A mathematical model of steel sample stretching: establishing a unified nature and mechanism of the relationship between deformation stages

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Abstract. The relevance of creating a mathematical model of deformation of various steels is dictated not only by the fact that it can be used to determine their physical and mechanical data used under certain external loads, but also because the mathematical model itself contributes to understanding the processes that occur in the deformed sample itself, allowing us to reveal the essence of the structure of the metal itself. Moreover, the current studies of this issue, despite the fact that they were carried out on the basis of a physical and mathematical approach, could not model the real process of this mechanism due to the fact that they did not find a single essence of this process. However, these studies have significantly helped in identifying the real mechanism of deformation and creating a new mathematical model. Using the data of existing experimental studies on uniaxial tension of steel rods, establish a single relationship between the stages of their deformation, taking into account the discrete structure of the crystal lattice of the metal, based on elastic and plastic shifts of its kinetic units. Based on the results obtained, develop a mathematical model that allows you to determine the necessary loads when using certain steels in specified conditions of their operation.

1 Introduction

The process of studying the physical and mechanical properties of metals is primarily dictated by the practical need associated with their use in industrial practice. As a rule, changes in the physical and mechanical properties of different metals are associated with changes in the structure of the crystal lattice of the material. Therefore, the study of this process is carried out on the basis of various methods - X-ray structural analysis, thermal analysis, molecular analysis and others.

Each research method, and first of all the method of studying physical and mechanical properties, has its own set of theoretical and methodological provisions that determine the mathematical model of a solid. Moreover, to study the process of deformation of the same material, several different mathematical models (MM) are often used. For example, models

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that separately evaluate its elastic or plastic deformation, which leads to the division of one general, inherent in nature, physical process into parts. Therefore, only a joint assessment of both elastic and plastic characteristics of materials based on the discrete structure of matter – atoms or molecules, for example, as was shown in the creation of MM of various liquids, solutions, melts and polymer systems [1], will allow us to describe with high accuracy the mechanism of stress occurrence in a particular deformation region.

2 Materials and methods

This study is based on the theoretical analysis of the accepted characteristics of elastic deformation. The main task of constructing an MM for deformation of different metals based on the discrete structure of the metal crystal lattice is to establish an accurate relationship between the values of mechanical characteristics of metals adopted in experimental studies and to establish the dependence of various modules and coefficients, based on the diagram (see Figure 1), obtained experimentally, on the displacement of atoms, which are the kinetic units of the material (KU), under the action of an external force.

It is known that to obtain a diagram, steel rods of standard sizes are stretched until complete destruction (rupture). A special device records the dependence of the absolute elongation of the sample on the longitudinal tensile load applied to it, and the recorder automatically plots a curve characteristic of a given material in the coordinate system $F - \Delta l$, where: F is the longitudinal tensile force, (kg·m/s^2); $\Delta\ l$ is the absolute elongation of the working part of the sample, (mm).

At the very beginning of the tensile test, the tensile (external) force $F_{ext}$and the deformation of the rod Δl are equal to zero, so the diagram starts from the intersection point of the corresponding axes (point *O*). In the section *I* to the point E, the diagram is automatically drawn as a straight line. This indicates that in this section of the diagram, at a strictly specified speed, the deformation of the rod Δl increases proportionally to the increasing load F_{ext}. Which clearly characterizes the mechanics of establishing the modulus of longitudinal elasticity or the so-called Young's modulus at the first stage of deformation O-E (see Figure 1).

3 Results and discussion

The diagram shows that the Young's modulus adopted for the evaluation of different metals reflects the relationship between the characteristics of the experiment, on the basis of which it can be written as:

$$E = \frac{\sigma}{\varepsilon} \quad (1)$$

In order to clearly show the relationship between the characteristics, we substitute into equation (1) the initial characteristics that determine the normal stress σ in the cross-section of the rod and its relative elongation ε_1. As a result, we obtain the equation:

$$E = \frac{\sigma}{\varepsilon} = \frac{F_{ext} * L}{S_{st} * \Delta l} = \frac{A}{\Delta V} = P(кг/м * c^2) \quad (2)$$

Equation (2) reveals the possibility of refining the assessment of the mechanism of the sample deformation process. Namely, that the constancy of the value of Young's modulus E is determined by the constant energy $A = F * L, (kg * m^2/s^2)$, expended on the relative elongation $\varepsilon_1 = \Delta l/L, (-)$ of the sample during its elastic deformation within the limits $\varepsilon \leq 3\%$; $\Delta V = S_{st} * \Delta l, (m^3)$ is the volume determining the degree of displacement of the KU of the material during elastic deformation of the sample; $\sigma = F_{ext}/S_{st}(kg/m * s^2)$ is the normal stress in the cross-section of the sample, determined under the condition of constancy of the cross-sectional area of the rod $[S_{st}] = Const$.

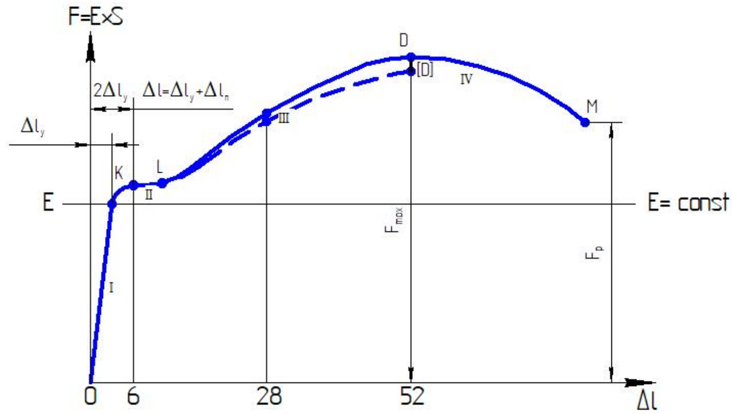

Fig. 1. Tensile stress diagram of a steel specimen, including stages characterized by: *I* - Young's modulus; *II* - yield strength; *III* - self-hardening; *IV* - destruction.

However, the assumption that the cross-sectional area is constant $S_{st} = Const$ does not correspond to the actual reality of the process, as evidenced by the Poisson ratio ν. Since this coefficient reflects the change in the transverse dimensions of the rod $\varepsilon = \frac{\Delta S}{S_{st}}$ to its relative elongation $\varepsilon_1 = \frac{\Delta l}{L}$ as:

$$\nu = -\frac{\varepsilon}{\varepsilon_1} \quad (3)$$

It should be noted that Poisson's ratio ν not only reflects the change in the linear dimensions of the rod, but also links this process with changes in the elementary volume of the crystal lattice of the material through the shift of an atom from the equilibrium position in the elastic region.

In this case, we pay special attention to the fact that, based on equations (1) and (2), written on the basis of experimental data obtained within the limits of elastic deformation $\varepsilon \leq 3\%$ (see Figure 1), the dimension and value of normal stress in the cross section of the sample (4)

$$\sigma = \frac{A*\Delta l}{V*L} = [E]*\varepsilon, (kg/m*s^2) \quad (4)$$

acquires the same dimension of pressure $P = A/V = A/S*L(kg/m*s^2)$, inherent in the dimension of Young's modulus. And only because the elastic deformation is defined in dimensionless form $\varepsilon_1 = \Delta l/L, (-)$.

After passing point E, the diagram changes its direction, since at point *E* the line begins to turn in the direction parallel to the axis *Δ l*. That is, further on the entire section *EKL* the deformation of the rod increases at practically the same value of the load and deformation rate. Thus, irreversible changes in the dimensions of the rod material begin to occur, accompanied by plastic deformation. It is believed [2] that the process of plastic deformation corresponds to the process of pure shear of parts of the crystal (grains) relative to each other along slip planes, which can only be achieved under the action of shear stress. In turn, studies of shear stress τ [3,4] show that shear stress is proportional to shear strain $\gamma = \frac{\Delta l}{h}(-)$, which is also defined in relative units, that is, again in dimensionless form, which in turn leads to the fact that the elastic shear constant of the metal has the dimension of pressure:

$$G = \frac{\tau}{\gamma}(кг/м*c^2) \quad (5)$$

At the same time, the question naturally arose: why did the desire to clarify the assessment of the mechanism of the sample deformation process lead us to such a result, when all the established characteristics - Young's modulus and shear modulus, used to assess the process of deformation of the metal sample, turned out to be associated with the dimension that determines the constancy of pressure.

To answer this question let us start with the fact that the researchers themselves, based on the set goal of determining the characteristic of the metal that ensures its constancy over time, "threw out" the time parameter $t(c)$ from the assessment of the research results. As a result, restrictions arose - the introduction of a limitation on the speed of application of the external load and the limitation of the "scale" factor - the establishment of conditions for choosing the size of the sample in the form of a ratio d/l. That is, the scientists themselves prepared the ground for the adoption of a non-inertial reference system (NIRS), in which they established that it is the change in the geometric dimensions of the rod to the original length, in relative units, that reflects the change in stress in the rod during elastic deformation.

Thus, the scientists themselves, as a result of the adopted method of testing the sample, left only the account of the change in external force per standard surface unit [S] in NIRS. Which leads us to consider all processes at the level of the balance of pressure of external and internal forces, which formed the basis of all further research

$$E = P = F_{ext}/[S] = (kg/m * s^2) = Const \tag{6}$$

In fact, the value of Young's modulus, according to equation (2), can be expressed based on the balance of pressures (7) of external and internal forces as:

$$E = P = F_{ext}/[S] = \frac{\sigma}{\varepsilon}(kg/m * s^2), \tag{7}$$

By substituting $\varepsilon_1 = \frac{\Delta l}{L}$ and $\sigma = F_{ext}/[S]$, equation (7) can be transformed into an equation that determines the Hooke coefficient:

$$K = E * \Delta l = F_{ext} * L/[S] = A/[S](кг/c^2) \tag{8}$$

In (8) for values of $E = Const$ and $K = Const$, the value Δl Δl should determine the maximum permissible change in the length of the rod - the displacement of KU (atoms) in the crystal lattice from the equilibrium position.

From expression (8) it follows that taking into account the dimension of Young's modulus and the dimension of existing characteristics at stage *II* in the rod, when irreversible changes in the geometric dimensions of the rod begin to occur, should be realized only by increasing the shift of KU in the crystal lattice. That is, at point E (Figure 1), the elastic displacement reaches its limit and in section *II*, plastic displacement of KU (atoms) begins to occur, which requires additional energy ΔA. In this case, the balance of internal $E * \Delta l$ and external $A/[S]$ interactions in equation (8) shows that the deformation process should no longer be considered on the basis of Poisson's ratio, but on the basis of the dimension of Hooke's coefficient $\left[\frac{kg}{s^2}\right]$. In this case, based on the assessment of the displacement, on the basis of equation (8), the external force can be expressed as:

$$F_{ext} = E * \Delta l * S/L(kg * m/s^2) \tag{9}$$

Further, taking into account that according to the Van der Waals law the entire volume of the rod material V_{st} participates in the deformation, we can determine the internal resistance force of each KU included in the volume of the entire sample. We assume that the cross-sectional area of the rod S_{st} consists of a certain number n_1 of atomic areas s_{at}:

$$S_{st} = n_1 * s_{at}, \tag{10}$$

and the length of the sample $L_{st} = n_2 * a$ consists of a certain number of lengths of the crystal lattice parameter - a, then the volume of the crystal lattice of one KU is expressed as:

$$V_{k.u} = s_{at} * a(m^3) \tag{11}$$

Accordingly, the volume of the rod V_{st} will contain the total number $N_{k.u}$ of kinetic units, defined as:

$$N_{k.u} = \frac{V_{st}}{V_{k.u}} = \frac{S_{st} * L_{st}}{s_{k.u} * a}(-) \tag{12}$$

Considering that the value of Young's modulus (if we do not take into account the period allocated for metal fatigue) can be considered a constant value over time $E = Const$, then the 0-E section, the elastic deformation section (see Figure 1), can be considered independent of the duration of application of the external load. Indeed, having written out equation (1) as:

$$\frac{E}{\sigma} = \frac{L}{\Delta l} ,$$

can be further transformed as:

$$E * \Delta l = \sigma * L (kg/s^2) \tag{13}$$

(13) allows us to characterize the process of changing the Young's modulus only by a direct dependence on the value of the elastic displacement of the sample Δl, leading to the expression of the Young's modulus in the dimension of the Hooke's coefficient (10).

In other words, the accepted condition of constancy of the Young's modulus $[E] = Const$ leads to the fact that each time in tests, deforming the standard volume of the rod $[V_{st}] = Const$ within the elastic deformation, we obtain that with a minimum spread of experimental values, the energy of interatomic interaction (resistance) of kinetic units ζ_{at}, enclosed in the standard volume $[V_{st}]$, should also be a constant value $\zeta_{at} = Const$. It is this condition, according to equation (2), that allows us to express the Young's modulus as a characteristic that does not depend on the deformation of the sample, but is associated only with the energy of interatomic interaction ζ_{at}, as a constant value in time *t* at all stages of deformation of the rod:

$$E = \frac{A}{V} = \frac{\zeta_{at} * N_{k.u}}{V_{k.u} * N_{k.u}} = \frac{\zeta_{at}}{V_{k.u}} = Const \tag{14}$$

This allows the entire process of sample deformation (see Figure 1) to be characterized as consisting of two parts, when the first stage of elastic displacement KU remains unchanged, limited by the limit of the elastic displacement value (see Figure 1, line O-E), and all other stages depend on the displacement value - λ_{plast} atoms after overcoming the "potential barrier" - some limiting displacement of the atom from the equilibrium position λ_{elast}.

Considering the deformation process at stages II and III, it is necessary to take into account that in metals the energy spent on elastic deformation tends to return the atom to its original position in the crystal lattice node. That is, during plastic deformation, external energy must be spent both on overcoming elastic deformation KU and on the shift of kinetic units, which leads to an increase in the expended external energy (force). This is why the diagram at stages II and III "goes up". At point *D* in the working part of the tested rod, a so-called "neck" appears, probably caused by disturbances in the structure of the material (formation of defects, voids, microcracks, etc.). Due to the reduction in the cross-sectional area of the neck, the tensile force required to stretch it decreases, and the curve of the diagram "goes down". At point *M*, the rod ruptures. Of course, the rod is destroyed in the section where the "neck" was formed.

Such a detailed examination of the deformation stages is necessary for us to clarify the assessment of the relationship between the deformation stages. Since only understanding the relationship of all these stages - I, II and III, sequentially occurring in the deformed volume will allow us to assess the mechanism of the process through the values of different modules and establish the limits of their study and application. It should be noted that the assessment of plastic deformation and the assessment of elastic deformation led researchers to the conclusion that in an isotropic material the values E, G, ν do not depend on the direction in which the sample is “cut out” from the medium [9]. It should be added that, based on equation (2), the pressure *P*, ($kg/m \cdot s^2$) inside the metal also does not depend on the direction of its application.

Thus, from the above it can be understood that the stages of deformation of the rod, as a successive displacement of KU, are associated with the definition of Young's modulus and Poisson's ratio as an assessment of the elastic deformation, and Hooke's coefficient establishes a relationship with the elastic-plastic shift of KU. Therefore, for a real assessment of the value of the total external energy, the expended work, it is necessary to sum up the reversible (elastic) Δl_{elas} and irreversible (plastic) Δl_{plast} deformations, representing them in the form of a total displacement of the atom:

$$\Delta l = \Delta l_{elas} + \Delta l_{plast} \tag{15}$$

Condition (16) corresponds to the fact that plastic displacement in metals Δl_{plast} can begin only after the expenditure of external energy A_{ext} on elastic displacement KU of the metal:

$$A_{ext} = F_{plast} * \Delta l_{plast} = F_{elas} * \Delta l_{elas} \tag{16}$$

Expressing the external elastic displacement force KU in the rod F_{ext}according to equation (10) as:

$$F_{ext} = K * S/L = E * \Delta l * S/L(kg * m/s^2), \tag{17}$$

the elastic displacement Δl_{elas} of the sample can be determined from equation (17) as:

$$\Delta l_{elas} = K/E = F_{elas} * L/E * S \tag{18}$$

Further, taking into account that in metals the energy of the relationship between all KU will be determined at the level of the balance of energy spent on elastic-plastic mixing KU - λ_{plast}, which is achieved after overcoming the "potential barrier" by atoms λ_{elas}, that is,

$$\lambda_{elas}/\lambda_{plast} = 1, \tag{19}$$

all KU of the rod, according to equation (19), will pass into the region of elastic-plastic displacement. In this case, we will obtain that only after overcoming the potential barrier determined by the value (20), the total displacement within the elastic deformation, we can write the following equality of shifts of KU:

$$\Delta l_{elas} = \Delta l_{plast} \tag{20}$$

It can be expressed as:

$$\frac{F_{ext} * L}{E * S} = \frac{A_{ext}}{F_{ext}} \tag{21}$$

From where, under the condition of balance of energies (17), forces (18) and displacements (21), we can transform equations (22) as:

$$F_{ext}{}^2 = A_{ext} * E * S/L \tag{22}$$

Where the value of the external force F_{ext} at a particular stage of deformation of the sample in general form is determined by the following equation:

$$F_{ext} = \sqrt{A_{ext} * E * S/L} \tag{23}$$

Accordingly, when substituting the energy value expressed according to equation (17) as:

$$A_{ext} = F_{ext} * \Delta l = E * S * \Delta l\ , \tag{24}$$

where the external force is expressed through the value of Young's modulus $F_{ext} = E * S(kg * m/s^2)$, in equation (24) we obtain an equation that determines the additional external force:

$$F_{ext} = E * S * \sqrt{\frac{\Delta l}{L}} \tag{25}$$

necessary for a real assessment of the value of the deformation of the sample, taking into account reversible (elastic) and irreversible (plastic) deformations. Taking into account equation (16), we represent equation (26) as:

$$F_{ext} = E * S * \sqrt{\frac{\Delta l_{elas} + \Delta l_{plast}}{L}} \tag{26}$$

As a result, we obtain an equation that allows us to describe the nature of the additional resistance of the sample at the *II*-nd and *III*-rd stages of its deformation, determined by the development of the ratio of elastic (reversible deformation) and deformation associated with the plastic displacement of KU, estimated in their relative displacement, at the limiting value of their displacement $\frac{\Delta l_{elas} + \Delta l_{plast}}{L} = \frac{L}{L} = 1$. In this case, according to the obtained equation

(27), the initial value of Young's modulus can be calculated based on the value of the required external force at the first stage of its deformation, characterized only by the development of elastic displacements of KU, determined by the structure of the crystal lattice of the material. In other words, Young's modulus is the main initial characteristic of the material, practically unchanged over time, determining all other stages associated with the deformation of KU and their displacement during a given test.

4 Discussion

The results of the above analysis clearly show that at the *I*-st stage of elastic deformation, the theoretical value of Young's modulus *E* can be determined according to equation (27) as:

$$E = \frac{F_{ext}}{S_{st} * \sqrt{L/L}} = \frac{F_{ext}}{S_{st}} (kg/m * s^2) \tag{27}$$

In other words, based on the fact that the obtained graph (see Figure 1) accurately reflects all parameters of sample stretching, we obtain that at point E the elastic displacement at the sample length L= 100 *мм* at the moment of stretching reaches $\Delta l_{elas} = 3мм$. However, after the tensile load is no longer applied, everything returns to the original sample length ($\Delta l_{elas} = 0$). It is this condition that allows us to express Young's modulus as an internal characteristic that does not depend on the sample deformation and is constant over time at all stages of deformation (15), and to present the sample deformation process as a diagram in Figure 1, where Young's modulus is the limiting value (potential barrier), after overcoming which one or another stage of plastic displacement is realized.

At stage *II*, based on the fact that, as was assumed when constructing MM, plastic displacement of KU during sample stretching in the direction of the axis $0 - \Delta l$ will begin only after overcoming the potential barrier of elastic displacement at $\Delta l_{elas} = \Delta l_{past}$, as shown in Figure 1. As a result, the total elongation deformation of the sample will be $\Delta l = 6$*мм*. Substituting these values into equation (27):

$$F_{ext} = E * S_{st} * \sqrt{6/100} = E * S_{st} * 0{,}245 \ . \tag{28}$$

We will obtain that an additional external force was expended to overcome the potential barrier when shifting F_E from point *E* to point *K* (stage *II*).

Guided by the fact that the graph shown in Figure 1 was recorded by the device and is a classic example of an experimentally established dependence of all the characteristics of the experiment, we use it in our calculations. For example, with a force corresponding to Young's modulus, amounting to $190(kg/m * s^2)$, the additionally expended external force at stage II will be $\Delta F_{ext} = 46{,}5(kg * m/s^2)$, which, taking into account the scale of plotting the graph along the O-F axes, agrees well with the experiment (see Figure 1).

Further, after stage *II*, an increase in the sample elongation, when the material begins to change the geometry of the sample, that is, when the crystal lattice is rebuilt and the total shift KU $\Delta l = \Delta l_{elas} + \Delta l_{past}$ increases due to a significant increase in the value of Δl_{past}, for example, when the ratio $\Delta l = 28$ in the root expression is 0.28, with an initial force corresponding to Young's modulus equal to, for example, $E = 190(kg/m * s^2)$, the additionally expended external force will be:

$$F_{(28)} = E * S_{st} * \sqrt{0{,}28} = 190 * 0{,}53 = 100{,}7(kg * m/s^2) \tag{29}$$

Accordingly, when the maximum tensile force (point D) $\Delta l = 52$ is reached, the ratio $\Delta l/L_{st}$ in the expression under the root will be 0.52, and with an initial force corresponding to Young's modulus, the additional external force expended will be:

$$F_{(D)} = E * S_{st} * \sqrt{0{,}52} = 190 * 0{,}721 = 137(kg * m/s^2) \tag{30}$$

Putting the values of all these parameters on a scale corresponding to the scale of the tensile diagram of the sample shown in Figure 1, we obtain that the theoretical tensile diagram of the standard sample, shown by the dotted line in Figure 1, agrees well with the experiment.

Moreover, the fact that the calculated point $[D]$ of reaching the maximum force value has a value slightly less than the experimental one can be explained by the fact that the experiment still showed the effect of changing the area of the rod, or the rate of rod extension, which was established in the works [6, 7].

The assessment of the last IV stage can be carried out based on the constancy of the entire deformable volume and elongation of the sample, calculating the value of the cross-section of the sample. And since the data of the last stage are not used for practical purposes, this stage is not considered in this paper.

In this case, it is necessary to pay special attention to the fact that Young's modulus with its dimension of pressure has the property of resisting both tensile and compressive external loads. That is, it has a property whose mechanism can be explained only on the basis of one or another constant inherent in each material, determined by its structure, for example, the constancy of the interaction energy between the KU of the material $\xi_{at} = Const$ during their discrete displacement $\Delta l = Const$ which is the same for both stretching and compression of samples. As shown by the experimental studies of Bauschinger [8], shown in Figure 2.

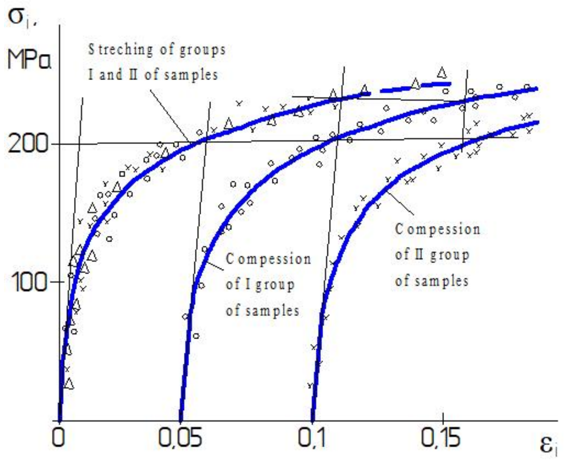

Fig. 2. Hardening curves of steel 10 at 20^0 C; Δ −compression testing of control samples; x-group of samples (ε =0.05); o-group of samples (ε =0.1).

To test the samples in [8], 3 cylindrical samples per tensile test point with a working part diameter of 10 and a length of 100 mm were made from annealed steel 10. These samples were then subjected to stretching to various strain values: 0.05; 01; 0.15. Next, four samples were turned from each sample for compression tests with a size of $d8{,}5 * 12$mm: 2 samples from the heads and two from the working part, which were then tested. A comparison was made based on the test results of these two pairs. Thus, in these experiments, the same metal in the form of different samples was subjected to both elastic stretching and compression deformation. Judging by the test results, Young's modulus confirmed the ability of the test samples to almost completely restore not only their geometric dimensions, but also their physical and mechanical properties. In other words, it is clear from the figure that if the obtained curves are shifted to the origin along the axis $0 - \varepsilon_i$, they will coincide, thus confirming the constancy of Young's modulus in the region of elastic displacements.

5 Conclusion

The created mathematical model allowed to theoretically substantiate that Young's modulus is indeed a constant characteristic of the NIRS, directly determining the value of the potential barrier of elastic displacement preceding plastic deformation at its different stages. At the same time, a new approach to assessing the properties of various metals made it possible to model the manifestation of the properties of a steel sample to resist deformation at the level of the pressure balance dimension. In addition, it was possible to show that the characteristics included in the definition of MM and all these stages should be associated with the discrete structure of the kinetic units that form the crystal lattice of a particular steel. As a result, the dimension of Young's modulus and its mechanism of action are associated with the internal resistance to external loads.

The resulting idea of the Young's modulus and the mechanism of the process, based on the understanding that to determine the amount of external energy or work expended on the deformation of a particular metal (with its characteristic crystal lattice) enclosed in the volume of the sample under consideration V_{st} (m^3), allowed the action of an external force F_{ext} and a change in the elongation Δl of the entire sample to be associated with the internal energy of the material

$$A = F_{ext} * \Delta l = \frac{V_{st}}{V_{k.u}} * \frac{\Delta l}{L_{st}} * \zeta = N * \frac{\Delta l}{L_{st}} * \zeta \qquad (31)$$

where $V_{k.u} = S_{st} * \Delta l(m^3)$ determines the volume of displaced KU; $N_{k.u} = \frac{V_{st}}{V_{k.u}}$ determines the number of KU in the volume of the sample (13); the energy ζ spent on the relative displacement of KU is the energy that is spent on changing the energy of the interatomic bond. Therefore, for a complete understanding of the deformation process itself, it is very important to consider the results of not only the equation (27) itself, but also the relationship of the characteristics included in the radical expression of equation (24), written out in full form of all the characteristics

$$F = \sqrt{A_{ext} * E * S_{st}/L_{st}} = \sqrt{E * S_{st} * \Delta l * E * S_{st}/L_{st}} \qquad (32)$$

where S_{st} is the cross-sectional area of the cylindrical sample; $S_{st} * \Delta l = \Delta V_{k.u}(m^3)$ essentially represents the displaced volume of KU, during one or another stage of deformation, which can change by changing the number of involved electrons (KU) located in the volume of the deformed sample with the initial cross-sectional area $[S]$ and the length L of the sample; $\frac{[S]}{L}$ is the ratio that determines the initial state of KE in the sample, characterizing the influence of the so-called “scale factor” on the results of the experiment.

In addition, in equation (34), the first radical expression $\frac{E*[S]}{L}$ determines the constancy of the Hooke coefficient value $K = Const$, multiplied by the energy of internal resistance of the material, confirms the constancy of the force resisting external loads. Thus, knowing the experimental values F_{ext} and K, it is possible to determine the value of the energy of internal resistance of the material contained in the volume of the sample.

$$A_{k.u} = \frac{F_{ext}^2}{K}, \qquad (33)$$

In this case, the essence that determines this constancy, which follows from substituting the expression for the Hooke coefficient into equation (33), is very important for understanding the constancy of the value of force, normal stress, Young's modulus, pressure, elastic shear constant, Hooke's coefficient and displacement energy of KU of the material, which determines the total resistance of the material during its deformation.

$$A = \frac{F^2}{K} = \frac{F^2}{E*\Delta l} \qquad (34)$$

After the transformation we obtain the equation:

$$A * \Delta l = \frac{F^2}{E} = F * S(kg * m^3/s^2) \tag{35}$$

(36) brings the consideration of the process at the level of the charge dimension, written as:

$$A_{k.u} * \Delta l = e^2 \left(\frac{kg * m^3}{s^2}\right) = Const \tag{36}$$

When the physical, mechanical and other properties of the KU that form the structure of metals (the structure of crystal lattices) are established on the basis of the constancy of the electromagnetic charge, which does not change its value either in time or in space. On this basis, a physical model and method for calculating the Young's modulus for various metals and their alloys were created, given in a separate article.

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