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Correcting for neutron width fluctuations in Hauser-Feshbach gamma branching ratios

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Abstract. Porter-Thomas fluctuations of neutron widths skew compound nuclear decay probabilities from their statistical Hauser-Feshbach values. We present a straightforward method to correct Hauser-Feshbach calculations for these fluctuations, useful for modeling near-threshold competition between gamma and neutron emission following beta decay or when standard width fluctuation corrections are inadequate.

Beta decay plays a crucial role in nucleosynthesis, making it one of the central focuses of the nuclear theory community and the new Facility for Rare Isotope Beams (FRIB) [1, 2]; in the laboratory, beta decay also serves a mechanism to study short-lived isotopes [3]. In neutron-rich systems, beta-delayed neutron emission (BDNE) may occur, in which case the gamma-channel branching ratio becomes a quantity of interest. Previous studies, such as those by Valencia et al. [4], have highlighted the limitations of Hauser-Feshbach (HF) theory in predicting BDNE gamma branching ratios. These studies have shown that incorporating neutron width fluctuations, as described by Porter-Thomas statistics, can significantly improve the accuracy of these predictions. However, the approach demonstrated by Valencia et al. [4] required a custom implementation of a `DICEBOX`-like [5] code, a Monte Carlo cascade simulation, including the effects of neutron width fluctuations in addition to gammas.

In these proceedings, we confirm that the gamma channel branching ratio can be significantly enhanced relative to the HF prediction, and we propose a straightforward correction factor to be applied to existing HF calculations. Our correction is inspired by the width fluctuation correction (WFC) factor [6–8] which allows one to avoid the costly Monte Carlo cascade calculations demonstrated in Ref. [4], but is not currently implemented in any HF code known to the authors.

Theory. As a function of the product excitation energy E_x, the gamma branching ratio is:

$$\left\langle \frac{\Gamma_{\gamma:i}}{\Gamma_{\gamma:i}+\Gamma_{n:i}} \right\rangle = \left\langle \frac{\sum_{f=1}^{k_\gamma} \Gamma_{\gamma:fi}}{\sum_{f=1}^{k_\gamma} \Gamma_{\gamma:f'i} + \sum_{f=1}^{k_n} \Gamma_{n:fi}} \right\rangle, \tag{1}$$

where $\Gamma_{\gamma:fi}$ is the partial decay width for a gamma transition from a level i to f, and $\Gamma_{n:fi}$ is the partial neutron decay width. $\Gamma = \hbar T$ for a transition probability T. The sums cover all k final states allowed by energy, angular momentum, and parity rules. The average is taken over all initial states in a specific initial energy bin. In principle, equation (1) can be calculated with a HF reaction code. However, HF theory assumes that:

$$\left\langle \frac{\Gamma_{\gamma:i}}{\Gamma_{\gamma:i}+\Gamma_{n:i}} \right\rangle \approx \frac{\langle \Gamma_{\gamma:i} \rangle}{\langle \Gamma_{\gamma:i} \rangle + \langle \Gamma_{n:i} \rangle}, \tag{2}$$

which is an approximation only valid for a sufficiently large number of partial widths.

Porter-Thomas theory posits that the partial decay widths $\Gamma_{fi} \propto |\langle \Psi_f | \hat{O} | \Psi_i \rangle|^2$ between initial states $|\Psi_i\rangle$ and final states $|\Psi_f\rangle$ follow a chi-squared distribution with one degree of freedom. Consequently, the total decay width for an initial level i with k partial widths, $\Gamma_i = \sum_{f=1}^{k} \Gamma_{fi}$, follows a chi-squared distribution with k degrees of freedom [7]. The corresponding probability density function for Γ_{fi} can be written: $P(x,k) = G(r)^{-1} r (rx)^{r-1} e^{-rx}$, where $x = \Gamma_{fi}/\langle \Gamma_{fi} \rangle$ is the partial width normalized to its mean, $G(r)$ is the gamma-function, and $r = k/2$.

Simulation. It is not obvious that Porter-Thomas fluctuations of neutron widths will increase the average gamma branching ratio relative to the HF prediction given by Eq. (2). To illustrate the effect, we conducted a numerical experiment by calculating the gamma branching Eq. (1) for randomly generated partial widths. In the first round of simulations, we arbitrarily assume that $\langle \Gamma_n \rangle = \langle \Gamma_\gamma \rangle$. We relax this assumption later. We work in units of the averages so that $P(x,k)$ given above applies directly to the partial widths ($x = \Gamma_{fi}$). Centrally important is the assumption that the neutron total widths include only a few terms k_n, so that $P(\Gamma_n) = P(\Gamma_n, k_n)$. We further assume the gamma total widths include many terms (owing to the high excitation energy required for neutron emission), so that $P(\Gamma_\gamma) \approx P(\Gamma_\gamma, k_\gamma = \infty) \approx \delta(\Gamma_\gamma - 1)$. From here onward $k_n = k$ refers to the number of neutron partial widths. With these assumptions, the purely HF estimate of $\langle \Gamma_\gamma / \Gamma_{\text{total}} \rangle$ for any k neutron partial widths is always:

$$\frac{\langle \Gamma_\gamma \rangle}{\langle \Gamma_\gamma \rangle + \langle \Gamma_n \rangle} = \frac{1}{1 + \langle P(\Gamma_n, k) \rangle} = \frac{1}{2}, \tag{3}$$

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regardless of the number of neutron partial widths. Next, we simulate the "true" gamma branching ratio by generating random neutron total widths $\Gamma_{n:i}$ from the chi-squared distribution $P(x, k)$ and computing the exact branching ratio:

$$\frac{\Gamma_\gamma}{\Gamma_{\text{total}}} = \frac{1}{1 + \Gamma_n}. \tag{4}$$

After generating $n = 10^6$ samples of the neutron widths and exact branching ratios, we compute the mean ratio $\langle \Gamma_\gamma / \Gamma_{\text{total}} \rangle$ of all the samples. The final results are relatively insensitive to the number of samples, but we use a large number to produce smooth histograms.

Figure 1 shows the results of the numerical simulations for $k = 1$ and $k = 100$. The gamma total widths are constant (black dashed line), while the neutron total widths are randomly distributed (blue histograms). The resulting width ratio distributions are shown in the narrower red histograms. At $k = 1$, we observe the maximum effect of PT

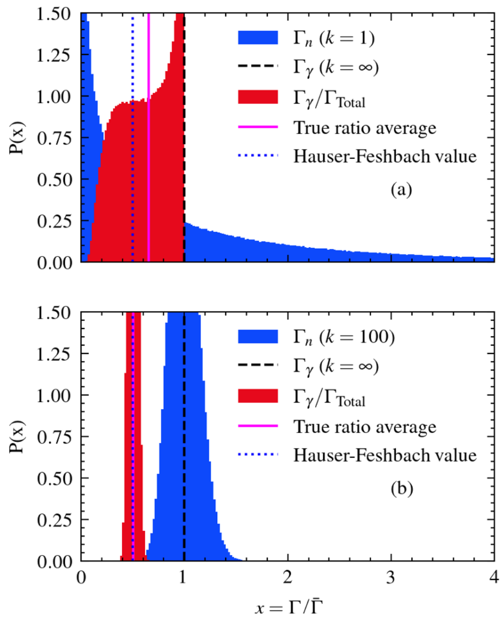

Figure 1. Porter-Thomas fluctuation toy model wherein the average neutron and gamma widths are equal. See text for discussion.

fluctuations. We obtain $\langle \Gamma_\gamma / \Gamma_{\text{total}} \rangle = 0.66$. As anticipated, the gamma branch is enhanced with respect to the HF prediction of 0.5; the increase is about 33 percent. From the $k = 100$ simulation, $\langle \Gamma_\gamma / \Gamma_{\text{total}} \rangle = 0.503$, which is close to the HF prediction. As expected, the effect of PT fluctuations are suppressed as the number of neutron partial widths increases.

We have shown how Porter-Thomas fluctuations of the neutron partial widths can enhance the gamma branching ratio. In the second round of simulations, we explore how the enhancement depends on the number of partial widths and the size of the HF estimate. We varied the number of neutron partial widths from $k = 1$ to $k = 100$ and relaxed the arbitrary assumption that $\langle \Gamma_n \rangle = \langle \Gamma_\gamma \rangle$. To preserve the generality of our findings, we normalize the average gamma width relative to the average neutron width. We set $P(\Gamma_\gamma) \approx \delta(\Gamma_\gamma - g)$ for some constant g in units of the average neutron total widths $\langle \Gamma_n \rangle$. The simulated branching ratios become:

$$\frac{\Gamma_\gamma}{\Gamma_{\text{total}}} = \frac{g}{g + \Gamma_n}, \tag{5}$$

where, as before, the neutron total widths are randomly drawn from $P(\Gamma_n, k)$. Since $\langle \Gamma_n \rangle = 1$ by construction, changing g is equivalent to changing the ratio

$$y \equiv \frac{\langle \Gamma_\gamma \rangle}{\langle \Gamma_\gamma \rangle + \langle \Gamma_n \rangle} = \frac{g}{g + 1}, \tag{6}$$

which is the HF estimate of the gamma branching ratio. We simulated HF gamma branching ratios between $y = 10^0$ and $y = 10^{-5}$ to span the range encountered in Ref. [4].

Solution. To model the impact of the neutron width fluctuations, we define a Porter-Thomas width fluctuation correction (PT WFC) factor, which relates the exact gamma branching ratio computed with Eq. (5) to the HF estimate, Eq. (6):

$$W(k, y) \equiv \frac{\langle \Gamma_\gamma / \Gamma_{\text{total}} \rangle}{\langle \Gamma_\gamma \rangle / (\langle \Gamma_\gamma \rangle + \langle \Gamma_n \rangle)} = \frac{\text{True ratio}}{\text{HF estimate y}}. \tag{7}$$

Importantly, this correction factor is independent of the absolute value of either the average neutron decay width or average gamma decay width; it depends only on the number of neutron partial widths k and the HF gamma branching ratio y.

Figure 2 shows the smooth decay of the PT WFC factor, Eq. (7), from its maximum at $k = 1$ where the PT enhancement can be up to two orders of magnitude at $y = 10^{-5}$. By $k = 5$, all curves are below an enhancement of 2. For all curves to be below an enhancement of

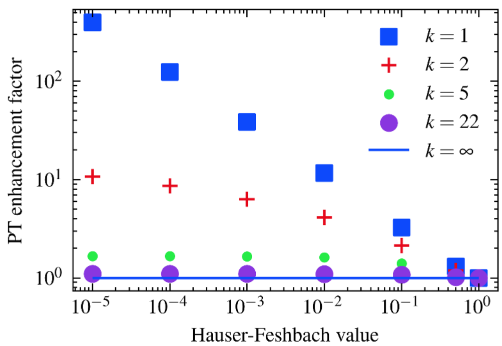

Figure 2. Porter-Thomas (PT) correction factor versus HF prediction y for different numbers of neutron partial widths k.

1.1, one requires $k > 22$. We find that the enhancement factor follows a smooth and systematic trend. This effect is independent of any energy dependence of the absolute strength of the neutron partial widths (which are known to have $\sqrt{E}$ dependence [9]) and depends only on the number of neutron partial widths k and the gamma branching ratio y given by Eq. (6). We can therefore compute the correction factor $W(k, y)$ *a priori* and apply it to our HF estimate to approximate the true ratio $\langle \Gamma_\gamma / \Gamma_n \rangle$.

Figure 3 shows an application of the PT WFC to beta-delayed gamma emission from Ref. [4]. We show our original HF calculation (HF) which used gamma strength functions from Ref. [10, 11], the corrected calculation using the $W(k, y)$ correction factor (HF+PTWFC), and the Monte-Carlo cascade simulation (Valencia 2017, MC) from Ref. [4]. All three consider only those decays from $J^\pi = 3^-$ states. At each excitation energy we determine y from the HF calculation, then apply the correction Eq. (7) from Figure 2. k is equal to the cumulative number of levels in the residual nucleus available for neutron emission. We reproduce the same enhancement produced by

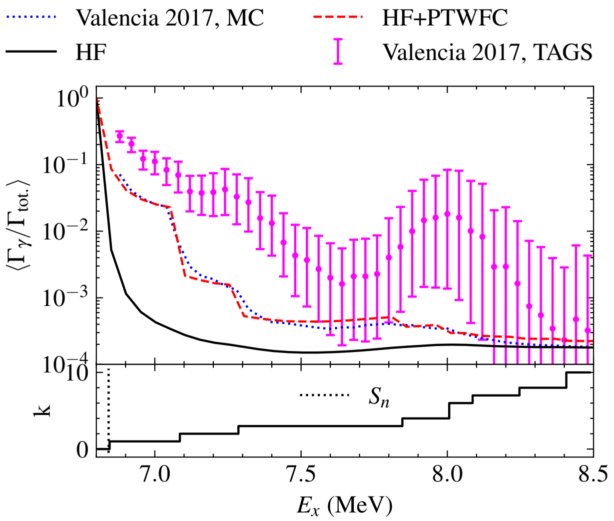

Figure 3. Gamma branching ratio with and without the PTWFC factor (7). The lower panel shows the number of neutron partial widths k available at each excitation E_x. The discontinuities in the HF+PTWFC calculation line up with changes in k.

the Monte Carlo decay simulation, within some margin of error attributable to differences in the details of the nuclear level densities and gamma strength functions used.

Conclusion

A simple Moldauer-type correction factor can correct standard HF calculations for neutron width fluctuations. This correction is broadly applicable to branching ratios near particle emission thresholds and is easy to implement. The correction factor can be precomputed, allowing adaptation of existing HF codes without *in situ* Monte Carlo simulations. Future work should produce a closed form for the correction factor.

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