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Consideration of shear deformation in the calculation of bent structural elements

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Abstract. The article studies the effect of experimental stresses on the deformation of structural elements subject to bending. It is known that the bending element of the structure is a beam, the main problem is to check their uniformity, for this it is necessary to determine the displacements caused by external forces, this task is carried out on the basis of integrating the main differential equation of the bent axis of the beam. But in general, the bending of the beam is significantly affected by tensile stresses arising on the surfaces of its cross section. Shear deformation caused by these stresses can in some cases significantly affect the deformation of the beam, that is, its composure. It is known that when deriving the main differential equation of the bent axis of the beam, only the effect of the bending moment is taken into account. In this case, the axis of the beam is curved, there is a stretching of the longitudinal layers from the neutral on one side and compression on the other. It is based on the hypothesis that the cross sections of the beam are straight, and the straight sections remain perpendicular to its bent axis. The article derives a differential equation for a curved beam axis taking into account the influence of shear deformation.

1 Introduction

It is known that the element that working on bending of the structure [1] is the beam, and checking their stiffness is the main problem, for this it is necessary to determine the displacements caused by external forces, this problem is carried out based on the integration of the main differential equation of the bent axis of the beam. But in general, the bending of the beam is affected by the tangential stresses generated on its cross-sectional surfaces. The shear deformation caused by these stresses can, in some cases, significantly affect the deformation of the beam, that is, its deflection. When deriving the basic differential equation of the bent axis of the beam, only the influence of the bending moment - M is taken into account [2,3].

$$EJ\frac{d^2y}{dx^2} = \pm M \tag{1}$$

The bent axis of the beam occurs as a result of stretching of the longitudinal layers on one side of the neutral layer and compression of the other side. It is based on the hypothesis that

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the cross-sections of the beam are straight and perpendicular to its bent axis, that is, the hypothesis of straight sections (Bernoulli's hypothesis).

2 Materials and methods

But in general, in addition to the longitudinal tensile and compressive stresses, the tensile stresses generated on its cross-sectional surfaces also affect the bending of the beam [4]. This is the displacement caused by stresses deformation can, in some cases, significantly affect the deformation of the beam, that is, its deflection. We define this shear deformation effect as follows. If we determine the deflection of the beam due to the bending moment - y_1, and the transverse force - y_2, then the resulting stiffness of the beam is equal to the following expression (2).

$$y = y_1 + y_2 \tag{2}$$

Then, taking into account (2), we can express the differential equation of the bent axis of the beam by the following relation:

$$\frac{d^2y}{dx^2} = \frac{d^2y_1}{dx^2} + \frac{d^2y_2}{dx^2} \tag{3}$$

The first additive on the right side of the expression (3) represents (1) and the second additive represents the conditional bending resulting from displacement [4].

Thus, we are interested in the second first joiner of (3), the shear deformation resulting from the action of experimental stresses on the center of gravity of the cross-sectional surface. We express the relative displacement - b generated on these surfaces by the following formula:

$$\beta = \frac{kQ}{GF} = \frac{k}{GF}\frac{dM}{dx} \tag{4}$$

Here, k is a coefficient that depends on the shape of the cross-sectional surface.

3 Results and discussion

For example, the maximum value of tensile stresses on the surface of a circular section, is equal to - and $\tau_{max} = \frac{4}{3}\frac{Q}{F}$ reaches at - from the neutral axis according to its height (diameter) $\varphi = \frac{\pi}{2}$ [3,5,7]. If b is constant, then the displacement does not significantly affect the bending of the beam, the additional stiffness - y_2 - is shown linearly with respect to the x-axis. - $\frac{d^2y_2}{dx^2}$ since $\frac{d\beta}{dx}$ the curvature is the result of displacement along the longitudinal axis of the beam - b (usually, the value of the ratio is positive if the bubble of the longitudinal axis of the beam is bent upwards). In the future, it will be necessary to take into account the displacement deformation caused by the tangential stress on the curvature of the beam relative to the adopted coordinate system, which is related to the following connection.

$$\frac{d\beta}{dx} = \frac{d^2y_2}{dx^2} \tag{5}$$

If we put (4) instead of b - and consider the connection (1) between y_1 and M_1, we get the following:

$$\frac{d^2y_2}{dx^2} = -\frac{k}{GF}\frac{d^2M}{dx^2} = -\frac{kEJ}{GF}\frac{d^2}{dx^2}\left(\frac{d^2y_1}{dx^2}\right) \tag{6}$$

Then the differential equation for the bent axis of the beam will have the following form

$$EJ\left(\frac{d^2y_1}{dx^2} + \frac{d^2y_2}{dx^2}\right) = E\frac{d^2y}{dx^2} = M - \frac{kEJ}{GF}\frac{d^2M}{dx^2} \tag{7}$$

Example. Below we see a cantilever beam loaded with a distributed load and a concentrated force at the free end with one end clamped and the other end free.

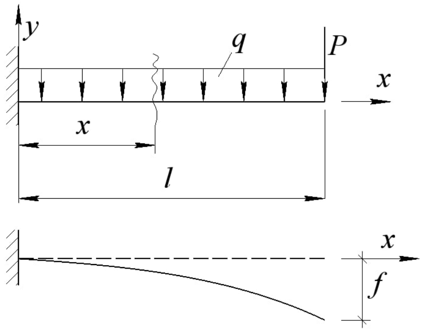

Fig. 1. General view of the bent position of the cantilever beam.

For this beam, the bending moment at an arbitrary x-section is

$$M = -P(l-x) - \frac{q(l-x)^2}{2}$$

Then the differential equation (7) of the bent axis of the beam will have the following. form.

$$EJ\frac{d^2y}{dx^2} = -P(l-x) - \frac{q(l-x)^2}{2} + \frac{kEJ}{GF}q \quad (8)$$

Integrating (8) twice, we get the following.

$$EJy = -P\left(\frac{l \cdot x^2}{2} - \frac{x^3}{6}\right) - \frac{q}{2}\left(\frac{l^2 \cdot x^2}{2} - \frac{l \cdot x^3}{3} + \frac{x^4}{12}\right) + \frac{kEJ}{GF}\frac{qx^2}{2} + Cx + D \quad (9)$$

The invariant integral constant is equal to D = 0, since the deflection is zero at the clamped point. The second integral constant C is equal to the value of the slope angle between the X axis, that is, the inclined axis of the beam and the test.

For this angle to be equal to zero: If the axis of the beam at the point of clamping does not bend, then it is equal to $k\frac{P+ql}{GF}$, if the isolated elementary surface at the point of clamping does not allow rotation with respect to the center of gravity.

In the last case, the element of the beam axis in the clamped place is equal to the angle formed by the transverse force P+ql with the X axis, i.e.

$$\left|\frac{dy}{dx}\right|_{x=0} = -k\frac{P+ql}{GF} \quad (10)$$

Based on the condition of fixing the beam, the differential equation of the bent axis is expressed by one of the following equations (11).

$$EJy = -P\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) - \frac{q}{2}\left(\frac{l^2x^2}{2} - \frac{lx^3}{3} + \frac{x^4}{12}\right) + k\frac{EJ}{GF}\frac{qx^2}{2} \quad (11)$$

$$EJy = -P\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) - \frac{q}{2}\left(\frac{l^2x^2}{2} - \frac{lx^3}{3} + \frac{x^4}{12}\right) + k\frac{EJ}{GF}\frac{qx^2}{2} - kEJ\frac{(P+ql)}{GF}x$$

If the beam is only under the influence of the accumulated force - P, then the complete deflection of the beam is based on the condition of the second connection, i.e., fixation.

$$f = \frac{Pl^3}{3EJ}\left(1 + \frac{3kE}{G}\frac{r^2}{l^2}\right) \quad (12)$$

where r^2 is the square of the radius of inertia, the second additive in conditionally corresponds to the deflection due to the conditional transverse force. For rectangular and circular sections, this additional deflection is so small that it can be ignored.

For example, for beams with a straight cross-section, the ratio of the height of the cross-section surface to the length of the prolet:

$$\frac{h}{l} = 0{,}1, \text{ if } \frac{r^2}{l^2} = \frac{h^2}{12l^2} = \frac{1}{1200}, \quad k = 1{,}5; \quad \frac{E}{l} = 2(1+\mu) = 2{,}6$$

Additional deflection is less than 1% of the deflection caused by bending moment.

However, the effect of the tangential stress on the double cross-section surface, as can be seen from the following example, is significantly greater.

l equal to that shown in Figure 2 below:

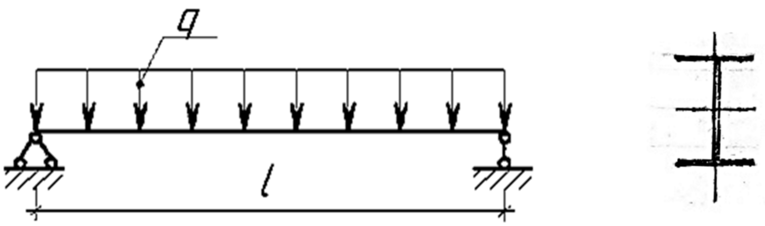

Fig. 2. Computational form of a simple beam with a I-beam cross-sectional surface resting on two supports.

For such a beam, (7) - the differential equation of the bent axis of the beam is written as follows

$$EJ\frac{d^2y}{dx^2} = \frac{ql}{2}x - \frac{qx^2}{2} + \frac{kEJ}{GF}q \tag{13}$$

Integrating (13) twice, we get the following.

$$EJy = \frac{qlx^3}{12} - \frac{qx^4}{12} + \frac{kEJ}{GF}\frac{qx^2}{2} + Cx + D \tag{14}$$

The boundary conditions in (14) are: y = 0 at X=0 and X= λ , the gradients are zero.

$$\mathrm{D} = 0,\ C = -\frac{ql^3}{24} + \frac{kEJ}{GF}\cdot\frac{ql}{2}$$

Extra deflection, conditional effort from tangential stress

$$y_2 = \frac{k}{GF}\left(\frac{qx^2}{2} - \frac{qlx}{2}\right) = \frac{kM}{GF} \tag{15}$$

Therefore, the additional deflection beam is proportional to the ordinate of the bending moment. This conclusion is also true for the edges drawn differently and for different loading cases. However, the edges are an exception for beams loaded with a couple of forces M.

It is known that the displacement in bending from the couple force - M is determined as follows.

$$\beta = \frac{k}{GF}\cdot\frac{M_0}{l} \tag{16}$$

(16) remains constant over the length of the beam. It can be seen that the tangential stresses [5-6] do not bend the axis of the beam, because its edges on fixed supports, so no additional stiffness is created from the tangential stresses.

Now we return to the beam loaded with a uniformly distributed load. The deflection between these beams is equal to:

$$f = \frac{5}{384}\frac{ql^4}{EJ} + \frac{k}{GF}\cdot\frac{ql^2}{8} \tag{17}$$

The second additive in (17) represents the additional deflection due to stress. Let this size, that is, the size of the additional deflection, be selected for the double-section beam through the allowable stresses R_t and R, maximum normal and experimental stresses, on the basis of the following stability conditions (18) and (19).

$$\frac{kQ_{max}}{F\frac{kql}{2F}t} \quad \text{and} \quad \frac{M_{max}}{W} = \frac{ql^2h}{8J2} = R \tag{18}$$

$$\frac{5}{384}\frac{ql^4}{EJ} = \frac{5}{24}\cdot\frac{Rl^2}{Eh} \quad \text{and} \quad \frac{k}{GF}\frac{ql^2}{8} = \frac{R_tl}{4G} \tag{19}$$

The ratio of the shear stress to the torque produced by the shear stress

$$f_2 : f_1 = \frac{R_tl}{4G} : \frac{5}{24}\frac{Rl^2}{Eh} = \frac{6}{5}\frac{R_1}{R}\frac{E}{G}\cdot\frac{h}{l} \tag{20}$$

where $R_t = 0{,}5R$ and $E = 2{,}6\cdot G$ if (20) the ratio is equal to the following

$$f_2 : f_1 = 1{,}56\frac{h}{l} \tag{21}$$

4 Conclusion

Thus, the effect of tangential stresses in short bars is significant, and it is necessary to take it into account.

Therefore, in practice, in order to ensure sufficient priority of the wall of double-section beams, it is necessary to try to increase the width of the double-section I-beam in calculations on the effect of tangential stresses. The beam reduces the belt from the base to the center of its span.

This reduces both of the above conditions $f_2 : f_1$ and f_2 justifies ignoring the traditional deflection determination with respect to the value of f_1.

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