).

4 Conclusion

Based on the study of physical phenomena in the deformation zone of roller devices, analytical expressions for contact curves are obtained.

The results of the study show that the contact curve consists of two parts: the input part, described by formula (7), and the output part, described by one of formulas (18), (19) and (20).

It has been established that the factor that most influences the mathematical model of the contact curve of the rolls is the ratio of the deformation rates of the roll coating to the deformation rate of the processed material during compression.

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