

Microscopic calculations with noniterative finite amplitude methods and the application to neutron radiative captures and inelastic scatterings

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Abstract. We derive the fully self-consistent quasiparticle random-phase approximation (QRPA) equations with noniterative finite amplitude methods and calculate the transition strengths of giant resonances. Then, we apply the QRPA results to both neutron radiative capture calculations based on the statistical Hauser-Feshbach theory and inelastic scattering calculations based on distorted-wave Born approximation (DWBA). We compare the calculated results with available experimental data and demonstrate how our approach can reproduce giant resonances and various nuclear reactions.

1 Introduction

Progress of microscopic theories for nuclear many-body systems is continuously required for basic science, and it is important for nuclear energy production and nuclear security. Stellar evolution and heavy element nucleosynthesis are typical research subjects where the nuclear theory is indispensable due to the missing experimental data for many radioactive unstable nuclei in stars. The accurate nuclear reaction rates reduce the uncertainties of astrophysical simulations and enable more reliable predictions of observational signals. This paper overviews our noniterative finite amplitude method (FAM) [1, 2] and shows the application to neutron radiative captures and neutron-induced inelastic scatterings.

2 Methods and Results

FAM is an efficient microscopic calculation method for treating residual interactions and solving linear response equations in nuclear structure calculations [3]. It has been applied to calculate photoabsorptions, β -decays, and spontaneous fission. The residual interaction is derived from the Time-Dependent Hartree-Fock Hamiltonian, which includes nuclear forces such as the Skyrme force. Explicit linearization of the residual interaction removes the need for the iterative procedures used in other conventional FAM methods [1]. The random-phase approximation (RPA) matrices A and B are derived through this linearization of the residual interaction. The amplitudes X and Y are obtained by solving fully self-consistent quasiparticle random-phase approximation (QRPA) equations with an external field F , such as E1 and M1 operators, which trigger giant resonances [1, 2]. Transition strengths and cross sections are calculated using the obtained amplitudes. The FAM result of photoabsorption cross sections for E1 and M1 transitions are used

to calculate the γ -ray strength function. We use the γ -ray strength function to calculate the transmission coefficient and neutron capture cross sections with the coupled-channels Hauser-Feshbach code CoH₃ [2]. Furthermore, we apply the FAM/QRPA framework to distorted-wave Born approximation (DWBA) calculations for neutron-induced inelastic scatterings on spherical nucleus by using a nuclear force v_{12} between an incident neutron (index 1) and nucleons in the target nucleus (index 2) as the external field F instead of E1 and M1 operators,

$$F_{\text{DWBA}} = \chi_{\alpha}^{(-)\dagger} v_{12} \chi_{\beta}^{(-)}, \quad (1)$$

where $\chi_{\alpha}^{(-)\dagger}$ and $\chi_{\beta}^{(-)}$ are distorted waves of outgoing and incoming neutrons calculated with an optical model in CoH₃. We employ the Skyrme force (SLy4) for v_{12} . The calculation does not require an adjustment of the deformation parameter as is typically done in the collective model for vibrations within the DWBA framework.

Figure 1 shows E1 and M1 dipole response functions for ¹⁵⁶Gd, which include giant resonances (E1 giant dipole resonance, M1 scissors mode, and M1 spin-flip) as well as smaller individual resonances for M1. The Skyrme force SLy4 is used to calculate the RPA matrices. The FAM calculation reproduces the split of resonance energies of the E1 transition for the deformed nucleus without any adjustable parameters. The energy-weighted sum rule computed from both Hartree-Fock ground-state density and the integration of the E1 photoabsorption cross section is consistent within a few percent. The M1 transitions in Fig. 1(b) are calculated by substituting the M1 operator with the external field F . As shown in the dotted line, low energy transitions (<4 MeV) are associated with the orbital operator that can be seen as macroscopic M1 scissors mode. This FAM calculation overestimates the total strength for the scissors mode ($\sum B(M1)_{\text{exp}} \sim 3\mu_N^2$), which can be improved by introducing the quench of the spin g -factor [5].

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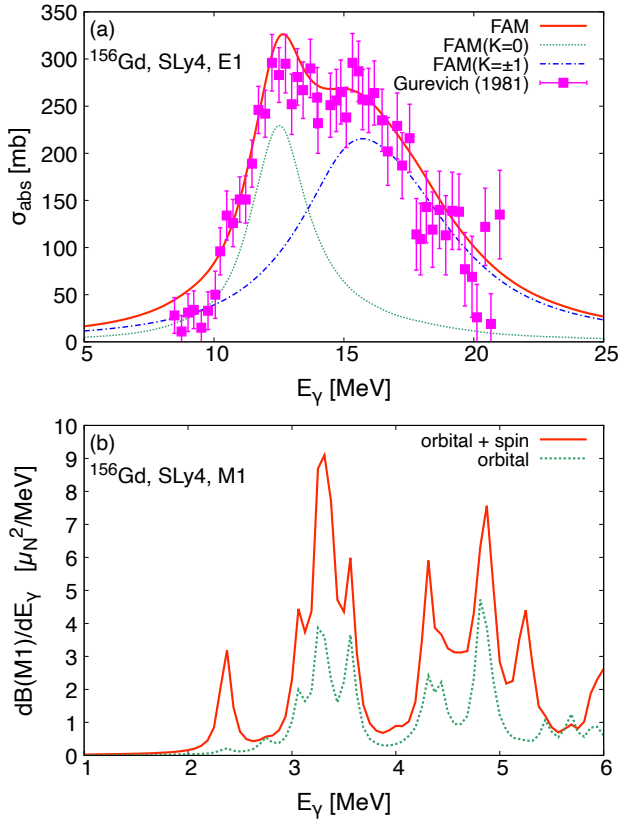


Figure 1. (a) The calculated electric giant dipole resonance (E1) for ^{156}Gd . The solid line shows the results of the noniterative FAM. The dotted and dash-dotted lines show the partial contributions from $dB(E_\gamma, D_0)/dE_\gamma$ and $\sum_{K=\pm 1} dB(E_\gamma, D_K)/dE_\gamma$. The symbol represents the reported and evaluated experimental data [4]. (b) The M1 transition of ^{156}Gd . The solid line shows the FAM result. The dotted line shows the partial contribution associated with the orbital angular momentum in the M1 operator.

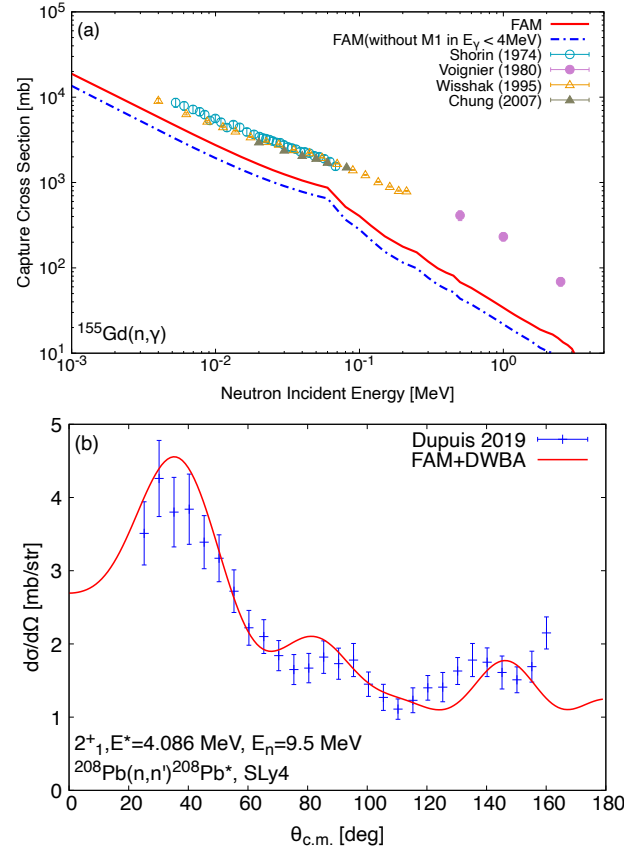


Figure 2. (a) The neutron capture cross sections of ^{155}Gd . The thick solid line shows the calculation result with photoabsorption cross sections of E1 and M1 transitions obtained from FAM. The dash-dotted line shows the result without the low energy M1 transition ($E_\gamma < 4\text{MeV}$). The symbols are experimental data in EXFOR. (b) The differential cross section for neutron-induced inelastic scattering up to the low energy state 2^+_1 ($E^*=4.086\text{MeV}$, $E_n=9.5\text{MeV}$) of ^{208}Pb with an incident neutron energy of 9.5 MeV. The symbols are the experimental data [7].

Figure 2(a) shows the cross section of $^{155}\text{Gd}(n, \gamma)$ with the γ -ray transmission coefficient by FAM. The strength of low energy M1 transition ($< 4\text{MeV}$) contributes to about half of the total calculated capture cross section (solid line). The underestimation of the cross section could be improved by uncertainties of the low energy E1 transition neglected in our QRPA calculation. Figure 2(b) shows one of the FAM+DWBA results of an inelastic scattering on ^{208}Pb . The FAM+DWBA calculation successfully reproduces both the amplitude and shape of the differential cross section. The FAM calculations reproduce the transition strengths to low-lying states of ^{208}Pb within 10%. Following Ref. [6], we apply a factor of $(\alpha + 2)(\alpha + 1)/2$ to a three-body term in the Skyrme where α is the density dependence of the Skyrme force. Additionally, we consider the nonlocal Perey effect and modify the distorted waves. These effects reduce the amplitude of the calculated differential cross section and make the results more consistent with experimental data.

3 Conclusion

We derive the fully self-consistent QRPA equation based on a noniterative FAM. The calculation based on such a noniterative FAM can reproduce the photoabsorption cross sections for E1 and M1 transitions. We apply the obtained cross sections to the calculation of neutron radiative capture cross sections and find the enhancement of the cross section induced by the low energy M1 transitions relevant to M1 scissors modes. The FAM+DWBA calculation can reproduce the amplitude and the shape of differential cross sections of neutron-induced inelastic scatterings up to low-lying discrete levels.

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