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Numerical modelling with analytical estimation of the free parameter space of GRB afterglows: the case of GRB 221009A

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Abstract. On date 9th of October 2022, an exceptionally bright Gamma-Ray Burst, GRB 221009A, was detected by several observatories, both ground- and space-based. It has been the highest flux Gamma-Ray Burst ever observed, with an energy $E_{iso} \sim 10^{55}$erg, and its detection was followed by many studies. In our work, we model the very high energy (E> 100 GeV) afterglow light curve published by the LHAASO Collaboration using a numerical modelling and an analytical approach to constrain the Gamma-Ray Burst free parameter space. The light curve in the (0.3 - 5) TeV range, as detected by LHAASO, shows a simple broken power law shape, with the peak around ~11 s after the trigger. We estimated the afterglow parameters using a Maximum Likelihood Estimation followed by a Markov-Chain Monte Carlo. In this way, we obtain the fiducial confidence intervals. We found some interesting preliminary results about the parameter distributions, in reasonable agreement with other studies.

1 Introduction

Gamma-Ray Bursts (GRBs) are violent extragalactic explosions, characterized by a two-phases emission: a *prompt* emission followed by an *afterglow* [1]. The latter follows the prompt by tens of seconds and can last much longer, in most cases up to hours or even days. In addition, it extends in the whole electromagnetic (EM) spectrum, from radio up to TeV energies [2–4]. Our interest is focused on the study of what has been recorded as the most energetic GRB ever detected, GRB 221009A, or the "Brightest Of All time" (BOAT) [4–6]. It has been an exceptional event, in particular due to its detection by the LHAASO Collaboration [4] which reveals photons up to ~ 10 TeV.
In this work, we have applied an afterglow numerical modelling coupled with an analytical estimation of the parameter space of GRB afterglows to describe the very high-energy (VHE) light curve (LC) of GRB 221009A. We found some preliminary results about the emission parameters.

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2 Methods

2.1 Theoretical Bases

Afterglow emission refers to the broad-band radiation observed from a GRB on longer timescales (up to months) following the prompt emission. Its temporal evolution is usually well described by simple power laws. We estimate the best-fit GRB afterglow parameters applying a two-step method: (a) we reproduce the GRB afterglow emission by means of a numerical model described in [7] extracting a range of possible solutions; (b) we apply Maximum Likelihood Estimation (MLE) and we run a Markov-Chain Monte Carlo (MCMC) to derive the best-fit parameters and their corresponding confidence intervals. The GRB modelling makes use of a numerical method to solve the kinetic equation of accelerated particles and photons, in order to estimate the time-evolution of the multi-wavelength afterglow emission within the GRB external forward shock scenario. A set of free parameters is introduced to describe the unknown features of the jet dynamics, particle acceleration, shocks micro-physics and external medium. We assume the blast wave to expand into a constant circumburst medium. We focused on modelling the LC provided by the LHAASO Collaboration in [4] estimating the simulated flux in the 0.3 - 5 TeV energy range, collected in the first ~3000 s from the trigger. Once obtained a correct representation of the VHE LC through our *physical parameters*, we studied its shape and reproduced it with a made-up function, depending on some *fit parameters*: this allows us to find a relation between fit and physical parameters, so to describe the functional form of the LC, ultimately, in terms of the physical parameters. To conclude, after a MLE [8], we ran a MCMC [8] to find the best estimation for the parameters with their associated confidence intervals.

2.2 Analytical modelling and MCMC

The adopted numerical model requires a set of input parameters, five of which are left free to vary: the electron energy fraction, ϵ_e, the magnetic energy fraction, ϵ_b, the initial bulk Lorentz factor, Γ_0, the circumburst medium density, n_0, and the spectral index of the distribution of accelerated electrons, p. The LC, parametrized by a smooth Broken Power Law (BPL), depends on these parameters, as in the following:

$$F(t) = \Phi \left(\frac{t}{\tau}\right)^{a_1} \left[\frac{a_1 \left(\frac{t}{\tau}\right)^{\frac{1}{s}} + a_2}{a_1 + a_2}\right]^{-(a_1+a_2)s} , \qquad (1)$$

where ϕ is the flux at the peak, τ is the time corresponding to such a peak, a_1 and a_2 are the indeces of the broken power law (assumed to be positive) and s is the smoothing factor. We refer to these parameters as *fit parameters* from now on.
The set of physical parameters found through the numerical model, being so our initial values (*hat* values), is: $\widehat{\epsilon_e} = 6.5 \times 10^{-2}$, $\widehat{\epsilon_b} = 1.0 \times 10^{-2}$, $\widehat{\Gamma_0} = 650$, $\widehat{n_0} = 0.75\ cm^{-3}$, $\widehat{p} = 2.1$, while the relation between physical and fit parameters ξ_i is of the form:

$$\xi_i = A_i \times \left[\left(\frac{\epsilon_e}{\widehat{\epsilon_e}}\right)^{b_e^i} \left(\frac{\epsilon_b}{\widehat{\epsilon_b}}\right)^{b_b^i} \left(\frac{\Gamma_0}{\widehat{\Gamma_0}}\right)^{b_G^i} \left(\frac{n_0}{\widehat{n_0}}\right)^{b_n^i} \left(\frac{p}{\widehat{p}}\right)^{b_p^i}\right] \qquad (2)$$

Once derived the relations between fit and physical parameters, it is possible to express the functional form of the flux in terms of the physical parameters: thus, assuming the errors in

LHAASO data as Gaussian, the log-likelihood function can be expressed as in the following:

$$\ln P(y|t,\sigma,\epsilon_e,\epsilon_b,\Gamma_0,n_0) = -\frac{1}{2}\sum_n \left[\frac{(y-F)^2}{\sigma^2} + \ln\sigma^2\right], \tag{3}$$

where F represents the functional form for the LC, as in Eq. 1, t is the time, and σ the errors on the flux in LHAASO data. Then, after the MLE procedure, we obtain this set of values: $\epsilon_e^{ml} = 9.8\times10^{-2}$, $\epsilon_b^{ml} = 2.4\times10^{-2}$, $\Gamma_0^{ml} = 630$, $n_0^{ml} = 7.7\times10^{-1}\ cm^{-3}$, $p^{ml} = 2.0$.
At this point, we could proceed with the MCMC, with this configuration: the initial values are the *ML* ones, the number of walkers is 32 times the number of parameters (five), and the number of steps is 2×10^4. Moreover, the MCMC has been run over a chosen normalization of the parameters, called θ (Figure 1).

3 Results and Conclusions

The advantage of this technique lies in its speed: running a MCMC producing each LC using a numerical method would have taken months, given the precision required for each curve (several hours to produce one). In contrast, thanks to the analytical description of the functional form, the MCMC has taken only ~55 s. The results of the MCMC for each parameter are:

$$\epsilon_e = 0.095^{+0.013}_{-0.013}\,,\ \epsilon_b = 0.024^{+0.007}_{-0.006}\,,\ \Gamma_0 = 670^{+210}_{-140}\,,\ n_0 = 0.75^{+2.9}_{-0.66}\,,\ p = 2.01^{+0.08}_{-0.07},$$

these values representing the average value of the MCMC, with the errors indicating the confidence intervals and chosen as 2σ.
In Figure 1, we show the results of the MCMC as the trajectories of the walkers (left) and the relative distribution of parameters (right). From the latter, in particular, we get the most interesting information: for ϵ_e, ϵ_b and p the MLE already allows us to retrieve good results, while for Γ_0 and n_0 we got slightly different results because of the presence of other minima.
Figure 2 shows the LCs produced using the ML parameters (magenta) and the average MCMC parameters (blue). Moreover, all the sets explored by the MCMC (fading blue) and the LHAASO data with errors (fading black) are shown. We can see that the MLE provides a good estimation of the emission parameters, but is not able to estimate the errors. On the contrary, the MCMC shows that slightly different sets can reproduce LHAASO data, providing for each parameter a confidence interval. Note that this work is still in progress, and new results are expected to come soon.

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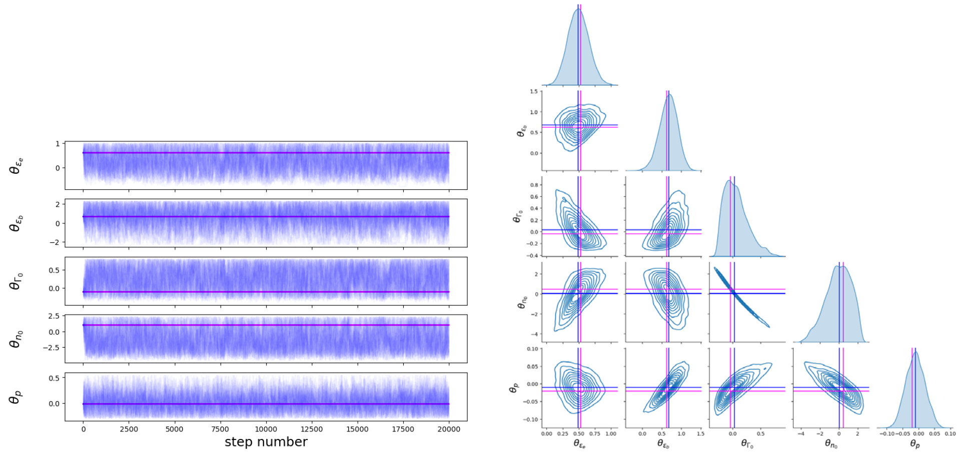

Figure 1. Left: In blue, the MCMC trajectories for each opportunely normalized parameter; in magenta, the horizontal line representing the initial (normalized) value. Right: corner plot representing the relative distributions of MCMC parameters, with the magenta line representing the initial values and the blue line the average value of the MCMC.

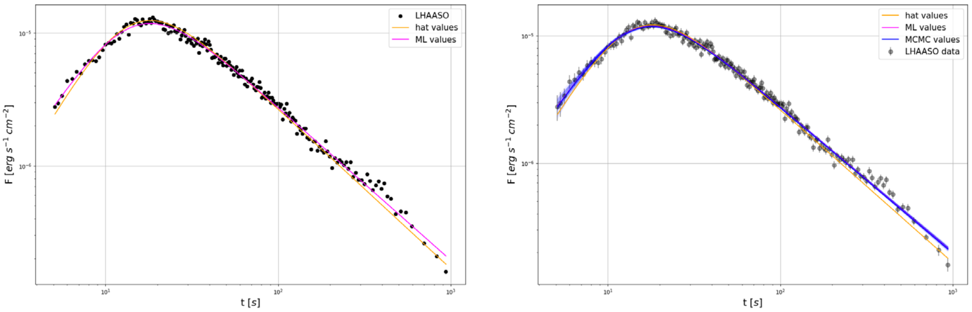

Figure 2. Left: In yellow, the LC produced with the initial *bar* values; in magenta, the LC produced with the values optimized with the MLE (*ML* values); the black dots represent LHAASO data. Right: same two LCs in yellow and magenta; in blue, the LC produced with the average MCMC values; in fading blue, all possible LCs explored by the MCMC; fading black points represent the LHAASO data with errors.

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