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Multi-phonon γ vibrational bands and chiral vibrations in the A$\sim$100 neutron-rich region

E.H. Wang[1,2,*], *M.* Abushawish[3,**], *J.H.* Hamilton[2], *J.* Dudouet[3], *A.* Navin[4], *S.* Bhattacharyya[5,6], *O.* Stezowski[3], *G.* Bhat[7], *J.A.* Sheikh[8,9], *S.* Jehangir[8], *M.* Rejmund[4], *A.* Lemasson[4], *E.* Clément[4], *S.* Sun[10], *H.* Zhang[10], *W.Z.* Xu[10], *S.Y.* Wang[1,10,11], *B.* Qi[10], *C.F.* Jiao[12], *Y.X.* Luo[13], *J.O.* Rasmussen[13], and *S.J.* Zhu[14]

[1]School of Energy and Power Engineering, Shandong University, Jinan 250061, China
[2]Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37235, USA
[3]Université de Lyon 1, CNRS/IN2P3, UMR5822, IP2I, F-69622 Villeurbanne Cedex, France
[4]GANIL, CEA/DRF-CNRS/IN2P3, BP 55027, 14076 Caen cedex 5, France
[5]Variable Energy Cyclotron Centre, 1/AF Bidhannagar, Kolkata 700064, India
[6]Homi Bhabha National Institute, Training School Complex, Anushaktinagar, Mumbai-400094, India
[7]Department of Physics, Government Degree College Shopian, Jammu and Kashmir 192303, India
[8]Department of Physics, Islamic University of Science and Technology, Awantipora 192122, India
[9]Department of Physics, University of Kashmir, Srinagar 190 006, India
[10]School of Space Science and Technology, Institute of Space Sciences, Shandong University, Weihai 264209, China
[11]WeiHai Research Institute of Industrial Technology, Shandong University, Weihai 264209, China
[12]School of Physics and Astronomy, Sun Yat-sen University, Zhuhai 519082, China
[13]Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
[14]Department of Physics, Tsinghua University, Beijing 100084, China

Abstract. We present experimental evidence for the one- and two-phonon γ vibrational bands in the isotopes $^{103-108}$Mo by analyzing high-statistics γ-γ-γ and γ-γ-γ-γ coincidence data from the spontaneous fission of ^{252}Cf. The consistency of the one- and two-phonon γ bands in both odd- and even-mass isotopes within this range provides a robust test for microscopic theories. Additionally, we have identified evidence for one-, two-, and possibly three-phonon γ bands in ^{103,105}Nb. Triaxial projected shell model calculations successfully reproduce the level structures and align well with the excitation energies of the observed one- and two-phonon γ bands. The potential existence of three-phonon γ bands in ^{103,105}Nb is further supported by recent particle-vibration coupling model calculations. Furthermore, chiral vibrational bands have been observed in ^{104,106}Mo.

1 Introduction

In nuclear structure studies, the low- and medium-energy excitations of nuclei can be explained as single particle motion, collective rotation and vibration [1]. Among these, quadrupole vibrations in deformed nuclei can be classified into β and γ vibrations, depending on the angular momentum projection of a quadrupole phonon on the symmetry axis, which is 0 or 2 $\hbar$, respectively. The γ vibrational band structure serves as a measure of triaxiality and its softness [2, 3]. Unlike widely observed one-phonon γ vibrations, two-phonon γ vibrations are rare and provide valuable tests of Pauli principle [4]. Early theoretical models predicted two-phonon γ bands, including the quasi-particle-phonon nuclear model [4, 5], multi-phonon method [6–8], extended interacting boson model [9], and self-consistent collective coordinate method [10]. Assuming K is the band-head quantum number of a quasiparticle band, two one-phonon γ states exist with $K_\gamma = K \pm 2$. Additionally, three possible two-phonon γ bands may emerge with $K-4$, K and $K+4$, where the $K_{2\gamma} = K+4$ state is the most purely vibrational.

The first experimental observation of two-phonon γ vibrations was reported in ^{168}Er [11] and subsequent studies expanded to regions such as A$\sim$ 100 [12], 130 [13], 230 [20], etc. In the neutron-rich A$\sim$100 region, two-phonon γ vibrational bands were first identified in ^{106}Mo [12]. Experimental indicators of K+4 two-phonon γ vibrational bands include: (i) allowed E2 transitions from the one-phonon to ground-state bands and between successive phonon states, while transitions from two-phonon to ground-state bands are forbidden; (ii) comparable B(E2) values for interband transitions; (iii) consistent properties among the 0γ, 1γ and 2γ bands, including moments of inertia, rotational parameters and g-factors; (iv) pairing gap energies and Potential Energy Surface (PES) calculations ruling out two- or three-quasiparticle states for even-even and odd-A nuclei, respectively.

The study of fission fragments from heavy isotope reactions provides insights into neutron-rich nuclei across broad isotopic chains, spanning from Z=32 to 64 [15].

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[*] e-mail: ehwang@sdu.edu.cn

[**] e-mail: m.abushawish@ip2i.in2p3.fr

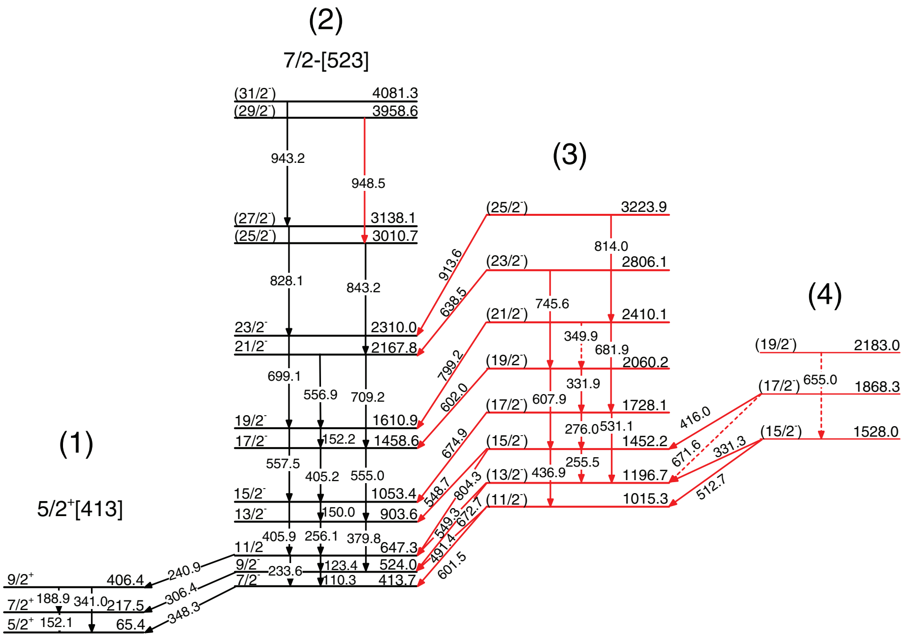

Figure 1. Level scheme of ^{107}Mo obtained in this study. Newly identified transitions are highlighted in red. The level energies are normalized to the 65.4 keV isomer. This figure is adapted and modified from Refs. [19, 20]. Note that the 5/2$^+$[413] band head, previously reported as ground state in Ref. [19], was reassigned as a 65.4 keV isomer in Ref. [20].

This approach reveals diverse nuclear shapes and structures, including single-particle and collective degrees of freedom. Our collaboration identified the first candidate for $\gamma\gamma$ bands in an odd-A nucleus, ^{105}Mo [16] in the A∼100 triaxial region providing the coupling of single particle motion, collective rotation and vibration. We also substantially extended the findings of $\gamma\gamma$ bands in Nb, Mo, Tc, Ru, Pd isotopes. Here, we present new findings on the A∼100 triaxial region, populated via the spontaneous fission (SF) of ^{252}Cf. Rapid shape changes in neutron-rich nuclei of this region are linked to the filling of the $\nu h_{11/2}$ and $\pi g_{9/2}$ orbitals. We provide evidence of triaxiality through the observation of multi-phonon γ vibrational bands in $^{103-108}$Mo, ^{103,105}Nb, as well as chiral vibrations in ^{104,106}Mo.

2 Experimental setup

The experiments were conducted at the Lawrence Berkeley National Laboratory (LBNL). A 62 μCi ^{252}Cf source was placed between two iron foils, each 10 mg/cm^2 thick. Using 101 Ge detectors from the Gammasphere array, data were collected and sorted into 5.7×10^{11} γ-γ-γ and higher-fold γ events, as well as 1.9×10^{11} γ-γ-γ-γ and higher-fold γ coincidence events. The γ coincidence data were analyzed using the RADWARE software package [17].

3 Results

3.1 Multi-phonon γ vibrational bands in $^{103-108}$Mo and ^{103,105}Nb

In the recent study [18–20], we proposed $\gamma\gamma$ bands in ^{103,107}Mo. The new level scheme for ^{107}Mo is illustrated in Fig. 1. We identify two bands, with band (1) exhibiting significant extension, though only the first three levels are displayed. The newly proposed bands (3) and (4) correspond to the one- and two-phonon γ vibrational bands of the 7/2$^-$[523] orbital. Coincidence spectra demonstrating the γ bands in ^{107}Mo are shown in Fig. 2.

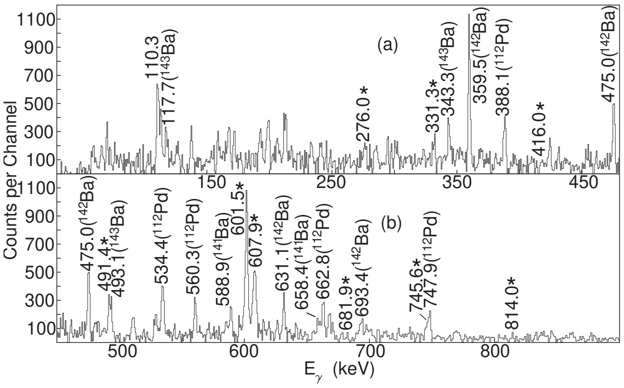

Figure 2. Partial γ-ray coincidence spectra gated on 436.9 and 348.3 keV transitions in ^{107}Mo. Part (a) and (b) show different regions of the spectrum within this gate. Newly identified transitions are marked with asterisks. This figure is adapted and modified from Ref. [19].

A comparison of Triaxial Projected Shell Model (TPSM) energies, obtained through shell model diagonalization, with observed energies shows strong agreement, particularly in the bandhead energy of the $\gamma\gamma$-band in ^{103,107}Mo (Fig. 3). The TPSM is now widely used to study the band structures of deformed and transitional nuclei [21–27]. Notably, this model has provided new insights into the nature of the observed γ-band structures.

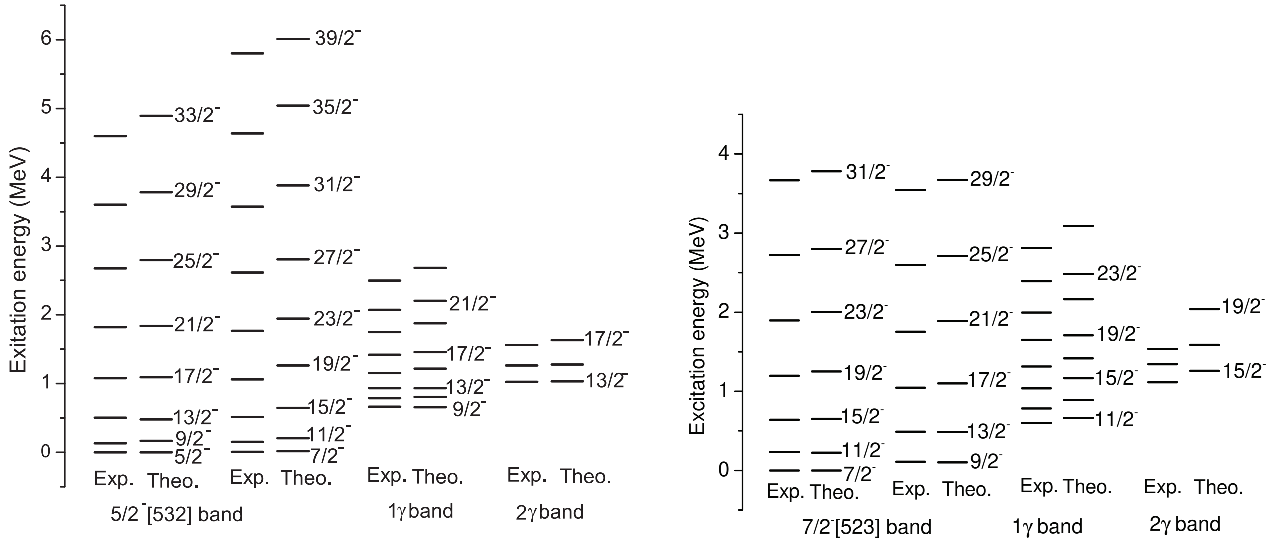

Figure 3. Comparison of the TPSM calculations with experimental data for ^{103}Mo (left) and ^{107}Mo (right). Level energies are normalized to the $5/2^{-}$[532] and $7/2^{-}$[523] 0γ bands, respectively. This figure is adapted and modified from Refs. [18, 19]. Note that the $5/2^{+}$[413] band head in ^{107}Mo, previously reported as the ground state in Ref. [19], was reassigned to a 65.4 keV isomer in Ref. [20]. This revision does not affect the current figure.

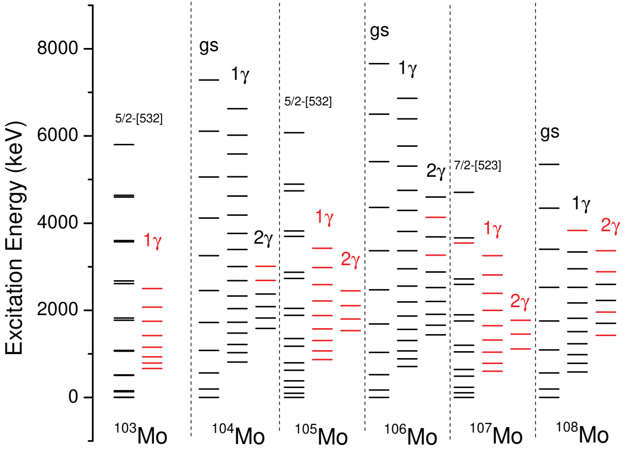

Figure 4. Systematic comparison of the 0-, 1- and 2- phonon γ-vibrational bands in $^{103-108}$Mo. Newly observed levels are highlighted in red. This figure is adapted and modified from Ref. [19].

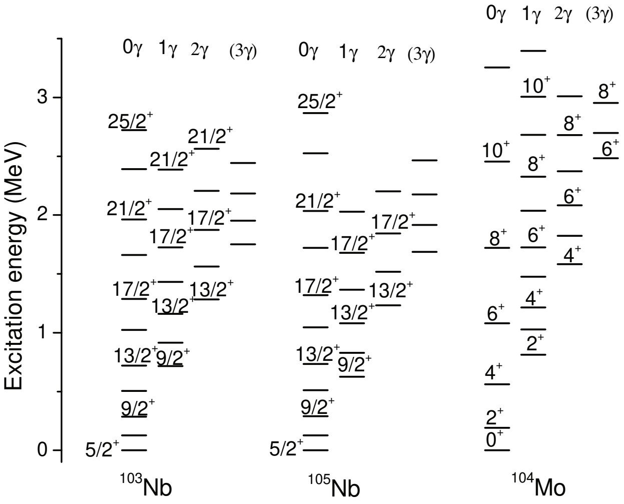

Figure 5. Systematic comparison of the 0-, 1- and 2- and possible 3-phonon γ-vibrational bands in ^{103,105}Nb and ^{104}Mo. Data are taken from Refs. [28, 29, 31].

Systematic studies of the 0-, 1-, and 2-phonon γ vibrational bands across $^{103-108}$Mo are presented in Fig. 4, highlighting newly observed levels in red. One- and two-phonon γ vibrational bands have previously been documented in the molybdenum isotopic chains. The identification of 1- and 2-phonon γ bands in ^{103,107}Mo helps bridge gaps in the odd-A molybdenum isotopic chains. Notably, among odd-A Mo nuclei, the $(E_{2\gamma} - E_{0\gamma})/(E_{1\gamma} - E_{0\gamma})$ ratios exhibit an increasing trend from ^{103}Mo (1.55) to ^{107}Mo (1.85), suggesting a harmonic relationship as neutron numbers increase in multi-γ phonon vibrational bands, all of which couple to the $\nu h_{11/2}$ orbital.

In ^{103,105}Nb we observed bands coupled exclusively to their respective $\gamma\gamma$ bands [28, 29]. Recent theoretical studies have suggested the presence of three-phonon γ bands in ^{103,105}Nb [30]. Furthermore, our collaboration has identified a similar band structure in ^{104}Mo [31]. As shown in Fig. 5, these nuclei exhibit systematic trends, highlighting the need for further microscopic unified theoretical investigations.

Currently, there are no experimental reports on multi-phonon γ-vibrational bands in odd-odd nuclei, nor has a specific microscopic unified model been discussed. However, preliminary results from our ^{252}Cf SF and ^{238}U induced fission studies suggest evidence of such bands in odd-odd Nb nuclei. Data analysis is ongoing to achieve a comprehensive identification.

3.2 Chiral vibrations in ^{104,106}Mo

In ^{104}Mo and ^{106}Mo, we also identified doublet bands coupled to the γ and $\gamma\gamma$ bands, exhibiting the same parity and nearly degenerate energies for corresponding states [31]. Additionally, the energy difference between the doublet bands in ^{104}Mo is approximately 60 keV, which is smaller than that observed in ^{106}Mo. Both nuclei display small, nearly constant signature splittings, which belong characteristics of chirality.

Theoretical Tilted Axis Cranking (TAC) and TPSM calculations suggest a distinct mechanism in these two nuclei, differing from known chiral examples. In this case, a high-j particle generates angular momentum along the short axis, while a high-j hole contributes along the long axis. The interplay of neutrons in the open shell leads to the emergence of soft chiral vibrations.

4 Conclusion

The investigation of the high-spin states of Mo isotopes has resulted in the establishment of new level schemes. The behavior of 0-, 1-, and 2-γ vibrational bands in $^{103-108}$Mo, as well as potential 3-γ bands in ^{103,105}Nb and ^{104}Mo has been systematically analyzed. Those observations, along with the identification of chiral bands in ^{104,106}Mo, provide strong evidence for triaxiality in this region.

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