Estimation of the error in 3D control of products with complex geometric profiles using the method of multi-view phase triangulation

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Abstract. The analysis of achievable measurement errors of three-dimensional geometry was performed during the implementation of the phase triangulation method in a multi-view setup. The essence of the method lies in measuring a stationary object from different perspectives using one or multiple measurement systems, and then merging the measured point clouds into a single closed three-dimensional model. The main focus was on measuring convex objects, as the phase triangulation method predominantly operates with convex objects. The limitation arises from the fact that for complex-profile objects with arbitrary geometry, "dead" zones will inevitably be present, which cannot be measured using methods based on phase-based triangulation with spatially modulated illumination. The primary attention in the study was directed towards elongated objects, as they are challenging to measure from a single viewpoint, and multi-view methods of phase triangulation enable measurements with virtually no fundamental limitations on the geometric dimensions of the object being measured. Mathematical modeling was conducted in the study to assess measurement errors when merging point clouds into a unified coordinate system. The obtained results demonstrated the potential use of multi-view phase triangulation methods even in industrial conditions for measuring elongated objects with complex three-dimensional geometry and arbitrary light-scattering properties.

1 Introduction

Improving methods for 3D control of products with complex geometric profiles is an important direction of development in the field of energy technologies. This is driven by increasing demands for the geometric accuracy of elements in energy systems. Accurate three-dimensional diagnostics enable more effective monitoring and analysis of complex objects, such as turbines, rotors, blades, and other components of energy installations. By utilizing modern three-dimensional diagnostic methods, specialists can more precisely identify defects, wear, as well as conduct stress and deformation analysis within objects, enhancing the efficiency and reliability of energy systems. Thus, the advancement of three-

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dimensional diagnostic methods plays a key role in ensuring high performance and safety of energy technologies [1].

When measuring 3D geometry of products with complex geometric profiles using phase-based triangulation with spatially modulated illumination [2-3], difficulties arise in ensuring complete measurement of the entire surface of the object. One solution to this problem is the use of multiple observations iterative methods for measuring 3D geometry. These methods allow measurements to be taken from different viewpoints, providing a more comprehensive and accurate coverage of geometrically complex objects. Multi-view measurement methods for three-dimensional geometry help improve the quality and accuracy of the obtained data, as well as provide a more complete representation of the shapes and sizes of objects. This significantly enhances the efficiency and reliability of measuring the three-dimensional geometry of complex-profile objects, which is an important aspect in the field of scientific and engineering research.

In general, all known solutions can be reduced to methods that involve the use of special markers to align various measured fragments in the final three-dimensional model [4]; methods that entail rotating the object being measured by a predetermined angle [5]; methods that involve rotating elements of the optical system [6]; methods that utilize spatial cross-correlation algorithms to determine the alignment parameters of measured surface fragments [7].

All existing approaches and methods are based on one or more principles: the use of special markers to connect different dimensions in a 3D model [4]; application of methods for rotating an object at a certain angle [5]; use of rotating components of an optical measuring system [6]; application of multiparameter spatial correlation algorithms to estimate surface measurement alignment parameters [7].

Existing approaches to solving the problem of measuring the three-dimensional geometry of complex-profile and elongated objects are typically based on several main methods: the use of special markers to connect different measured fragments in the single 3D model [4] (this method involves using markers or reference points to establish connections between different measured sections of the object and create a unified three-dimensional model); the method of rotating the object being measured by a predetermined angle [5] (by rotating the object by a specific angle and then measuring, a more complete and accurate three-dimensional model is obtained); the method of rotating components of the optical measuring system [6] (this approach involves rotating optical elements to obtain additional information and improve measurement quality); using multiparameter spatial correlation algorithms to estimate surface measurement alignment parameters of measured surface fragments [7] (this method is based on analyzing overlapping fragments of measured data to determine the precise position and correspondence of surface elements).

The method based on using markers to align sectors into a unified three-dimensional model provides low dependency of the alignment results on the surface light-scattering properties and shape. However, a significant drawback of this method is the physical limitation on the number of markers that the experimenter can apply to the surface of the object being measured. Considering that the measurement accuracy directly depends on the number of markers used, this method has significant limitations for high-precision measurements.

An advancement of the marker-based method is the multiple observations measurement method using markers, generated by light illumination (RU Patent N 2708940). The essence of this approach is that markers are used to align the measured surface fragments of the object, which are not directly mounted to the object's surface itself but illuminate specific local areas of the surface of the measured product. These markers must be uniquely observable and dissemble from different viewpoints during measurements. This approach is more versatile and flexible compared to traditional methods of using markers, as it does not require direct contact with the object's surface. However, successful implementation of this method requires additional data processing and recognition of the coordinates of the illuminated markers on the object's surface, which may necessitate additional efforts and resources.

The research employed the phase triangulation method using a multi-view approach, which allows merging measured areas based on data about local intensity parameters on the object's surface registered by photodetectors. The main idea of the method is that two photodetectors are placed in a known by operator position relative to each other, and spatially modulated light from a source located in a third position relative to the photodetectors is used for illumination. Three-dimensional geometry measurement is performed step by step, with each surface fragment being measured separately. This approach provides high texture resolution and reduces possible errors in the measurement process, making the method effective and accurate for obtaining three-dimensional data about the object's surface.

To align the measured fragments, a solution similar to the described method, using special markers on the surface of the product being measured, which was discussed earlier, is used. In our implementation, the difference lies in using spatial parameters of the structured illumination, generated by the light source, as the "markers." The illumination is registered by photodetectors on the measured surface at each point and the software is able to match areas in different photos that correspond to the same section of the object's surface. This approach aims to minimize alignment errors when 3D control of products with complex geometric profiles using the method of multi-view phase triangulation. This method allows for effectively and accurately merging measured fragments based on the parameters of structured illumination, contributing to increased accuracy and quality of the obtained data on the geometry of objects.

The aim of this research is to estimate the accuracy of the multiple-view spatially modulated light triangulation method in merging fragments obtained from measurements taken at different viewing angles.

2 Method descripton

When merging surface sectors, the assessment of errors can be carried out in two ways. The first method is based on evaluating the maximum deviation of a point's position on the object's surface after aligning the measured fragments. This allows for estimating the maximum displacement of points on the surface of the complex-profile object being measured. The second approach involves examining how much the geometric center of the measured fragments deviates spatially. This method enables the assessment of the displacement of the center of mass of the merged fragment algorithms and its impact on the excellence of reconstructing the final 3D model. Both approaches allow for assessing the accuracy and reliability of the process of aligning measured surface fragments and determining the level of error in the obtained data of the three-dimensional geometry of the object's surface being measured.

Below is a theoretical estimation of the error when merging measured surface fragments obtained at different viewing angles. This analysis assumes that the results of the measurements of the surface fragments used follow a normal distribution, which allows for estimating the probability of deviations and determining the level of error when combining measurement data into a unified three-dimensional model.

Let Cl1 and Cl2 be two sets of points obtained from measuring the surface using phase triangulation from different perspectives. It is important to note that the analyzed sets of points should have a significant overlap of surface fragments in space. The attitude between the points in the measured 3D models will be as follows:

$$P_i^{Cl2} = (P_i^{Cl1} + E_i) \cdot M \, , \tag{1}$$

where PiCl1 is a [3x1] vector describing the i-th point in the set Cl1, PiCl2 is a [3x1] vector describing the i-th point in the set Cl2, Ei is the difference in the measurement results of the i-th point in set Cl1 compared to the i-th point in set Cl2 determined by the transformation matrix M. For a three-dimensional Cartesian coordinate system, the matrix M will have dimensions of 4x4 and account for linear displacement and rotation with respect to two orthogonal axes.

During the process of stitching surface fragments measured using the multi-view phase triangulation method, a matrix M' will be calculated, representing the transformation of the coordinate system of set Cl1 into the coordinate system of set Cl2.

In other words, after transforming the coordinates from the coordinate system of set Cl1 to the coordinate system of set Cl2, the coordinates of the i-th point will be determined as follows:

$$P_i^{Cl2}{}' = P_i^{Cl1}{}' \cdot M'. \tag{2}$$

The alignment error of the sets can be represented as a random variable, similar to how it is done when assessing other random processes, such as measurement errors, random noise in signals, or random deviations in data. The random variable can be represented as a three-dimensional vector with independent components:

$$E_i' = \begin{bmatrix} X(P_i^{Cl2}) - X(P_i^{Cl2}{}') \\ Y(P_i^{Cl2}) - Y(P_i^{Cl2}{}') \\ Z(P_i^{Cl2}) - Z(P_i^{Cl2}{}') \end{bmatrix}, \tag{3}$$

where X, Y, Z represent the respective Cartesian coordinates of the point in the reference system. Then, the total alignment error of the measured sets can be determined by calculating the highest deviation in the coordinates of the points $P_i^{Cl2'}$ from the coordinates of the points P_i^{Cl2}:

$$Err_{\max} = \max\left(\left\{E_i'\right\}_{i=1..N}\right), \tag{4}$$

where N – is the number of points in the sets $Cl1$ and $Cl2$. If the algorithm for merging measured surface fragments operates correctly, then it can be considered that condition (5) is met, considering that the set of points $Cl2$ was initially formed by adding a random vector E_i:

$$Err_{\max} \le \max\left(\left\{E_i\right\}_{i=1..N}\right). \tag{5}$$

If the distribution of a random variable E_i is normal, then the distribution of the random variable that reflects the difference in coordinates between point clouds after their stitching will also be normal and will be characterized by similar parameters. This means that the probabilistic deviations in the coordinates of points after stitching will be distributed according to a normal distribution, which facilitates the assessment and analysis of errors in the obtained data.

3 Results of mathematical modeling

To assess the effectiveness of the point cloud merging algorithm, mathematical modeling was conducted. In this study, a flat set of points $C1$ was created, where points were randomly distributed on a single plane within a specific volume. Subsequently, the second point set, $C2$, was created by altering the first set, $C1$: the coordinates of each point were adjusted by a random amount, and the entire set was rotated around the X and Z axes by two random angles. These spatial modifications can be represented by a linear matrix $M0$, characterizing the described linear transformations:

$$C2 = (C1 + E) \cdot M0. \tag{6}$$

After that, the surface fragment merging algorithm described in reference [3] is applied, resulting in the transformed set of points $C3$. This set is formed using the calculated transformation matrix $M1$, so the expression below is correct:

$$C3 = C2 \cdot M1. \tag{7}$$

After that, an analysis is conducted on the maximum displacement of the center of mass of the set of points $C3$ relative to the original set $C1$. Figure 1 shows the point sets $C1$ and $C2$ within the coordinate framework of set $C1$ for visual comparison and evaluation of the results.

During the execution of the merging algorithm, a distribution of points in the cloud $C3$ was formed (see Figure 2). The maximum deviation, denoted as $Err_{\max}$, was 3.63 mm. The root mean square deviation of the set, denoted as E, was 1 mm.

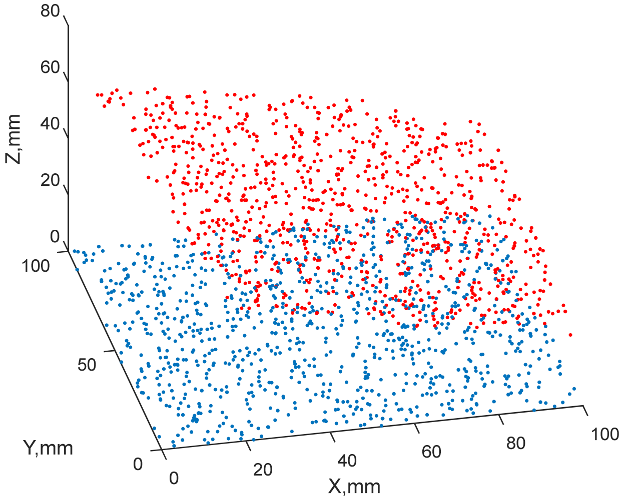

Fig. 1. The arrangement of point sets C1, represented by blue points, and C2, depicted as red points, within the coordinate framework of the C1 set.

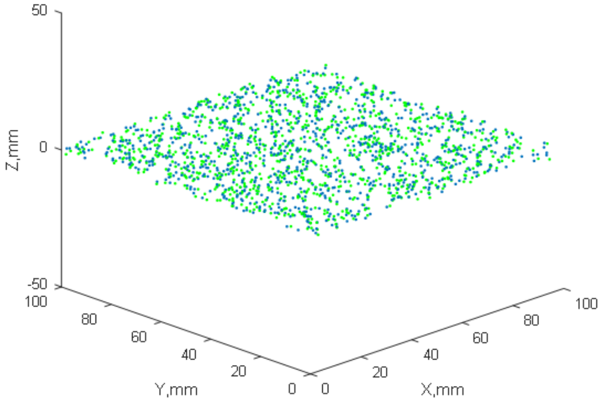

Fig. 2. The spatial arrangement of point sets C1, indicated by blue points, and C3, represented by green points, within the coordinate system of the C1 set following the amalgamation of sets C1 and C2.

Below is a graph representing the variation of the maximum error $Err_{\max}$ as a function of the experiment number under the conditions: standard deviation $E = 1$ mm, the point cloud comprises a total of 1000 points, and the dimensions of the planar region containing the located measurement points is 100 mm^2. This graph allows for a visual assessment of the dynamics of the maximum error magnitude during a series of experiments.

The average value of $Err_{\max}$ in the presented experiment was 3.55 mm.

Figure 4 shows the dependence of the average value of $Err_{\max}$ on the standard deviation (E).

Consequently, it was determined that the relationship of $Err_{\max}$ is closely approximated by a linear trend.

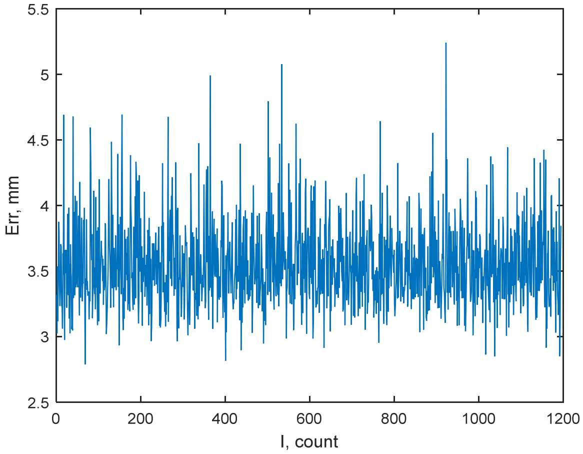

Fig. 3. Dynamics of the value $Err_{\max}$ depending on the experiment number at RMS (E) = 1 mm.

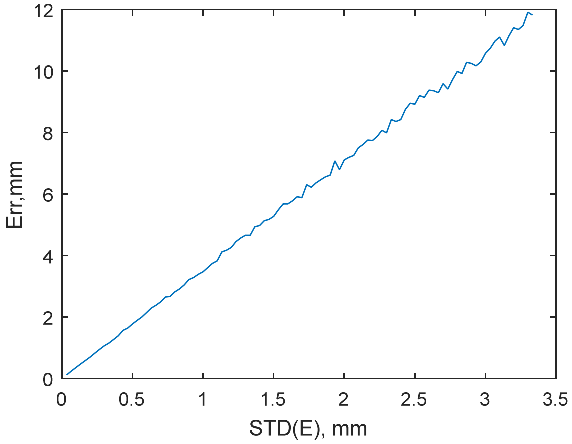

Fig. 4. The relationship between the maximum error, $Err_{\max}$, and the standard deviation of the variable E.

4 Conclusion

In the course of the study, theoretical error estimates of measurements were conducted, and the results of mathematical simulations were presented. Research has revealed that the multi-baseline phase triangulation technique, which is employed to gauge the three-dimensional shape of objects that are convex and elongated, ensures the required precision in measurements, enabling the acquisition of reliable and precise data on the geometry of objects with diverse shapes and sizes. This methodology showcases its ability to deliver high measurement accuracy even in conditions of complex object geometries. The method is successfully applicable for obtaining the 3D geometry of convex and elongated objects with complexity profile in industrial settings, confirming its effectiveness and reliability when working with various types of geometrically complex objects, making it a valuable tool for industrial 3D modeling tasks.

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