Microscopic analysis of giant monopole resonance in nuclear isotopic chains

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Abstract. A systematic study of the isoscalar giant monopole resonance (ISGMR) in a variety of nuclear systems is performed within the microscopic self-consistent Skyrme HF+BCS method and coherent density fluctuation model. The calculations for the incompressibility in finite nuclei are based on several energy density functionals for nuclear matter. This theoretical scheme is successfully proved, for instance, in calculations of the nuclear symmetry energy, as well as of the ratio of its surface to volume components. The good agreement achieved between the calculated centroid energies of the ISGMR and their recent experimental values for various nuclei demonstrates the relevance of the proposed theoretical approach. The latter can be applied to analyses of neutron stars properties, such as incompressibility, symmetry energy, slope parameter, and other astrophysical quantities.

1 Introduction

The isoscalar giant monopole resonance (ISGMR) plays an important role in constraining the nuclear equation of state (EOS) [1–6]. The energy of this resonance is closely related to the nuclear incompressibility. The latter can be connected to the incompressibility of the infinite nuclear matter [7], which represents an important ingredient of the nuclear matter EOS. To make the EOS isospin asymmetry term more precise, recent experimental measurements of isoscalar monopole modes are being extended in isotopic chains from the nuclei on the valley of stability towards exotic nuclei with larger proton-neutron asymmetry. For instance, different measurements have been conducted on Ni isotopes far from stability, namely ^{56}Ni [8, 9] and ^{68}Ni [10, 11]. In particular, the ^{68}Ni experiment is the first measurement of the isoscalar monopole response in a short-lived neutron-rich nucleus using inelastic alpha-particle scattering. The peak of the ISGMR was found to be fragmented, indicating a possibility for a soft monopole resonance.

In the present work (as well as in Ref. [12]), the incompressibility and the centroid energy of ISGMR are investigated for Ni (A=56-68), Sn (A=112-124), and Pb (A=204-208) isotopic chains on the basis of the Brueckner energy-density functional (EDF) for nuclear matter [13, 14] and using the coherent density fluctuation model (CDFM) (e.g., Refs. [15, 16]). The main reason to select these chains of nuclei is partly supported by their recent intensive ISGMR measurements so that we focus too on the comparison with the available experimental data for Ni [17], Sn [18], and Pb [19, 20] isotopes. The CDFM is a natural extension of the Fermi gas model based on the delta-function limit of the generator coordinate method [16, 21] and includes long-range correlations of collective type. During the years the CDFM has been successfully applied to calculations of nuclear structure and nuclear reactions characteristics. The efficiency of CDFM to be applied as a "bridge" for a transition from the properties of nuclear matter to the properties of finite nuclei studying the nuclear symmetry energy, the neutron pressure, and the asymmetric incompressibility in finite nuclei was demonstrated in our previous works (e.g. [22–25]).

Our main purpose is to validate the CDFM for studies of collective vibrational modes by using as a main theoretical ground the self-consistent Hartree–Fock (HF)+BCS method with Skyrme interactions. The mentioned above model gives a link between nuclear matter and finite nuclei in studying of their properties, such as binding energies and rms radii of light, medium, and heavy nuclei. In addition, new results for the excitation energies of ISGMR for Ca, Fe, Zn, Zr, Mo, and Cd nuclei are reported inspired by the new experimental data for them and the fully self-consistent quasiparticle random-phase-approximation (QRPA) calculations (e.g., in [26]).

2 Theoretical formalism

The centroid energy of ISGMR E_{ISGMR} is generally related to a finite nucleus incompressibility $\Delta K(N,Z)$ for a nucleus with Z protons and N neutrons ($A = Z + N$ is the mass number). Among the various definitions of E_{ISGMR} we will mention the one from, e.g., Ref. [27]:

$$E_{ISGMR} = \frac{\hbar}{r_0 A^{1/3}} \sqrt{\frac{\Delta K(N,Z)}{m}}, \tag{1}$$

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where r_0 is deduced from the equilibrium density and m is the nucleon mass. In the present work, describing the monopole vibrations in terms of harmonic oscillations of the nuclear size and assuming an $A^{1/3}$ law for it, we calculate E_{ISGMR} by using Eq. (1). Values of the parameter r_0 between 1.07 and 1.2 fm are adopted, which are determined from experiments on particle scattering off nuclei. The incompressibility (the curvature) for nuclear matter (NM) ΔK^{NM} of the symmetry energy $S(\rho)$ is given by

$$\Delta K^{NM} = 9\rho_0^2 \left.\frac{\partial^2 S(\rho)}{\partial \rho^2}\right|_{\rho=\rho_0}, \tag{2}$$

where ρ_0 is the density at equilibrium.

The CDFM was suggested and developed in Refs. [15, 16] (see also our recent papers [24, 25]). Within the model the one-body density matrix (OBDM) of the nucleus $\rho(\mathbf{r},\mathbf{r}')$

$$\rho(\mathbf{r},\mathbf{r}') = \int_0^\infty dx |F(x)|^2 \rho_x(\mathbf{r},\mathbf{r}') \tag{3}$$

is expressed by OBDM's $\rho_x(\mathbf{r},\mathbf{r}')$ of spherical "pieces" of nuclear matter ("fluctons") with radius x of all A nucleons uniformly distributed in it:

$$\rho_x(\mathbf{r},\mathbf{r}') = 3\rho_0(x)\frac{j_1(k_F(x)|\mathbf{r}-\mathbf{r}'|)}{(k_F(x)|\mathbf{r}-\mathbf{r}'|)}\Theta\left(x - \frac{|\mathbf{r}+\mathbf{r}'|}{2}\right). \tag{4}$$

In Eq. (4) j_1 is the first-order spherical Bessel function and

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_0(x)\right)^{1/3} \equiv \frac{\alpha}{x} \tag{5}$$

is the Fermi momentum with

$$\alpha \equiv \left(\frac{9\pi A}{8}\right)^{1/3} \simeq 1.52A^{1/3}. \tag{6}$$

It can be seen from Eq. (3) that the density distribution in the CDFM is:

$$\rho(\mathbf{r}) = \int_0^\infty dx |F(x)|^2 \rho_0(x)\Theta(x - |\mathbf{r}|) \tag{7}$$

with

$$\rho_0(x) = \frac{3A}{4\pi x^3}. \tag{8}$$

It follows from Eq. (7) that the weight function $|F(x)|^2$ of CDFM can be obtained in the case of monotonically decreasing local densities (*i.e.*, for $d\rho(r)/dr \leq 0$) by

$$|F(x)|^2 = -\frac{1}{\rho_0(x)} \left.\frac{d\rho(r)}{dr}\right|_{r=x} \tag{9}$$

being normalized as

$$\int_0^\infty dx |F(x)|^2 = 1. \tag{10}$$

In the case of the Brueckner method for nuclear matter energy [13, 14] the asymmetric incompressibility has the form [22, 23]:

$$\Delta K^{NM}(x) = -83.4\rho_0^{2/3}(x) + 4b_5\rho_0^{4/3}(x) + 10b_6\rho_0^{5/3}(x) \tag{11}$$

and contains the following values of the parameters: b_5=372.84 and b_6=-769.57. According to the CDFM scheme, the curvature for finite nuclei can be expressed in the following form:

$$\Delta K = \int_0^\infty dx |F(x)|^2 \Delta K^{NM}(x). \tag{12}$$

In our calculations we apply self-consistent deformed Hartree-Fock method with density-dependent Skyrme interactions [28] with pairing correlations. We use the Skyrme SLy4 [29], Sk3 [30], SGII [31], and SkM [32] parametrizations. The different Skyrme parameter sets used in the present calculations are chosen since they are characterized by different values of the nuclear incompressibility, ΔK^{NM} = 230, 217, 215, and 355 MeV for SLy4, SkM, SGII, and Sk3, respectively [33]. All necessary expressions for the single-particle functions and densities in the HF+BCS method can be found, e.g., in Ref. [22].

3 Results and discussion

The obtained centroid positions of the monopole mode for Ni, Sn, and Pb isotopes calculated using Brueckner EDF in the procedure [Eqs. (1), (11), and (12)] are compared with available experimental data in Tables 1–3, respectively. It can be seen from Table 1 that a very good agreement with the experimental data for ^{56,58,60}Ni is obtained, while the results with both Skyrme interactions underestimate the experimental energy of the soft monopole vibrations of ^{68}Ni. The excitation energy of this ISGMR in ^{68}Ni is located unexpectedly at higher energy (21.1 MeV) for the Ni isotopic chain, having at the same time large error bars. The reason is due to the large fragmentation of the isoscalar monopole strength in the unstable neutron-rich ^{68}Ni nucleus, much more than in stable nuclei [10, 11]. Recently, we have performed calculations by applying the latest version of the Barcelona-Catania-Paris-Madrid (BCPM) EDF [35–38] and obtained a value of 21.74 MeV for the monopole excitation energy in ^{68}Ni, close to the experimental value. The challenging picture of fragmentation of monopole resonance peak typical of lighter nuclei with $A \leq 28$ still remains poorly understood. The obtained values of E_{ISGMR} for Sn isotopes (A = 112–124) exhibit small difference regarding the Skyrme parametrization (see Table 2). The theoretical results for the centroid energies for the same Sn isotopes obtained in Ref. [18] by using the SkP (between 14.87 and 15.60 MeV), SkM* (between 15.57 and 16.23 MeV), and SLy5 (between 15.95 and 16.61 MeV) parameter sets are in good agreement with our results. Almost no dependence on the Skyrme forces used in the calculations of the centroid energies is found for Ni and Pb isotopes being slightly larger in the case of SkM interaction than when using the SLy4 one. The calculated excitation energies of the ISGMR within the procedure for the Ca isotopic chain compared with the available experimental data are listed in Table 4.

The collective (bulk) character of the giant resonances and nuclear incompressibility presumes a quite smooth

Table 1. The values of the centroid energies E_{ISGMR} (in MeV) of Ni isotopes obtained from HF+CDFM calculations in this work using SLy4 and SkM Skyrme forces compared with the experimental data found in the literature.

Nucleus	SLy4	SkM	Exp.
^{56}Ni	19.41	19.57	19.1 ± 0.5 [9]
			19.3 ± 0.5 [8]
^{58}Ni	18.95	19.18	18.43 ± 0.15 [17]
^{60}Ni	18.62	18.79	18.10(29) [17]
^{68}Ni	17.46	17.70	21.1 ± 1.9 [10, 11]

Table 2. The values of the centroid energies E_{ISGMR} (in MeV) of Sn isotopes (A=112-124) obtained from HF+CDFM calculations in this work using SLy4, SGII, and Sk3 Skyrme forces. The experimental data are taken from Table III of Ref. [18].

Nucleus	SLy4	SGII	Sk3	Exp.
^{112}Sn	15.04	15.30	14.89	16.2 ± 0.1
^{114}Sn	15.03	15.20	14.70	16.1 ± 0.1
^{116}Sn	14.94	15.08	14.56	15.8 ± 0.1
^{118}Sn	14.82	15.13	14.48	15.8 ± 0.1
^{120}Sn	14.69	15.08	14.58	15.7 ± 0.1
^{122}Sn	14.68	15.00	14.61	15.4 ± 0.1
^{124}Sn	14.68	14.96	14.51	15.3 ± 0.1

Table 3. The same as in Table 1 but for Pb isotopes.

Nucleus	SLy4	SkM	Exp.	Theory
^{204}Pb	12.16	12.29	13.98 [19]	
^{206}Pb	12.12	12.23	13.94 [19]	
^{208}Pb	12.10	12.15	13.96 [20]	14.453 [34]

Table 4. The values of the centroid energies E_{ISGMR} (in MeV) of Ca isotopes obtained from HF+CDFM calculations in this work using SLy4 and SkM Skyrme forces and r_0 = 1.2 fm compared with the experimental data found in the literature.

Nucleus	SLy4	SkM	Exp.
^{40}Ca	20.03	19.99	19.18 ± 0.37 [39]
^{42}Ca	19.83	19.98	19.7 ± 0.1 [40]
^{44}Ca	19.71	19.95	19.49 ± 0.34 [41]
^{46}Ca	19.69	19.91	
^{48}Ca	19.71	19.89	19.88 ± 0.16 [39]

variation of the properties of the ISGMR with mass, thus not expecting very strong variations related to the internal nuclear structure. The isotopic evolution of the centroid energies E_{ISGMR} for the Ni, Sn, and Pb isotopes is presented in Fig. 1. As a test of the role of the half-density radius parameter r_0 on the centroid energy [Eq. (1)], the results in the case of SLy4 force with r_0 = 1.2 fm (e.g., in Refs. [42, 43]), r_0 = 1.07 fm (for instance, in Ref. [44]), and r_0 = 1.123 fm [45] are presented. It is seen that with the increase of r_0 the agreement with the experimental data becomes better for lighter isotopes. Particularly, the value of r_0 = 1.123 fm leads to fair agreement of the ISGMR energies for Sn isotopes, while for Ni isotopes the experimental data are reproduced better with r_0 = 1.2 fm and for Pb isotopes with r_0 = 1.07 fm. Here we would like to note that the specific choice of the r_0 parameter values adopted to calculate the values of the centroid energies by using (1) is often used in the literature. The values of the measured nuclear radii are deduced from processes with strongly interacting particles or electron (muon) scattering. It is well known that the A-dependence of r_0 exhibits a smooth decrease with A being 1.07 fm for nuclei with $A > 16$ and increasing to 1.2 fm for heavy nuclei. The results for the calculated values of E_{ISGMR} and the corresponding ranges of change in respect to r_0 are illustrated in Fig. 1 by hatched areas. Thus, we find a sensitivity of the results for centroid energies of ISGMR to the radial parameter r_0 and this fact has to be taken into account when considering resonances in light, medium, and heavy nuclei.

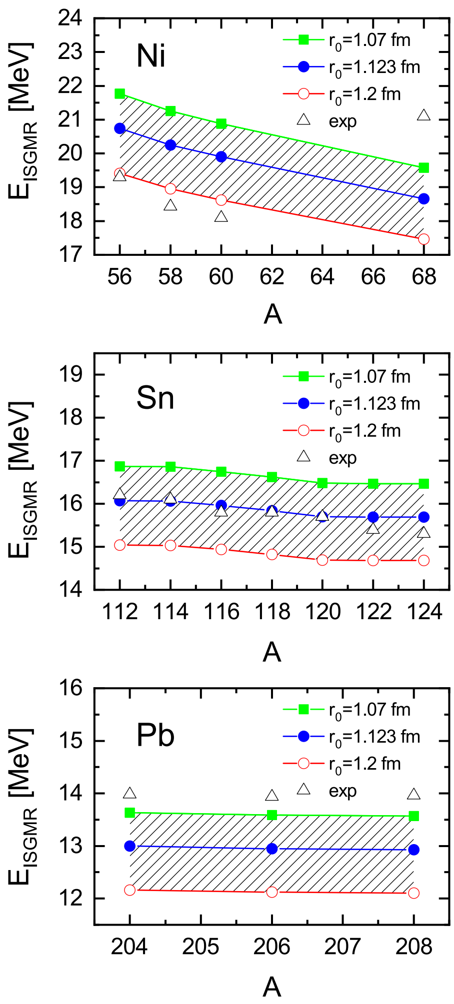

Figure 1. The centroid energies E_{ISGMR} as a function of the mass number A for Ni, Sn, and Pb isotopes in the case of SLy4 force obtained with three different values of the parameter r_0 = 1.07, 1.123, 1.2 fm [Eq. (1)] compared with the experimental data (see Refs. in Tables 1-3).

We present in Table 5 the calculated values of E_{ISGMR} with SLy4 force for Cd isotopes using the BCPM EDF.

Table 5. The values of the centroid energies E_{ISGMR} (in MeV) of Cd isotopes obtained from HF+CDFM calculations in this work using SLy4 force and the BCPM functional with r_0 = 1.123 fm in Eq. [1]. The experimental data are taken from Ref. [46].

Nucleus	SLy4	Exp.
^{106}Cd	16.15	16.27 ± 0.09
^{110}Cd	15.88	15.94 ± 0.07
^{112}Cd	15.73	15.80 ± 0.05
^{114}Cd	15.64	15.61 ± 0.08
^{116}Cd	15.49	15.44 ± 0.06

The role of microscopic three-body forces in the proposed approach to study the giant monopole resonances can be clearly revealed by applying this functional and particularly to treat successfully medium-heavy nuclei. An excellent agreement with the available experimental data is achieved for Cd isotopic chain. For this case our results fit very well the theoretical predictions from QRPA calculations for the ISGMR peaks obtained with the SV-bas Skyrme force [26].

The isotopic evolution of the centroid energies E_{ISGMR} for the Ca and Cd isotopes is presented in Fig. 2. A very good agreement with the experimental data is observed for the centroid energies of isotopes from the Cd chain. Concerning the Ca isotopes, a mild softening of the giant monopole resonance energy can be seen going from ^{40}Ca to ^{48}Ca, particularly for the calculated values with SkM force. Similar conclusion was drawn in Ref. [47], where a need for new measurements along the unstable neutron-rich Ca isotopes is noticed in order to look at the role of the continuum in the development of a soft monopole mode. On this line, the recent measurements performed in iThemba LABS [48] showed conflicting results with previous TAMU and RCNP data for the monopole resonance of Ca isotopes. From the theoretical side, it was demonstrated that the gross structure of the ISGMR in the Ca isotopes (A = 40 − 48) is caused by the complex configurations [49].

In the end, we would like to note that the values of the radial parameter r_0 used in the calculations are result of parametrization from the fit of E_{ISGMR} with the experimental data for two nuclei in both limits of the considered nuclear range, namely for ^{40}Ca and ^{208}Pb. At the same time we took care and checked whether the relation between the mean square mass radius and r_0 is fulfilled (see, e.g., Ref. [50]).

4 Conclusions

The main results of the present work can be summarized as follows:

i) A very good agreement is achieved between the calculated centroid energies of the ISGMR and corresponding experimental values for Ni isotopes when r_0 = 1.2 fm. Especially this concerns the exotic double-magic ^{56}Ni nucleus, for which the obtained (with SLy4 Skyrme force) value is 19.41 MeV, in consistency with the centroid position of the ISGMR found at 19.1 ± 0.5 MeV.

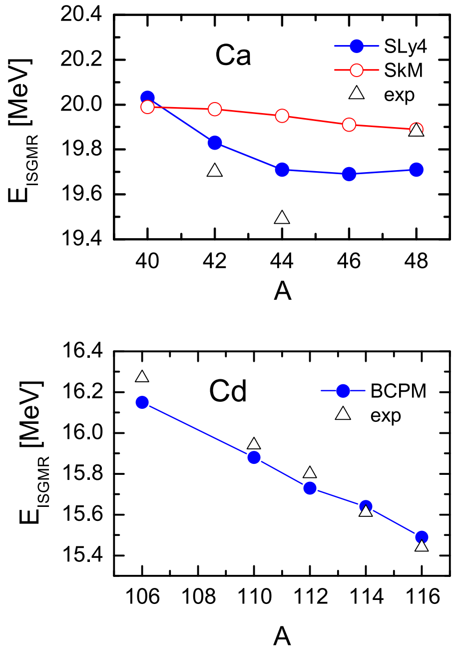

Figure 2. The centroid energies E_{ISGMR} as a function of the mass number A for Ca and Cd isotopes. In the case of Ca isotopes the HF+CDFM calculations are performed with Brueckner EDF using SLy4 and SkM Skyrme forces with r_0 = 1.2 fm [Eq. (1)], while in the case of Cd isotopes BCPM results with SLy4 force with r_0 = 1.123 fm are shown. Comparison with the experimental data is made (see Refs. in Tables 4 and 5).

ii) The comparative analysis of the centroid energies in the case of Sn and Pb isotopes shows less agreement with r_0 = 1.2 fm, but still in acceptable limits.

iii) The agreement with the experimental values of E_{ISGMR} can be improved also by varying the parameter r_0 in strong connection with the mass dependence of this parameter and its effect for the considered isotopes.

iv) In general, the obtained results demonstrate the relevance of our theoretical approach to probe the excitation energy of the ISGMR in various nuclei. The future goals are to extend this theoretical study by employing more realistic energy-density functionals for nuclear matter, on one side. For instance, a good choice could be the microscopic EOS derived by Sammarruca *et al.* [51] based on high-precision chiral nucleon-nucleon potentials at next-to-next-to-next-to-leading order (N^{3}LO) of chiral perturbation theory. On another side, to expand the nuclear spectrum to lighter and medium mass nuclei including isotopes with large proton-neutron asymmetry.

In conclusion, to extract the isospin dependence of the incompressibility coefficient, a key ingredient in astrophysical studies, further theoretical investigations are needed to carry out calculations of the ISGMR for neutron-rich nuclei and to compare the results with the available experimental data.

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