Semi-classical treatment of photon cascades in nuclei

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Abstract. A simple semi-classical treatment of photon cascades in nuclei has been developed. The basic assumption is that a nucleus with a classical spin vector $\boldsymbol{J}$ can be represented by the maximally aligned quantum state $|J, M{=}J\rangle$ with the quantization axis being the spin direction $\boldsymbol{J}/J$. It is furthermore assumed that a photon emission yields a daughter state of a similar form, $|J', M' = J'\rangle$, but with its alignment direction having been modified as a consequence of the angular momentum recoil. The overall good quality of the treatment is illustrated for a variety of $E1$ and $E2$ two-photon cascades having non-trivial angular correlations. The method is suitable for use in nuclear fission simulation codes, making it possible to address photon-photon correlation observables quantitatively.

Low-energy fission leads to primary fragments that each have about a dozen MeV of excitation and half a dozen units of angular momentum, on average. The fragments, after possible neutron evaporation, deexcite by sequential photon emission. The associated emission patterns may reveal interesting aspects of the fission process, such as the directions of the fragment angular momenta and their mutual correlations.

The theoretical treatment of these cascades is most conveniently carried out by means of event-by-event Monte Carlo simulations that yield large samples of complete final states from which any distribution and correlation can subsequently be extracted (see, for example, Ref. [1]). Several such treatments have been developed for this purpose, most notably CGMF [2, 3], FREYA [4, 5], FIFRELIN [6], and GEF [7].

Directional correlations between the photons emitted during a decay cascade arise because each emitted photon carries some angular momentum and that affects the direction of the angular momentum of the corresponding daughter nucleus, thereby influencing the angular distribution of the next photon emitted. The inclusion of such "spin-recoil" effects, while highly interesting, is complicated to accomplish in an exact quantal treatment. We have therefore developed a simple semi-classical treatment that includes the successive changes in the nuclear spin direction and makes it practical to simulate the correlated photon emissions in a Monte Carlo approach.

In the semi-classical treatment, it is generally assumed that the initial nucleus, before the photon emission, has a definite spin magnitude J (as well as a definite excitation energy E) *and* that its spin is maximally aligned along some direction $\hat{z}$, *i.e.* its quantum state has the form $|\mathcal{N}\rangle = |\alpha; J, M{=}J\rangle$ when $\hat{z}$ is used as the quantization axis.

The nucleus-photon state resulting from a decay of multipolarity λ therefore has the following form,

$$|f\rangle_{J',h} = \sum_{\mu=-\lambda}^{\lambda} \langle J', M'; \lambda, \mu | J, J\rangle \, |\alpha'; J', M'\rangle \, |\lambda; \mu, h\rangle, \quad (1)$$

where J' is the angular momentum of the daughter level and $h = \pm 1$ is the helicity of the emitted photon. Furthermore, $|\alpha'; J', M'\rangle$ is a nuclear angular-momentum eigenstate (with respect to $\hat{z}$) and $|\lambda, \mu h\rangle$ denotes the state of a photon having the total angular momentum λ, the projection μ on $\hat{z}$, and the helicity h. The amplitude for the photon to be moving in the direction $\hat{\omega} = \boldsymbol{p}/\varepsilon = (\theta, \phi)$ is given in terms of the Wigner function,

$$\langle \hat{\omega} | \lambda; \mu, h\rangle = \langle \lambda, \mu | \, e^{i\phi \hat{J}_z} e^{i\theta \hat{J}_y} |\lambda, h\rangle = d^{\lambda}_{\mu,h}(\theta) \, e^{i\mu\phi}. \quad (2)$$

Thus, for each J' and h, the emitted photon and the daughter nucleus are generally in an entangled state $\langle \hat{\omega} | f\rangle$, with the nucleus being left in a different superposition of rotational substates $|\alpha'; J', M'\rangle$ for each photon emission direction $\hat{\omega}$. It follows that the expectation value of the angular momentum in the nuclear daughter state generally has the following form,

$$\underline{\boldsymbol{J}}'(\hat{\omega}) = (\underline{J}'_{\perp}(\theta)\cos\phi, \underline{J}'_{\perp}(\theta)\sin\phi, \underline{J}'_{z}(\theta)). \quad (3)$$

This means that the expectation value of the angular momentum vector of the daughter nucleus, $\underline{\boldsymbol{J}}'$, is *tilted* relative to that of the mother nucleus, $\underline{\boldsymbol{J}} = J\hat{z}$. The azimuthal direction of the tilted vector is either opposite of ϕ (for $h = +1$) or equal to ϕ (for $h = -1$). Thus the tilting angle $\chi(\theta)$, which is determined by $\tan\chi = \underline{J}'_{\perp}(\theta)/\underline{J}'_{z}(\theta)$, can have either sign. It is elementary to derive the expressions for $\underline{J}'_{\perp}(\theta)$ and $\underline{J}'_{z}(\theta)$ for the various transitions of interest (see Ref. [8] on which the present condensed report is based).

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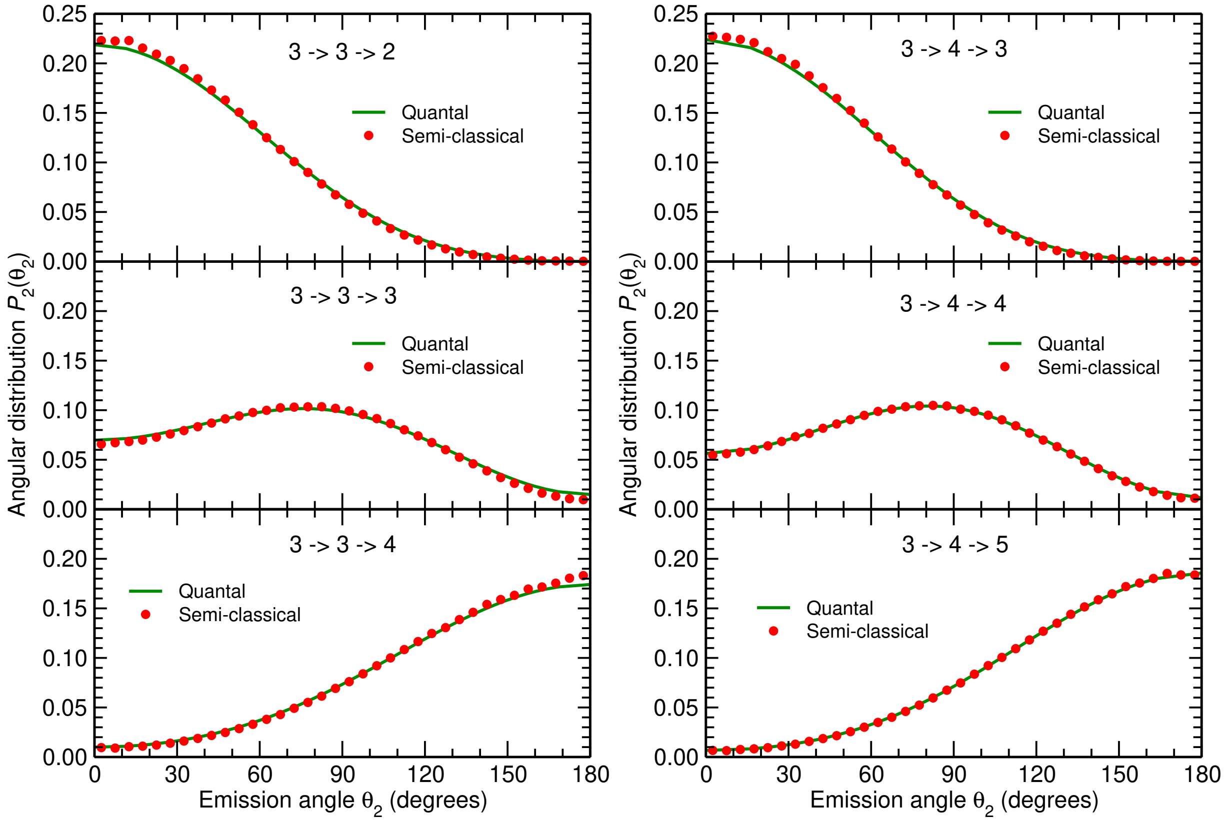

Figure 1. The angular distribution of the second $E1$ photon, $P_2(\hat{\omega}_2)$, for the three cascade types where the first $E1$ emission leads to $J' = J$ (left panels) and for the three cascade types where the first $E1$ emission leads to $J' = J+1$ (right panels), calculated for $J = 3$ and $h_1, h_2 = 1$. Solid (green) curve: the exact quantal distribution; solid (red) circles: the semi-classical simulation.

The principal approximation in the present semi-classical treatment is to replace the actual nuclear daughter state, $\langle\hat{\omega}|f\rangle$, by a state that is maximally aligned along $\underline{\boldsymbol{J}}'(\hat{\omega})$, *i.e.* it has the form $|\alpha'; J', J'\rangle$ in a reference system that has its polar axis $\hat{z}'$ directed along $\underline{\boldsymbol{J}}'(\hat{\omega})$.

The quality of the semi-classical approximation can be illustrated by cascades of two successive $E\lambda$ emissions. For each such two-photon cascade it is assumed that the nucleus is initially maximally aligned, $|\mathcal{N}\rangle = |\alpha; J, J\rangle$. The illustrations are made for relatively small angular momenta because the correlation effects decrease with J.

For each type of cascade, the exact expression for the joint directional distribution of the two emitted photons, $P_{12}(\hat{\omega}_1, \hat{\omega}_2) = d^2N_{12}/d^2\hat{\omega}_1 d^2\hat{\omega}_2$, can be derived in an elementary manner, while the approximate joint angular distribution is obtained by Monte Carlo simulation.

The approximate treatment is obviously exact for the angular distribution of the first photon, $P_1(\hat{\omega}_1)$, so only the angular distribution of the second photon, $P_2(\hat{\omega}_2)$, needs to be examined. Figure 1 shows that the semi-classical treatment is excellent for the distribution of the second photon in $E1 - E1$ cascades.

Figure 2 illustrates the semi-classical approximation for $E2 - E2$ cascades. Because $E2$ transitions are usually associated with collective modes whose energies increase with J, only transitions that decrease the angular momentum are considered, leaving only two cases to check: $J \to J-1 \to J-1$ and $J \to J-1 \to J-2$.

Although the quality of the approximation is less impressive than for $E1$ cascades, it may still be considered as acceptable because the deviations occur primarily in the polar region which is suppressed by the factor $\sin\theta_2$.

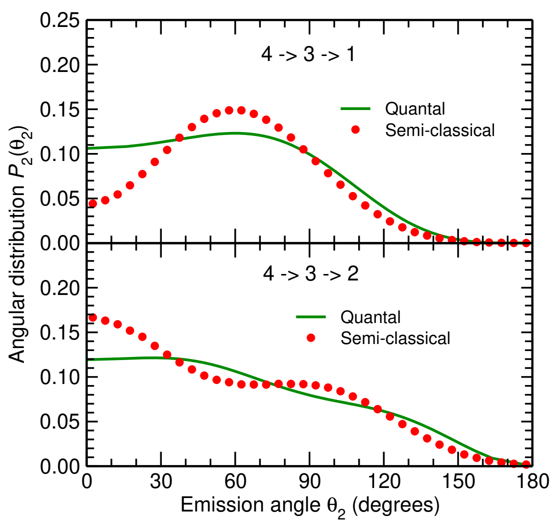

Figure 2. The angular distribution of the second $E2$ photon, $P_2(\hat{\omega}_2)$, for the two cases where the first $E2$ emission decreases the nuclear spin by one unit, $J' = J - 1$, for $J = 4$ and $h_1, h_2 = 1$. Solid (green) squares: the exact quantal distribution; solid (red) circles: semi-classical simulation.

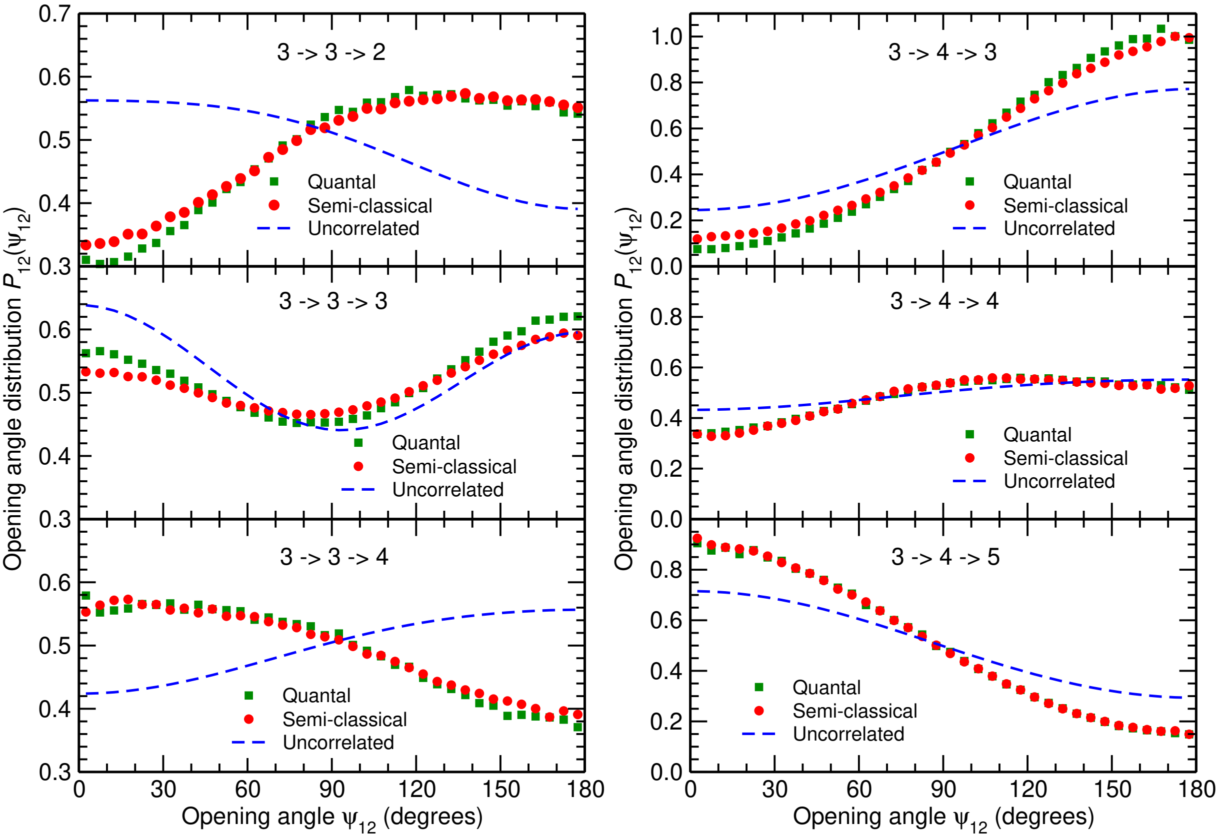

Figure 3. The distribution of the opening angle between the emission directions of two sequential $E1$ photons, $P_\psi(\psi_{12})$, for the the same cascade types as shown in Fig. 1. As there, the solid (green) curve shows the exact quantal distribution and the solid (red) circles show the semi-classical simulation; the dashed (blue) curve shows the result of ignoring the correlation, putting $P_{12}(\hat{\omega}_1, \hat{\omega}_2) = P_1(\hat{\omega}_1)P_2(\hat{\omega}_2)$.

It is of particular interest to determine how well the approximate treatment reproduces the correlations between the emission directions. To illustrate this crucial feature, we consider the distribution of the opening angle ψ_{12} between the two emission directions $\hat{\omega}_1$ and $\hat{\omega}_2$, where

$$\cos\psi_{12} = \hat{\omega}_1\cdot\hat{\omega}_2 = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos\phi_{12}, \quad (4)$$

with $\phi_{12} \equiv |\phi_1 - \phi_2|$. Figure 3 shows that the semi-classical treatment is very good for the distribution of the opening angle in $E1 - E1$ cascades and Fig. 4 shows that it is excellent for $E2 - E2$ cascades.

The distribution of the opening angle between two correlated emission directions differs notably from that arising from uncorrelated emission, as obtained by taking the joint distribution to be given by the product of the individual distributions, $P_{12}(\hat{\omega}_1, \hat{\omega}_2) = P_1(\hat{\omega}_1)P_2(\hat{\omega}_2)$ (indicated on Figs. 3 and 4 by the dashed blue curves).

The physical mechanisms responsible for the fission fragment angular momenta are currently a topic of active research. Although many recent correlation experiments have helped to illuminate the issue [11–18], a variety of mechanisms are being advocated [11, 19–28]. There is thus a need for careful calculation of the observable consequences of the various models proposed and photon correlation measurements may be particularly revealing. The semi-classical treatment may be helpful in this context.

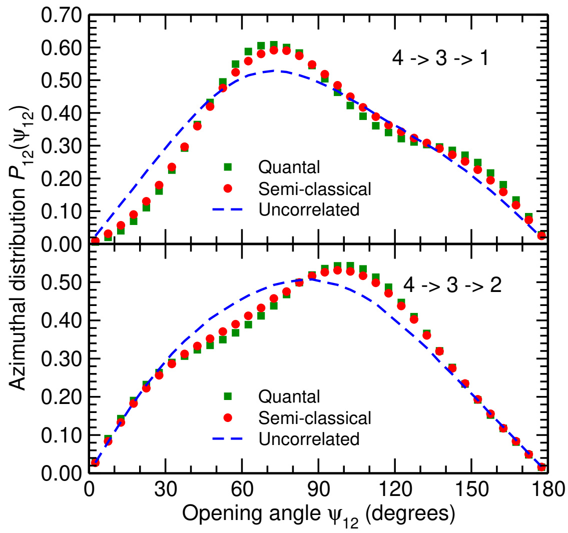

Figure 4. The distribution of the opening angle between the emission directions of two sequential $E2$ photons, $P_{12}(\psi_{12})$, for the two cases where the first $E2$ emission decreases the nuclear spin by one unit, $J' = J-1$, for $J=4$ and $h_1, h_2 = 1$. Solid (green) squares: sampling of the exact correlated distribution; solid (red) circles: semi-classical sampling; dashed (blue) curve: uncorrelated emission.

We have presented a novel semi-classical treatment of photon emission that is particularly well suited for Monte Carlo simulations of decay cascades, such as those occurring in fission fragments.

The key approximation is to represent each fragment by a maximally aligned quantum state, using the direction of its original classical angular momentum as the initial alignment direction and then modifying the alignment direction after each emission depending on the emission direction. The inclusion of this spin recoil effect at each stage of a cascade ensures that the resulting many-photon emission pattern (approximately) retains its inherent correlations.

The quantitative utility of this approximate treatment was illustrated by a variety of two-photon cascades. For $E1$ cascades it was shown that the treatment works very well for both the individual angular distributions and for the distribution of the opening angle between the two photons. For $E2$ cascades the reproduction of the individual distribution of the second photon is less perfect but still acceptable, while the opening-angle distributions are nearly perfectly reproduced.

The developed semi-classical treatment is straightforward to implement into existing fission simulation codes such as FREYA. Because the treatment keeps track of how the fragment angular momentum vector is affected by each photon emission, the evolution of the correlated spin-spin distribution can be followed through the cascade stage. In particular, the results presented suggest that such an augmentation would make it possible to address the photon correlations in a quantitatively meaningful manner and develop more refined observables for probing the fission fragment angular momenta.

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