EPJ Web of Conferences **324**, 00026 (2025)
NSD2024
https://doi.org/10.1051/epjconf/202532400026

A unified description of the shape phase transition, shape coexistence and mixing phenomena in nuclei

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Abstract. The shape phase transition, shape coexistence and mixing phenomena can be described in a unified way within the framework of the phenomenological Bohr-Mottelson model involving a sixth order anharmonic oscillator potential in the intrinsic deformation variable. This potential can have alternatively, depending on its parameters, a single spherical minimum, a single deformed minimum, a flat shape and simultaneously spherical and deformed minima, respectively. Thus, an entire phase transition from a spherical shape to a deformed one can be covered, crossing the critical point where the potential is flat (second order phase transition) or it has a small barrier (first order phase transition), while the shape coexistence and mixing phenomena emerge by simply increasing the barrier in the critical point. The type of axial deformations (prolate, triaxial, γ-unstable) are given here by the additional degree of freedom γ in relation to the γ potential shape.

1 Introduction

The nuclear shape phase transitions and their critical points [1–7], respectively, the nuclear shape coexistence and mixing phenomena [8–15] have been extensively studied in the recent decades. The aim of the present study is to show that these phenomena can be described in a unified way, and more specifically to present an interplay mechanism between the critical point of a shape phase transition and the coexistence and mixing of the two corresponding shapes, spherical and deformed ones. This is achieved in the framework of a phenomenological approach [16–19], based on the Bohr-Mottelson model [20], by involving a sextic potential for the β intrinsic deformation variable. A quasi-exactly solvable method [21] of this potential was widely applied to investigate the shape phase transitions [22–31], while a numerical diagonalization one [32] to study the shape coexistence and mixing phenomena [16–19, 33–35], respectively. All these results revealed this possibility of appearance of the shape coexistence and mixing in the vicinity to the critical point. They will be discussed and presented according to the following plan. The solutions of the Bohr-Mottelson Hamiltonian with sextic potential in the β variable are briefly introduced in Section 2. In Section 3, the phenomena of shape phase transition, coexistence and mixing are presented in a unified way by underlying their importance in considering them together for a better understanding of the structure of the lowest quadrupole collective states belonging to the ground, β and γ bands. Finally, the main achievements of these studies are indicated in Section 4.

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2 Bohr-Mottelson Hamiltonian with sextic potential

The Bohr-Mottelson Hamiltonian has the following expression [20]:

$$H = -\frac{\hbar^2}{2B}\left(\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma}\right) + \frac{\hbar^2}{8B\beta^2}\sum_{k=1}^{3}\frac{Q_k^2}{\sin^2\left(\gamma-\frac{2\pi}{3}k\right)} + V(\beta,\gamma), \quad (1)$$

where $\hbar$, B, β, γ and Q_k are: the reduced Plank constant, mass parameter, intrinsic deformation coordinates, angular momentum projections. The β variable can be separated by γ and the three Euler angles, associated with rotations, by considering the potential of the form [1]:

$$V(\beta,\gamma) = V(\beta) + \frac{1}{\beta^2}V(\gamma). \quad (2)$$

After the separation of variables, the eigenvalue problem is reduced to an equation only in β,

$$\left[-\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{\Lambda}{\beta^2} + v(\beta)\right]\psi(\beta) = \varepsilon\psi(\beta), \quad (3)$$

which contains also the contributions from the other four degrees of freedom by means of the Λ separation constant. The expression of Λ depends on the axial deformation considered (prolate, triaxial or γ-unstable one) and of course on the shape of the potential in the β variable [1, 5]. $\varepsilon = (2B/\hbar^2)E$ and $v(\beta) = (2B/\hbar^2)V(\beta)$ are the reduced total energy and potential in β, respectively. Further, there will be discussed two solvable methods of the sextic oscillator potential, namely the quasi-exactly solvable method and the numerical diagonalization approach.

2.1 Quasi-exactly solvable method

The expression of the potential in this case is chosen such that exact solutions of the Hamiltonian to be obtained [21]:

$$v(\beta) = \left[b^2 - 4a\left(s + \frac{1}{2} + m\right)\right]\beta^2 + 2ab\beta^4 + a^2\beta^6. \quad (4)$$

Here, a and b are parameters which define the shape of the potential, as in Fig. 1. s depends on some quantum numbers such as the seniority number (τ) [29, 31], total angular momentum (L) [22, 25, 26] or in some cases on its projection (K, R) too [27], while m gives the number of the solutions which can be exactly determined. The term $s + 1/2 + m \equiv c$, in the frame of the Bohr model, is constrained to remain constant preserving a state-independent potential. Nevertheless, one can still have two or more such constants depending on the shape phase transition analyzed. One can see, from Fig. 1, that by varying b

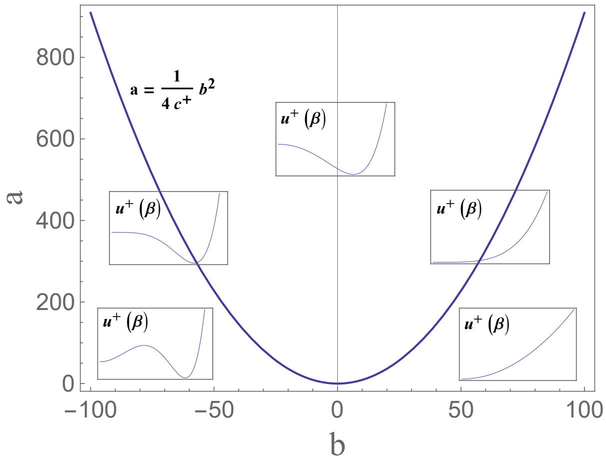

Figure 1. Possible shapes of the sextic potential in the (a, b)-plane. The parabola $a = b^2/4c^+$ separates different shape regions and indicates the critical point. c^+ is the constant for even seniority quantum number [31].

from 100 to 0, a phase transition is described from an approximately spherical shape to a deformed one crossing the critical point delimited by the parabola where the potential is flat. This situation corresponds very well with the second order shape phase transition from the spherical vibrator to γ-unstable rotor [36], but also can very well approximate the first order shape phase transition from spherical vibrator to prolate rotor [37]. The latter one requires a potential with a small maximum separating two degenerated minima. On the left side of the figure, for large negative values of b, a potential with two minima is obtained. Unfortunately, the minima are not degenerated and moreover, due to the quasi-exactly solvability method, the deformed minimum is always below the spherical one. Despite of this limitation, the quasi-exactly solvable method of the sextic potential, for the Bohr Hamiltonian, proved to be very appropriate to describe triaxial, prolate, spherical and γ-unstable deformed nuclei, respectively and phase transitions between these shapes and their critical points: $^{172,176-192}$Os, ^{228,230}Th, $^{180-196}$Pt, ^{170,182}W, ^{166,168,180}Hf, $^{116-132}$Xe, $^{98-108}$Ru, ^{100,102}Mo, ^{132,134}Ce, $^{146-150}$Nd, ^{150,152}Sm, ^{152,154}Gd, ^{154,156}Dy, ^{190}Hg, ^{222}Ra, ^{134}Ba, $^{102-110}$Pd, $^{106-116}$Cd [22–31, 38].

2.2 Numerical diagonalization

Within the numerical diagonalization method, there are no constraints imposed for the potential parameters:

$$v(\beta) = v_1\beta^2 + v_2\beta^4 + v_3\beta^6. \quad (5)$$

Instead, due to a scaling property of the polynomial potentials [27], v_1 can be extracted as a scaling parameter for energies through an appropriate change of variable. Moreover, the β equation can be reduced to a one-dimensional Schrödinger form using a change of function [17]. By these changes, one obtains an effective sextic potential depending on two free parameters denoted by A and B [17]:

$$v_{eff}(\beta) = \frac{W}{\beta^2} + \beta^2 + A\beta^4 + B\beta^6, \quad (6)$$

where W contains the rotational energy contribution. There is also possible to have a single free parameter if the two minima of the potential are constrained to be degenerated [16]. This condition with a small height for the maximum separating the two minima corresponds to the first order shape phase transition [37]. The corresponding Hamiltonian with this potential is numerically diagonalized in a basis of the functions [16, 19],

$$f_{n,\nu}(\tilde{\beta}) = \frac{\sqrt{2}}{\tilde{\beta}_w} \frac{\tilde{\beta}^{-\frac{3}{2}} J_{n,\nu}\left(\frac{z_{n,\nu}}{\tilde{\beta}_w}\tilde{\beta}\right)}{J_{\nu+1}(z_{n,\nu})}, \quad (7)$$

which in turn are written in terms of the Bessel functions of the first kind $J_{n,\nu}$, of index ν and Bessel zero $z_{n,\nu}$. The index ν depends on the considered collective conditions. For example, $\nu = \sqrt{[L(L+1) - K^2]/3 + 9/4}$ and $\nu = \tau + 3/2$ for prolate [16, 18] and γ-unstable system [19], respectively. Here, L and K are the total angular momentum and its projection on the z axis, while τ is the seniority quantum number. These functions are solutions obtained for an infinite square well potential, defined by the limit $\tilde{\beta}_w$, and correspond to the E(5) [36] and X(5) [37] models depending on the γ-stable/unstable conditions. The model was proposed for the first time to describe the critical point of the first order phase transition from the spherical vibrator to prolate rotor [17], and then further developed for the same phase transition [17, 18] and for the second order phase transition from spherical vibrator to γ-unstable system [19]. Other applications to the experimental data followed, especially to investigate new phenomena such as shape coexistence and mixing, respectively anomalous small $B(E2)$ transition rates within the yrast band [33–35]. An alternative phenomenological approach, known as the General Collective Model [39–41], treated the sixth order potential in β for a more general expression of the total potential (2) in respect with the γ variable. In this case, the basis for the numerical diagonalization is made up of the eigenstates of the five-dimensional harmonic oscillator. The main differences between the two methods were investigated and underlined in [19].

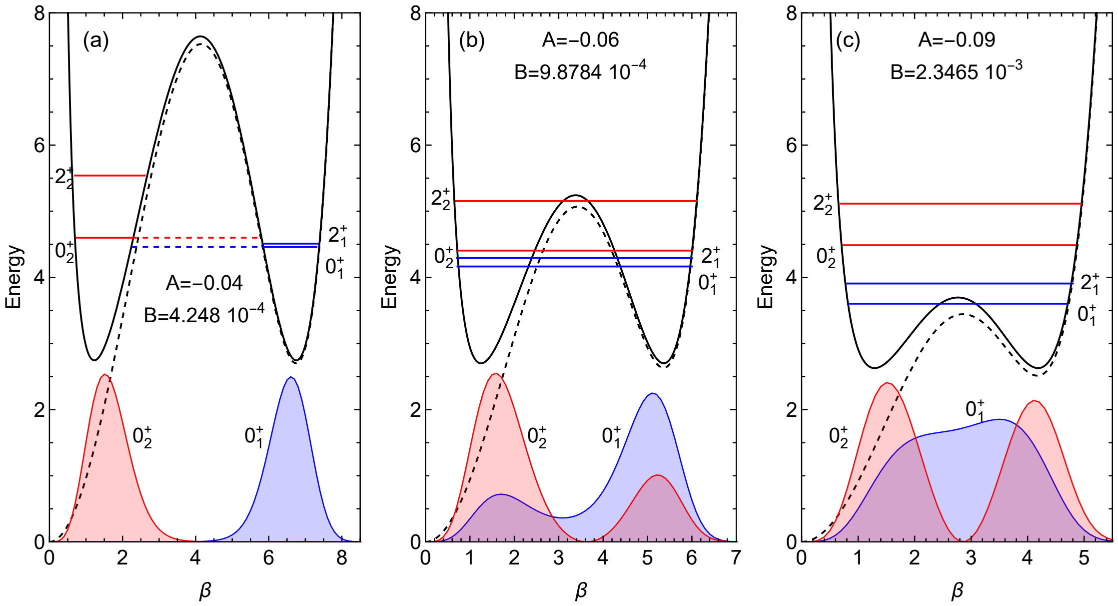

Figure 2. Regular (dashed curve) and effective ground state (full curve) potentials are plotted, as well as the probability density of deformation for the ground state (0_1^+) and excited 0_2^+ state for three distinct situations: shape coexistence without mixing (a), shape coexistence with mixing (b) and close to shape fluctuations (c). Here, γ-stable conditions are considered.

3 Shape phase transition, coexistence and mixing

The numerical diagonalization method of the sextic potential allows us to explore other features of the nuclear structure than shape phase transitions and critical point, namely shape coexistence and mixing phenomena. This can be achieved starting from the critical point of the shape phase transition and simply increasing step by step the barrier separating the two minima of the potential as in Fig. 2. A small increase of the barrier, as in panel (c), such that the ground state passes below the maximum, produces an incipient splitting in two peaks for the probability density of deformation, which is not the case for the critical point where only a single extended peak emerges [17]. The phenomenon appearing in the critical point, for a single extended peak, is that of shape fluctuation [17]. Instead, the two peaks of the excited 0_2^+ are related to the β-vibration and one node of the function ($n_\beta = 1$). Because for the ground state there is no β-vibration, one can interpret this situation as a presence of the shape coexistence with mixing. This aspect can be more clearly seen in panel (b), where the two plots look like in the mirror. Both states, 0_1^+ and 0_2^+, are now below the barrier as well as the two excited 2_1^+ states. This affects the vibration motion of the states of the β band and induces a shift of deformation of the ground and β-band states towards the second and first minimum, respectively. The process is completed for a very high barrier, shown in panel (a) where a single peak for the ground state and first excited 0^+ state, respectively for the states constructed above these ones is present. It should be noted that there is no β-vibration for the 0_2^+ state and for the states above it even if one has one node for these states, this band being characterized only by a rotational motion as the ground band. This case is associated within the present model to the shape coexistence without mixing.

In order to validate the model, several applications to the experimental data have been made so far, each of them coming to confirm all these calculations. In the following, these results will be briefly reviewed. In [16], the potential is constrained to the case of two degenerated minima corresponding to a first order shape phase transition and applied for ^{238}Pu, ^{152}Nd and ^{170}Hf nuclei selected as possible candidates. The experimental data are well reproduced, especially the large monopole transition strength between the first excited 0^+ state and the ground state for ^{238}Pu. It is well known that this observable can be considered as a good signature for the presence of the shape coexistence and mixing phenomena [42]. This result represented a first indication of the model ability to describe such phenomena and it was immediately confirmed by new applications, this time to nuclei suspected to manifest shape coexistence and mixing phenomena as ^{76}Kr [17, 43], $^{72-76}$Se [18, 43], $^{96-100}$Mo [19], ^{74}Kr, ^{74}Ge [33], ^{80}Ge [34] and ^{42,44}Ca [35]. High values for some $B(E2)$s of these nuclei, between states of different bands, which sometimes are even greater than the $B(E2)$ transitions in band, could be very well reproduced within the present model only if the shape coexistence with mixing was taken into account. The same thing is valid also for the presence of a very low energy for the first excited 0_2^+, another signature for shape coexistence. It is interesting that the model can specify if a nucleus manifest shape coexistence or not even if few experimental data are available. This was the case for ^{80}Ge [33] for which the model predicts the absence of the shape coexistence and an energy close to 2 MeV for the first excited 0^+ state. This is in agreement with a recent experiment [44], but also with microscopic models based on the Shell Model [45] and the Covariant Density Functional Theory [33, 34, 46]. Another surprising feature of the model proved to be its ability to explain anomalous reduced $E2$ transitions probabilities [47, 48] by assuming that the two involved states have different quadrupole deformation β_2, even if they belong to the same band, and are separated by a potential barrier. This behavior was observed for the transitions: $B(E2; 4_1^+ \to 2_1^+)$ in ^{72}Se [18], $B(E2; 6_1^+ \to 4_1^+)$ in ^{42,44}Ca [35] and $B(E2; 8_1^+ \to 6_1^+)$ in ^{80}Ge [33]. All these findings recommend the model for further applications to the experimental data indicating the presence of such phenomena.

4 Conclusions

The quasi-exactly and numerically solvable methods for the Bohr-Mottelson Hamiltonian with a sextic oscillator potential in the β deformation variable were proposed and applied to study in a unified way nuclear shape phase transitions and their critical points, shape coexistence and mixing phenomena. By varying the potential parameters, a phase transition takes place from an approximately spherical shape to a deformed one crossing the critical point. Increasing the barrier in the vicinity of the critical point, a transition from shape fluctuation towards shape coexistence with mixing and without mixing is found. Moreover, anomalous $E2$ transitions in the ground band are well explained by considering different quadrupole deformations for the states and a high barrier of potential separating them.

Acknowledgments

This work was supported by grant of the Ministry of Education and Research: PN-23-21-01-01/2023.

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